Article

Some aspects of animal behavior and community dynamics

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Abstract

We simulate the dynamical behavior of a few two – dimensional predator – prey systems in two – dimensional parameter spaces to gain insight into how functional responses affect community dynamics. The insight gained helps us design three dimensional systems. We construct models for a few ecosystems with three species and study them using computer simulations. The models have been developed by linking food chains which have both kinds of predators: *specialist as well as generalist.* The linking functions are *weakly non-linear*. The three dimensional model ecosystems have sexually reproducing top – predators. We perform extensive simulations to figure out dynamics of dynamical possibilities caused by changes in animal behavior. The animals change the foraging strategies and behave differently in different environments. At the end of the paper, we examine how diseases can govern transitions in meandering of dynamical models in bounded volume of their phase spaces.

Keywords prey interference; prey toxicity; extinction – persistence; diseases in plant and animals; health of ecosystems; human health.

1 Introduction

Natural populations do oscillate (Kendall et al., 1989). These oscillations are caused either by pure predator-prey or inter-specific competition (Damgaard, 2011; Zhang, 2011). Weak trophic interactions are mutualism and interference. In this article, we consider model systems which are designed by linking food chains by linear or weakly non-linear trophic interactions. There exist four basic type of weakly non-linear trophic interactions; Holling type II, III, IV, Beddington – DeAngelis (BD). The Crawely – Martin(CM) functional response function is derived from BD by incorporating an additional aspect of animal behavior. We do not study the effect of other interactions except competition. The only type of competition considered is *intra-specific.* Gause experiments have shown that competitive interactions can also generate oscillations. At this point, it should be noted that neutral stability of Lotka – Volterra models have been replaced by sustained periodic oscillations of predator – prey systems (Rosenzweig – MacAurthur, 1963). This is our subsystem A. Other kind of predator – prey systems which comprise generalist predators are known as Holling – Tanner class of systems (Pielou, 1977). This is subsystem B in the present article. The distinguishing feature of the latter is that the predator has alternative food items when its favorite food is in short supply. Various types of functional responses are used to couple these food chain modules to design food webs (Rai, 2009). The food chains are derived by coupling two subsystems A and B; the first with specialist predator and the second with generalist predator. The specialist predators are ones which die out exponentially in the situation when their most favorite food choice is not available. The generalist predators are ones who still survive such a situation by switching to alternative food options. We understand that these mathematical models are only caricatures of reality. However, these models do help us get an idea about which kind of interactions are key players to explain patterns present in an observed time – series; which is often short and noisy.

Rai and Upadhyay (2006) have shown that sustained periodic oscillations confine to a narrow region of parameter space in (a_2, K) of RM predator – prey systems. We investigate into how the region is affected when we try three functional responses which are proven to be better that the Volterra response function: Beddington – DeAngelis(BD), Crawley – Martin (CM) and Type IV.

Functional response functions are per capita predation rates. Five main functional responses are listed below. *Volterra response function* is the per capita predation rate which grows linearly with the prey density. Holling (1969) proposed a response function which saturates for high prey densities. Before reaching the plateau, it grows through linear and non – linear phases. This is known as *type II functional response*. *Beddington – DeAngelis (BD*) functional response can be considered as a modification to prey – dependent Holling's type II response function, which includes 'searching for prey' and 'handling prey'. BD function includes a third behavioral trait 'mutual interference' with competitors'. *Holling type IV* functional response is related to type II response. This is a functional response in which predator's per capita rate of predation decreases at sufficiently high prey density. This may be due to either prey interference (care for BD and CM functional responses), prey toxicity or group defense. It was used by Andrews (1968) as substrate uptake function. In the limit of large values of the immunity parameter, it reduces to type II functional response. We will discuss more about prey interference and prey toxicity in the 'Discussion' section.

2 Community Dynamics in Two – dimensional Model Ecosystems

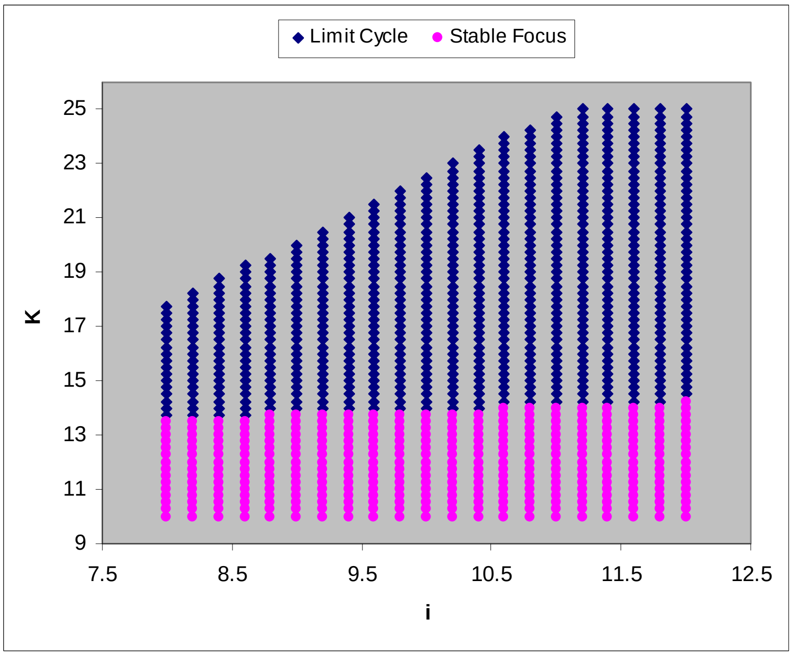
Crawley (1992) has argued for a new kind of functional response which is either due to group defense in prey species or prey toxicity. For large value of the immunity parameter i, it reduces to a type II functional response. Rosenzweig – MacAuthur predator – prey system is modified to following system.

$$\frac{dX}{dt} = rX\left(1-\frac{X}{K}\right) - \frac{wXY}{\frac{X^2}{i} + X + D}$$

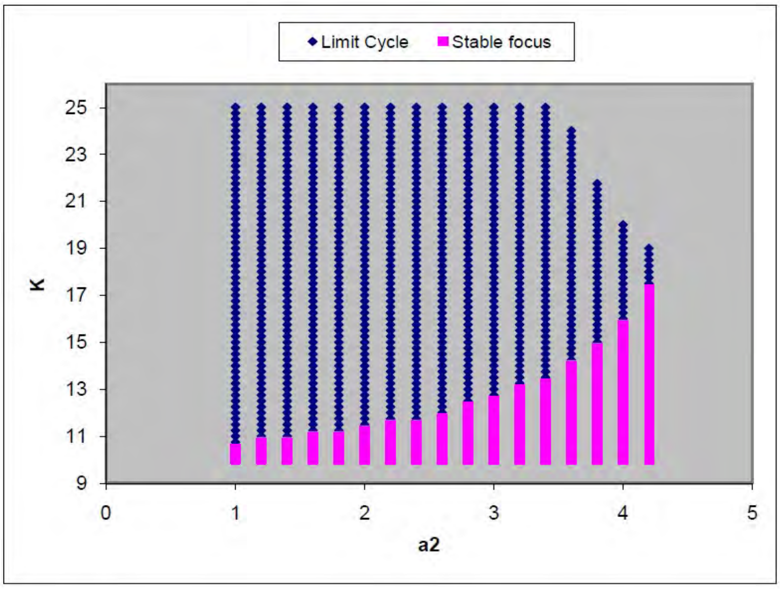
(Holling Type IV)

$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{\frac{X^2}{i} + X + D}$$

where *K* is the carrying capacity and *i* is the immunity parameter. *D* is the prey density at which the per capita predation rate attains half of its maximum value. This can also be interpreted as the protection provided by the environment to individuals of prey. *X* denotes prey density and *Y* that of the predator. We performed two – dimensional parameter scans to investigate frequencies of occurrence of oscillatory and equilibrial dynamics. The oscillatory dynamics is represented by stable limit cycles and equilibrial dynamics by stable focus in the phase space of the model system. Results of these scans are presented in Fig. 1.



(a)

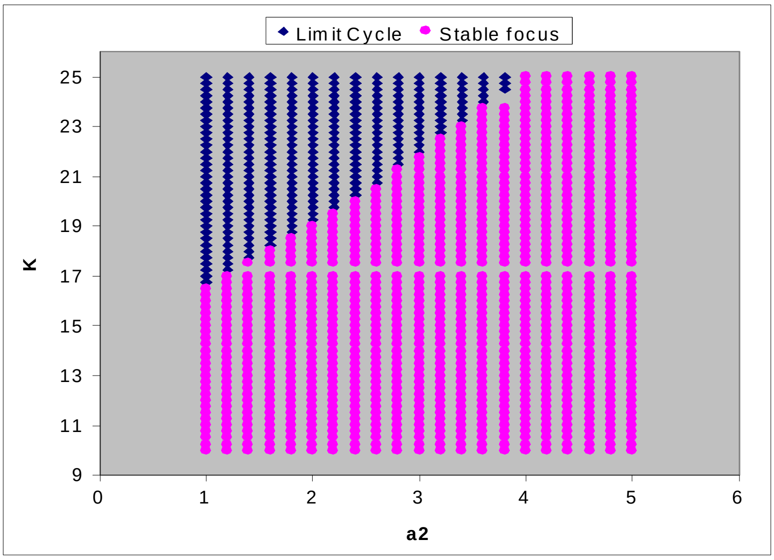


(b)

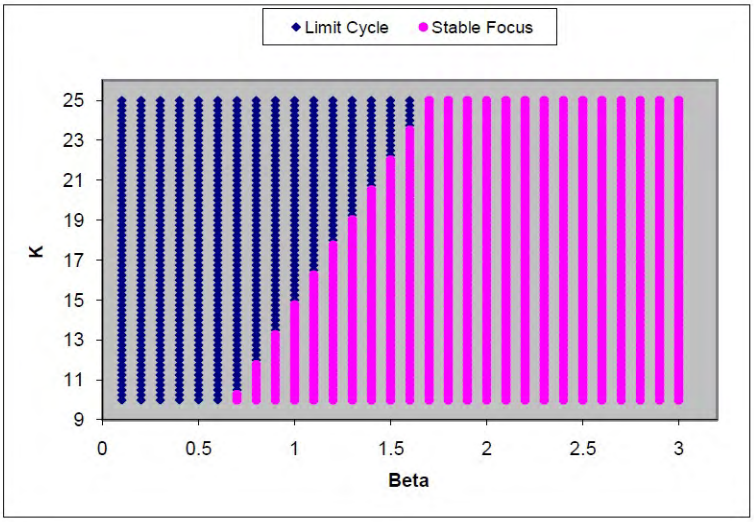
Fig. 1 Two – dimensional parameter scans showing the regions which are inhabited by limit cycle solutions and by stable equilibrium solutions for a model community.

If we compare Fig. 1b. with Fig. 1 (Rai and Upadhyay, 2006), it is found that the region housing periodic solutions is enhanced. Asymptotic equilibrium dynamics is confined to a narrow region of the parameter space.

2.1 Model system 2



(a)



(b)

Fig. 2 Two – dimensional parameter scans showing the regions which are inhabited by limit cycle solutions and by stable equilibrium solutions for second model community.

$$\frac{dX}{dt} = rX\left(1-\frac{X}{K}\right)-\frac{wXY}{\beta+\gamma Y+\delta X}$$

(Beddington – DeAngelis type)

$$\frac{dY}{dt} = -a_2 Y+\frac{w_1 XY}{\beta+\gamma Y+\delta X}$$

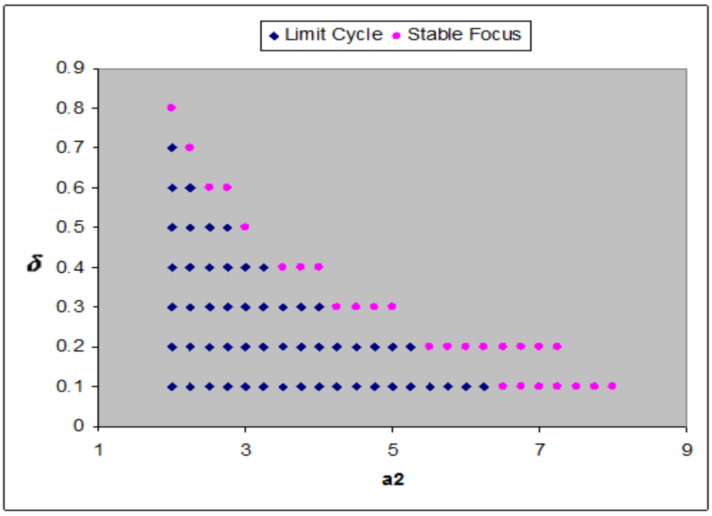
It is clear from Fig. 1(a) that the parameter structure remains the same; i. e., periodic solutions remain confined to a narrow region of the parameter space. For smaller values of the protection provided by the environment to the prey, one obtains oscillatory dynamics in this two species system.

2.2 Model system 3

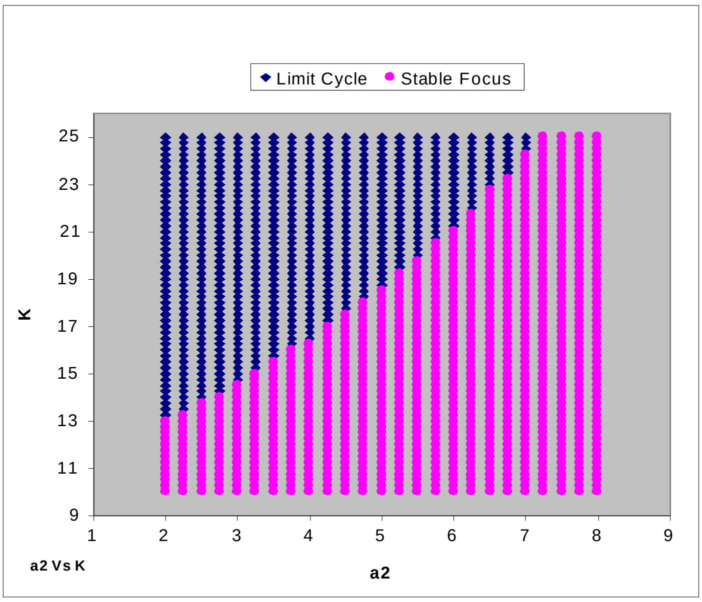
$$\frac{dX}{dt} = rX\left(1-\frac{X}{K}\right) - \frac{wXY}{1+\gamma Y+\delta X+\gamma\delta XY}$$

(Crowley - Martin type)

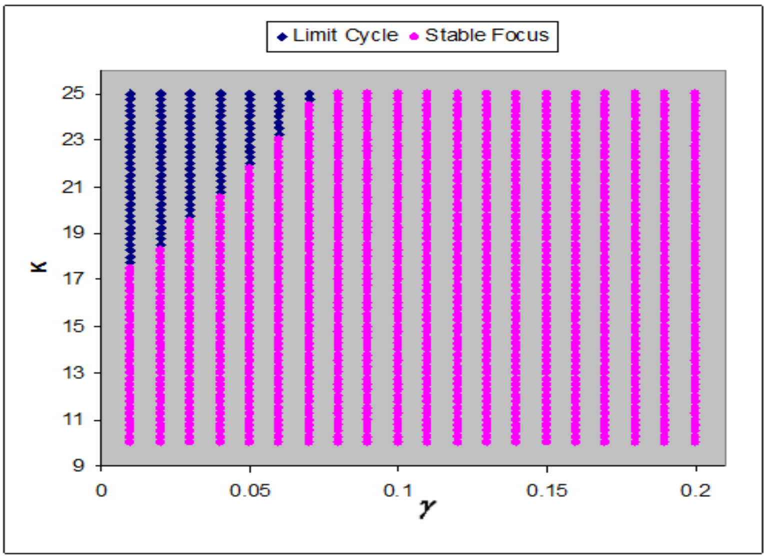
$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{1+\gamma Y+\delta X+\gamma\delta XY}$$



(a)



(b)



(c)

Fig. 3 Two – dimensional parameter scans showing regions of two types of dynamical behavior for third model community.

3 Community Dynamics in 3D Model Ecosystems

Two dimensional predator – prey systems were used as a template to build three – dimensional systems which support both oscillations and chaos.

We link two subsystems (cf. section 2) through weakly non-linear functions (Holling type II, BD, CM) to design three – dimensional (3D) model ecosystems which are non – linear food chains. Linear Food chains are the ones in which oscillations at one trophic level induce oscillations at the next level. The overall dynamics is governed determined by the coupling of these two oscillation: one original and other induced. In non – linear food chains, both oscillations are independent and represent intrinsic system dynamics. If coupled through type BD, CM and type IV response functions, dynamics in natural ecosystems are modeled. In what follows, we investigate the nature of dynamics in three model ecosystems by computer simulations. We have allowed transient to die out so that we capture the asymptotic dynamics instead of trajectory meandering. It is assumed that individuals of all the species are abundant.

These model ecosystems are similar in some sense to ones introduced by Rai (2004) and further studied by Rai and Upadhyay (2006). The present model systems are food chains constructed by linking the Rosenzweig – MacArthur (Rosenzweig and MacArthur, 1963) and Holling – Tanner (Pielou, 1977) 2- species systems using type II, BD and CM functional responses as linking mechanism.

3.1 Model system 1

$$\frac{dX}{dt} = a_1 X\left(1 - \frac{X}{K}\right) - \frac{wXY}{X + D} \quad (1.1)$$

$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{X + D_1} - \frac{w_2 YZ}{Y + D_2} \quad (1.2)$$

$$\frac{dZ}{dt} = cZ^2 - \frac{w_3 Z^2}{Y + D_3} \quad (1.3)$$

3.2 Model system 2

$$\frac{dX}{dt} = a_1 X\left(1 - \frac{X}{K}\right) - \frac{wXY}{X + D} \quad (2.1)$$

$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{X + D_1} - \frac{w_2 YZ}{Y + D_2 + bZ} \quad (2.2) \qquad \text{(B-D)}$$

$$\frac{dZ}{dt} = cZ^2 - \frac{w_3 Z^2}{Y + D_3} \quad (2.3)$$

3.3 Model system 3

$$\frac{dX}{dt} = a_1 X\left(1 - \frac{X}{K}\right) - \frac{wXY}{X + D} \quad (3.1)$$

$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{X + D_1} - \frac{w_2 YZ}{1 + dY + bZ + bdYZ} \quad (3.2) \qquad \text{(C- M)}$$

$$\frac{dZ}{dt} = cZ^2 - \frac{w_3 Z^2}{Y + D_3} \quad (3.3)$$

We mention that top predator Z is sexually reproducing. We assume that number of males and females are equal in any random sample drawn from populations of these species inhabiting any geographical area. c is the rate of per capita growth of the generalist predator.

In the next section, we present analyses which help us make choices for biologically realist parameter values. For these 3- dimensional system, we focus on existence persistence events. Persistence means coexistence all the species. Extinction means the situation in which any of the three species become extinct.

4 Analysis

We employ a method for the dynamical study of three-species ecosystems discovered by Upadhyay and Rai (1997). Species are related through trophic interactions. The top prey (X) and the middle predator (Y) give a biologically meaningful subsystem (subsystem A). In order to be a biologically meaningful system, a subsystem should qualify as a Kolmogorov system (Upadhyayet al., 2008). The last term in Eq. (1.2) is omitted to get this subsystem. The subsystem is

$$\frac{dX}{dt} = a_1 X\left(1 - \frac{X}{K}\right) - \frac{wXY}{X+D} \qquad (4.1)$$

(Subsystem A)

$$\frac{dY}{dt} = -a_2 Y + \frac{w_1 XY}{X+D_1} \qquad (4.2)$$

4.1 Kolmogorov analysis

For the above subsystem $F(X,Y)$ and $F(X,Y)$ are given by

$$F(X,Y) = a_1\left(1 - \frac{X}{K}\right) - \frac{wY}{X+D}$$

$$G(X,Y) = -a_2 + \frac{w_1 X}{X+D_1}$$

Applying the conditions of the Kolmogorov theorem, we obtain the following:

(i) $$\frac{\partial F}{\partial Y} < 0 \Rightarrow -w/(X+D) < 0$$

The condition is satisfied as w and D are positive constants.

(ii) $$X\frac{\partial F}{\partial X} + Y\frac{\partial F}{\partial Y} < 0 \Rightarrow a_1(X+D)^2 + KwDY > 0$$

This condition holds in the domain $X, Y > 0$ as a_1, D, K and w are positive constants.

(iii) $\frac{\partial G}{\partial Y} = 0$ is automatically satisfied.

(iv) $X\frac{\partial G}{\partial X} + Y\frac{\partial G}{\partial Y} > 0 \Rightarrow w_1 D_1 X > 0 \Rightarrow D_1 > 0$.

(v) $F(0,0) > 0$ gives $a_1 > 0$. This condition is automatically satisfied as $a_1 > 0$.

(vi) $F(0, A^*) = 0$ gives $A^* = a_1 D / w$. Since a_1, D, w are positive constants, this implies $A^* > 0$, which is true.

(vii) $F(B^*, 0) = 0$ gives $B^* = K$. Since K is a positive constant this implies $B^* > 0$, which is true.

(viii) $G(C^*, 0) = 0$ gives $C^* = a_2 D_1 / (w_1 - a_2)$. Since $C^* > 0$, we get a constraint $w_1 > a_2$.

(ix) The condition $B^* > C^*$ gives the constraint

$$K(w_1 - a_2) > a_2 D_1.$$

Thus, Kolmogorov theorem is satisfied when

$$w_1 > a_2, \quad D_1 > 0, \qquad (4.3)$$

$$K(w_1 - a_2) > a_2 D_1. \qquad (4.4)$$

4.2 Linear stability analysis

For *X*- isocline

$$\frac{dX}{dt}=0\Rightarrow a_1\left(1-\frac{X}{K}\right)-\frac{wY}{X+D}=0. \tag{5.1}$$

For *Y* - isocline

$$\frac{dY}{dt}=0\Rightarrow -a_2+\frac{w_1X}{X+D_1}=0. \tag{5.2}$$

There are three equilibrium points $E_{00}=(0,0)$, $E_{10}=(0,K)$ and $E_{20}=(X^*,Y^*)$. The intersection of the two isoclines is the equilibrium point (X^*,Y^*), where $X^*=(a_2D_1)/(w_1-a_2)$ and $Y^*=\frac{a_1}{w}\left(1-\frac{X^*}{K}\right)(X^*+D)$ exist under the Kolmogorov condition (4).

For the analysis of the equilibrium points of subsystem (3) we generate the variational matrix at the point (*X*, *Y*)

$$V(X,Y)=\begin{bmatrix} X\frac{\partial F}{\partial X}+F & X\frac{\partial F}{\partial Y} \\ Y\frac{\partial G}{\partial X} & Y\frac{\partial F}{\partial Y}+G \end{bmatrix}$$

where

$$F(X,Y)=a_1\left(1-\frac{X}{K}\right)-\frac{wY}{X+D}, G(X,Y)=-a_2+\frac{w_1X}{X+D_1},$$

$$\frac{\partial F}{\partial X}=-\frac{a_1}{K}+\frac{wY}{(X+D)^2},\frac{\partial F}{\partial Y}=-\frac{w}{(X+D)},$$
$$\frac{\partial G}{\partial X}=\frac{w_1D_1}{(X+D_1)^2},\frac{\partial F}{\partial Y}=0.$$

The variational matrix at $E_{00}=(0,0)$ is

$$V(0,0)=\begin{bmatrix} a_1 & 0 \\ 0 & -a_2 \end{bmatrix}.$$

This implies that the equilibrium point $E_{00}=(0,0)$ is a saddle point.

The variational matrix at $E_{10}=(0,K)$ is

$$V(K,0)=\begin{bmatrix} a_1 & \dfrac{-wK}{(K+D)} \\ 0 & -a_2+\dfrac{w_1K}{(K+D_1)} \end{bmatrix}.$$

This implies that the equilibrium point $E_{10}=(0,K)$ is saddle point if $w_1K > a_2(K+D_1)$ and is locally asymptotically stable if $w_1K < a_2(K+D_1)$.

The variational matrix at $E_{20}=(X^*,Y^*)$ is

$$V(X^*,Y^*)=\begin{bmatrix} X^*\left[-\dfrac{a_1}{K}+\dfrac{wY^*}{(X^*+D)^2}\right] & \dfrac{-wX^*}{(X^*+D)} \\ \dfrac{w_1D_1Y^*}{(X^*+D_1)^2} & 0 \end{bmatrix}.$$

The characteristic equation of above matrix is

$$\lambda^2+P\lambda+Q=0,$$

where $P=-(a_{11}+a_{22}), Q=a_{11}a_{22}-a_{12}a_{21}$.

Thus the equilibrium point $E_{20}=(X^*,Y^*)$ is locally asymptotically stable if the following condition hold if $(w+w_1)\left(\dfrac{a_2D_1}{w-a_2}\right)+Dw-w_1K>0$.

5 Simulations of Three Species Model Food Chains

We present results of simulation experiments in tables and graphs. The parameter sets are selected in accordance with inequalities in the last section.

Table 1 Holling Type II functional response (Extinction table)

Group	Range in which parameter varied		Outcome
(a_1,c)	a_1	c	
	0.01-4.0	0.04	Y becomes extinct
(K,c)	K	c	
	10	0.04	Y becomes extinct
	35	0.04	Y becomes extinct
(c,w_3)	c	w_3	
	0.001	0.02	Y becomes extinct
	0.01	0.2	Y becomes extinct

Simulation experiments of model system Eq. (1) with fixed parameter values $w=1.0, D=10, a_2=0.7, w_1=2.0, D_1=10, w_2=0.405, D_2=10, D_3=20$. The parameter values which are common in all the experiments are $a_1=1.93, K=36, c=0.027, w_3=0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60\le X,Y\le 60; -40\le Z\le 40$.

Table 2 Holling Type II functional response (Coexistence table)

Group	Range in which parameter varied		Species Coexistence Outcome
(a_1, K)	a_1	K	
	0.01-0.75	10-20	Coexists on Stable focus
		21-70	Coexists on Stable focus and Limit Cycle
	1.0	10-22	Coexists on Stable focus
		23	Coexists on Limit Cycle
		24-70	Coexists on Stable focus and Limit Cycle
	1.25	10-20	Coexists on Stable focus
		21-31	Coexists on Limit Cycle
		32-70	Coexists on Stable focus and Limit Cycle
	1.50	10-18	Coexists on Stable focus
		19-39	Coexists on Limit Cycle
		40-70	Coexists on Stable focus and Limit Cycle
	1.75	10-15	Coexists on Stable focus
		16-19	Coexists on Limit Cycle
		23-24	Coexists on Limit Cycle
		28-48	Coexists on Limit Cycle
		49-70	Coexists on Stable focus and Limit Cycle
	2.0	10-14	Coexists on Stable focus
		15-19	Coexists on Limit Cycle
		22	Coexists on Limit Cycle
		26-28	Coexists on Limit Cycle
		31-56	Coexists on Limit Cycle
		57-70	Coexists on Stable focus and Limit Cycle
	2.25	10-12	Coexists on Stable focus
		13-18	Coexists on Limit Cycle
		21	Coexists on Limit Cycle
		24	Coexists on Limit Cycle
		28	Coexists on Limit Cycle
		34-66	Coexists on Limit Cycle
		67-70	Coexists on Stable focus and Limit Cycle
	2.50	10-11	Coexists on Stable focus
		12-20	Coexists on Limit Cycle
		28	Coexists on Limit Cycle
		33	Coexists on Limit Cycle
		36-70	Coexists on Limit Cycle
	2.75	10-11	Coexists on Stable focus
		12-20	Coexists on Limit Cycle
		27	Coexists on Limit Cycle
		36	Coexists on Limit Cycle
		39-70	Coexists on Limit Cycle
	3.0	10	Coexists on Stable focus
		11-21	Coexists on Limit Cycle
		26	Coexists on Limit Cycle
		38	Coexists on Limit Cycle
		43-70	Coexists on Limit Cycle
	3.25	10	Coexists on Stable focus
		11-20	Coexists on Limit Cycle
		24	Coexists on Limit Cycle
		35-37	Coexists on Limit Cycle
		41	Coexists on Limit Cycle
		46-70	Coexists on Limit Cycle
	3.50	10-23	Coexists on Limit Cycle
		36-37	Coexists on Limit Cycle
		44	Coexists on Limit Cycle

		49-70	Coexists on Limit Cycle
		10-22	Coexists on Limit Cycle
	3.75	36	Coexists on Limit Cycle
		53-70	Coexists on Limit Cycle
		10-22	Coexists on Limit Cycle
	4.0	35	Coexists on Limit Cycle
		50	Coexists on Limit Cycle
		56-70	Coexists on Limit Cycle
(a_1, c)	a_1	c	
	0.01-0.75	0.001-0.03	Coexists on Stable focus and Limit Cycle
	1.0-2.0	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.3	Coexists on Limit Cycle
	2.25-2.50	0.001-0.02	Coexists on Stable focus and Limit Cycle
	2.75-3.50	0.001-0.01	Coexists on Stable focus and Limit Cycle
		0.02	Coexists on Limit Cycle
	3.75-4.0	0.001-0.01	Coexists on Stable focus and Limit Cycle
		0.02-0.03	Coexists on Limit Cycle
(a_1, w_3)	a_1	w_3	
	0.01-0.25	0.6-2.0	Coexists on Stable focus and Limit Cycle
	0.50-0.75	0.6	Coexists on Limit Cycle
		0.7-2.0	Coexists on Stable focus and Limit Cycle
	1.0	0.6-0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.25	0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.50-1.75	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	2.0-2.25	0.8-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	2.50-2.75	0.8-1.0	Coexists on Limit Cycle
		1.1-2.0	Coexists on Stable focus and Limit Cycle
	3.0	1.0-1.1	Coexists on Limit Cycle
		1.2-2.0	Coexists on Stable focus and Limit Cycle
	3.25	1.0-1.2	Coexists on Limit Cycle
		1.3-2.0	Coexists on Stable focus and Limit Cycle
	3.50	0.8	Coexists on Limit Cycle
		1.2	Coexists on Limit Cycle
		1.3-2.0	Coexists on Stable focus and Limit Cycle
	3.75	0.8	Coexists on Limit Cycle
		1.0-1.3	Coexists on Limit Cycle
		1.4-2.0	Coexists on Stable focus and Limit Cycle
	4.0	1.1-1.3	Coexists on Limit Cycle
		1.4-2.0	Coexists on Stable focus and Limit Cycle
(K, c)	K	c	
	15	0.03	Coexists on Limit Cycle
		0.04	Coexists on Stable focus
	20-25	0.04	Coexists on Stable focus
	30	0.03	Coexists on Limit Cycle
		0.04	Coexists on Stable focus
	35	0.03	Coexists on Limit Cycle
	40-70	0.03	Coexists on Limit Cycle
		0.04	Coexists on Stable focus
(K, w_3)	K	w_3	
	10	0.6	Coexists on Limit Cycle
		0.7-2.0	Coexists on Stable focus
	15	0.6-0.8	Coexists on Limit Cycle
			Coexists on Stable focus

		0.9-2.0	Coexists on Limit Cycle
	20	0.6	Coexists on Stable focus
		0.9-2.0	Coexists on Limit Cycle
	25	0.9-1.0	Coexists on Stable focus and Limit Cycle
		1.1-2.0	Coexists on Limit Cycle
	30	0.7-0.9	Coexists on Stable focus and Limit Cycle
		1.0-2.0	Coexists on Limit Cycle
	35	0.6	Coexists on Limit Cycle
		0.8-0.9	Coexists on Stable focus and Limit Cycle
		1.0-2.0	Coexists on Limit Cycle
	40-50	0.7-0.8	Coexists on Stable focus and Limit Cycle
		0.9-2.0	Coexists on Limit Cycle
	55-70	0.7	Coexists on Stable focus and Limit Cycle
		0.8-2.0	
(c, w_3)	c	w_3	
	0.001	0.03	Coexists on Limit Cycle
		0.04-2.0	Coexists on Stable focus and Limit Cycle
	0.006	0.2	Coexists on Limit Cycle
		0.3-2.0	Coexists on Stable focus and Limit Cycle
	0.01	0.2	Coexists on Stable focus and Limit Cycle
		0.3	Coexists on Limit Cycle
		0.4-2.0	Coexists on Stable focus and Limit Cycle
	0.06	1.2	Coexists on Stable focus
		1.7-2.0	Coexists on Limit Cycle
	0.1	2.0	Coexists on Stable focus

Simulation experiments of model system Eq. (1) with fixed parameter values $w = 1.0, D = 10, a_2 = 0.7, w_1 = 2.0, D_1 = 10, w_2 = 0.405, D_2 = 10, D_3 = 20$. The parameter values which are common in all the experiments are $a_1 = 1.93, K = 36, c = 0.027, w_3 = 0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60 \le X, Y \le 60; -40 \le Z \le 40$.

Table 3 Beddington-Deangelis type functional response (Extinction table)

Group	Range in which parameter varied		Outcome
(a_1, c)	a_1	c	
	0.01	0.04	Y becomes extinct
	0.50-0.75	0.04	Y becomes extinct
	1.25	0.04	Y becomes extinct
	2.0-2.25	0.04	Y becomes extinct
(K, c)	K	c	
	15	0.04	Y becomes extinct
	55-60	0.04	Y becomes extinct
(b, c)	b	c	
	0.001-0.1	0.04	Y becomes extinct
(c, w_3)	c	w_3	
	0.001	0.02	Y becomes extinct
	0.01	0.2	Y becomes extinct

Simulation experiments of model system Eq. (2) with fixed parameter values $w = 1.0, D = 10, a_2 = 0.7, w_1 = 2.0, D_1 = 10, w_2 = 0.405, D_2 = 10, D_3 = 20$. The parameter values which are common in all the experiments are $a_1 = 1.93, K = 36, b = 0.2, c = 0.027, w_3 = 0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60 \le X, Y \le 60; -80 \le Z \le 80$.

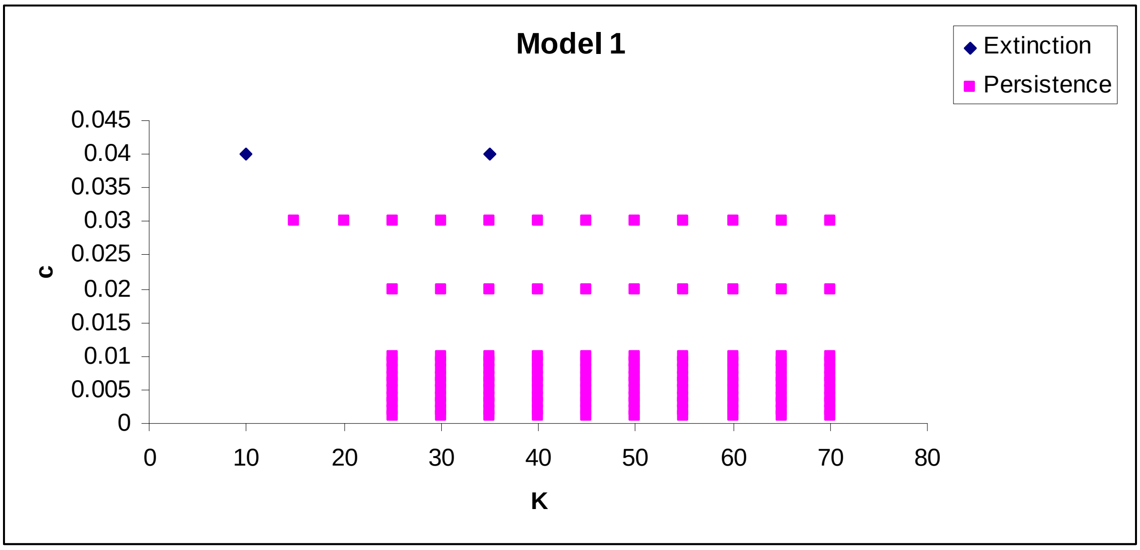
Extinction – Persistence Graphs: For the extinction – persistence graphs corresponding to model systems (1), (2) and (3), the base value of the parameters are

$$\left.\begin{aligned}&a_1 = 1.93, K = 36, w = 1.0, D = 10, a_2 = 0.7, w_1 = 2.0, D_1 = 10,\\&w_2 = 0.405, D_2 = 10, c = 0.027, w_3 = 0.8, D_3 = 20.\\&\text{Initial condition} = [0.5, 0.5, 0.5].\end{aligned}\right\} \tag{5.3}$$

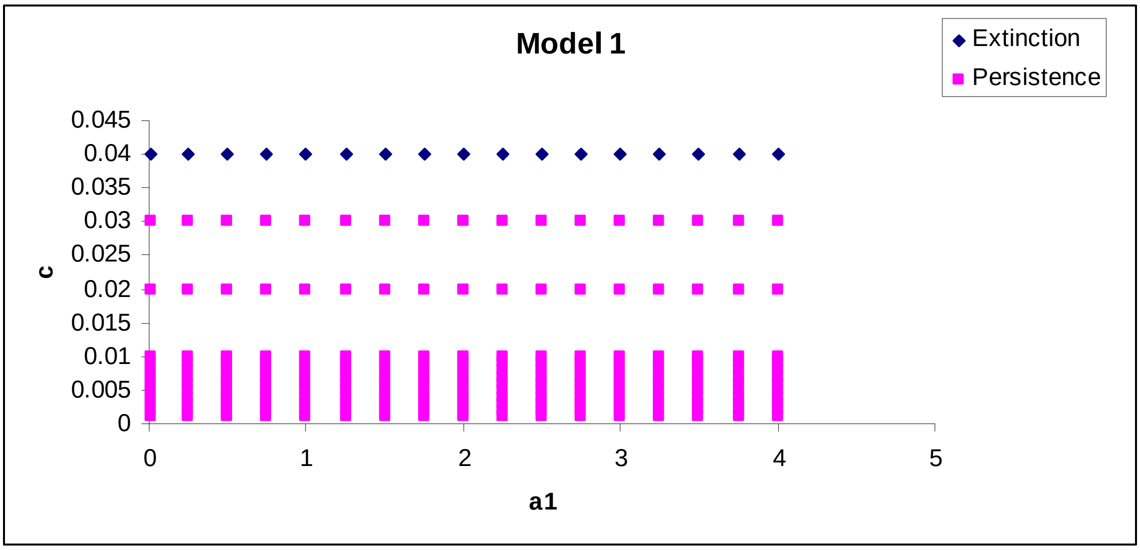
(a) For the extinction – persistence graphs of model system (1) with base (5.3), domain is $-60 \le X, Y \le 60; -40 \le Z \le 40$.

(b) For the extinction – persistence graphs of model system (2) with base (5.3), domain is $-60 \le X, Y \le 60; -80 \le Z \le 80$ and $b = 0.2$.

(c) For the extinction – persistence graphs of model system (3) with base (5.3), domain is $-60 \le X, Y \le 60; -10 \le Z \le 10$ and $b = 0.2$,

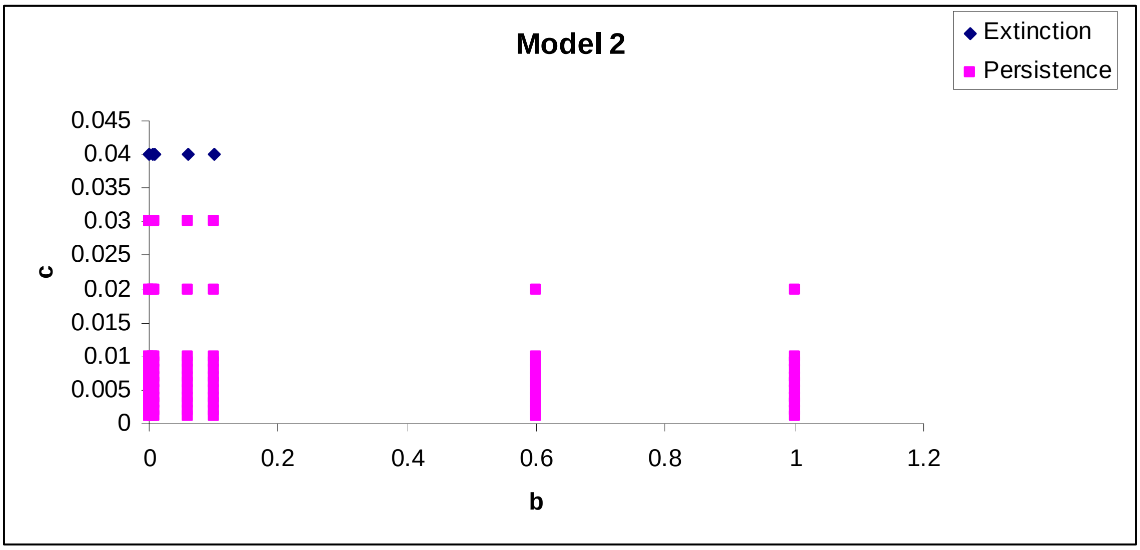


(a)

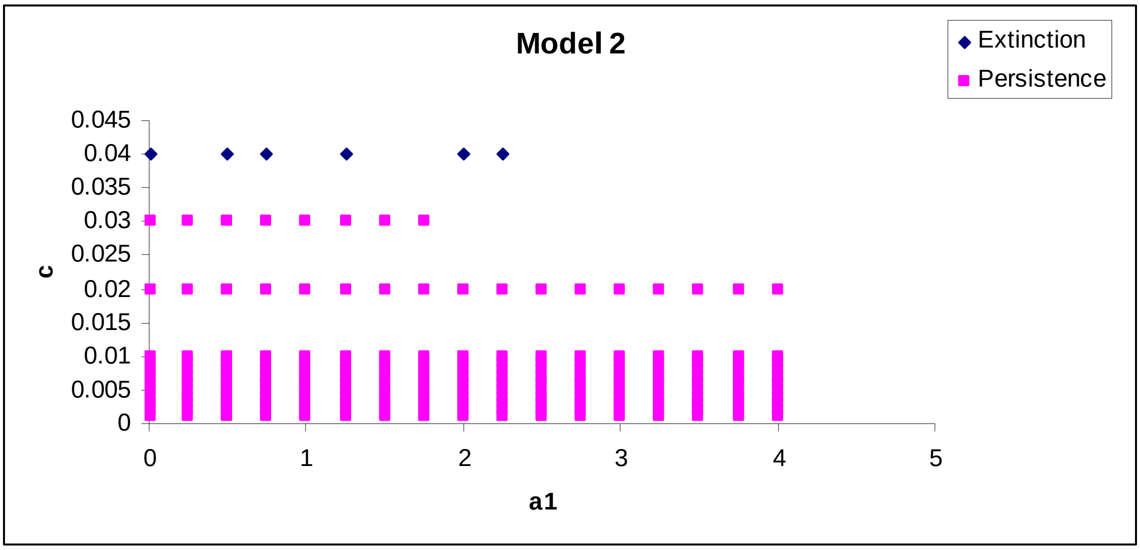


(b)

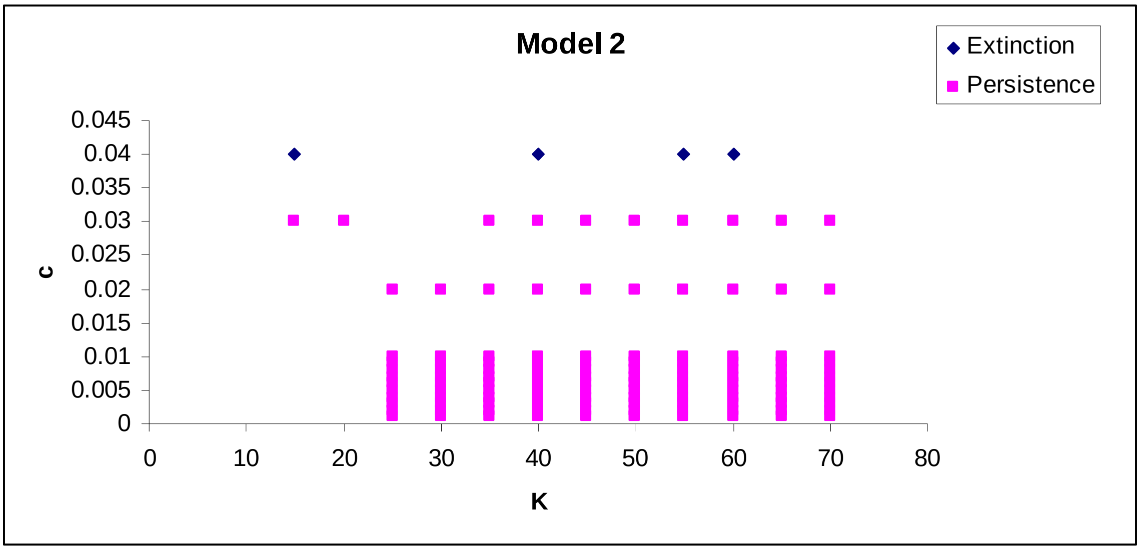
Fig. 4 Summary of results of simulation experiments on model ecosystem 1.



(a)



(b)



(c)

Fig. 5 Summary of results of simulation experiments on model ecosystem 2. We show extinction – Persistence events in different parameter spaces.

Table 4 Beddington - DeAngelis type functional response (Coexistence table)

Group	Range in which parameter varied		Species Coexistence Outcome
(a_1, K)	a_1	K	
	0.01-0.75	10-20	Coexists on Stable focus
		21-70	Coexists on Stable focus and Limit Cycle
	1.0	10-23	Coexists on Stable focus
		24	Coexists on Limit Cycle
		25-70	Coexists on Stable focus and Limit Cycle
	1.25	10-22	Coexists on Stable focus
		23-31	Coexists on Limit Cycle
		32-70	Coexists on Stable focus and Limit Cycle
	1.50	10-19	Coexists on Stable focus
		20-39	Coexists on Limit Cycle
		40-70	Coexists on Stable focus and Limit Cycle
	1.75	10-17	Coexists on Stable focus
		18-47	Coexists on Limit Cycle
		48-70	Coexists on Stable focus and Limit Cycle
	2.0	10-15	Coexists on Stable focus
		16-57	Coexists on Limit Cycle
		58-70	Coexists on Stable focus and Limit Cycle
	2.25	10-13	Coexists on Stable focus
		14-23	Coexists on Limit Cycle
		28	Coexists on Limit Cycle
		33-66	Coexists on Limit Cycle
		67-70	Coexists on Stable focus and Limit Cycle
	2.50	10-12	Coexists on Stable focus
		13-22	Coexists on Limit Cycle
		25-26	Coexists on Limit Cycle
		35-70	Coexists on Limit Cycle
	2.75	10-11	Coexists on Stable focus
		12-23	Coexists on Limit Cycle
		38	Coexists on Limit Cycle
		40-70	Coexists on Limit Cycle
	3.0	10-11	Coexists on Stable focus
		12-24	Coexists on Limit Cycle
		41	Coexists on Limit Cycle
		43-70	Coexists on Limit Cycle
	3.25	10	Coexists on Stable focus
		11-22	Coexists on Limit Cycle
		43	Coexists on Limit Cycle
		46-70	Coexists on Limit Cycle
	3.50	10-22	Coexists on Limit Cycle
		47-48	Coexists on Limit Cycle
		51-70	Coexists on Limit Cycle
	3.75	10-21	Coexists on Limit Cycle
		51-52	Coexists on Limit Cycle
		55-70	Coexists on Limit Cycle
	4.0	10-20	Coexists on Limit Cycle
		22-23	Coexists on Limit Cycle
		55-56	Coexists on Limit Cycle
		59-70	Coexists on Limit Cycle
(a_1, b)	a_1	b	
	0.01-1.25	0.001-1.0	Coexists on Stable focus and Limit Cycle
	1.5	0.001-0.4	Coexists on Stable focus and Limit Cycle
	1.75	0.001-0.2	Coexists on Limit Cycle
		0.001-0.2	Coexists on Limit Cycle

	2.0-2.25	0.001-0.006	Coexists on Limit Cycle
	2.5	0.02	Coexists on Limit Cycle
		0.05-0.2	Coexists on Limit Cycle
		0.09-0.1	Coexists on Limit Cycle
	2.75	0.03	Coexists on Limit Cycle
	3.0-3.25		
(a_1, c)	a_1	c	
	0.01-1.0	0.001-0.03	Coexists on Stable focus and Limit Cycle
	1.25-1.75	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
	2.0-4.0	0.001-0.02	Coexists on Stable focus and Limit Cycle
(a_1, w_3)	a_1	w_3	
	0.01-0.25	0.6-2.0	Coexists on Stable focus and Limit Cycle
	0.50-0.75	0.6	Coexists on Limit Cycle
		0.7-2.0	Coexists on Stable focus and Limit Cycle
	1.0	0.6-0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.25	0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.50-1.75	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	2.0-2.25	0.8-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	2.50	0.8-1.0	Coexists on Limit Cycle
		1.1-2.0	Coexists on Stable focus and Limit Cycle
	2.75-3.0	0.9-1.1	Coexists on Limit Cycle
		1.2-2.0	Coexists on Stable focus and Limit Cycle
	3.25-3.50	1.0-1.2	Coexists on Limit Cycle
		1.3-2.0	Coexists on Stable focus and Limit Cycle
	3.75-4.0	1.0-1.3	Coexists on Limit Cycle
		1.4-2.0	Coexists on Stable focus and Limit Cycle
(K, b)	K	b	
	10	0.001-1.0	Coexists on Stable focus
	15	0.001-0.06	Coexists on Limit Cycle
		0.07-1.0	Coexists on Stable focus
	20	0.07-0.4	Coexists on Limit Cycle
		0.5-0.6	Coexists on Stable focus
	25	0.001-0.002	Coexists on Limit Cycle
		0.04-0.4	Coexists on Limit Cycle
		0.5	Coexists on Stable focus
	30	0.001-0.07	Coexists on Limit Cycle
		0.1-0.2	Coexists on Limit Cycle
	35	0.001-0.2	Coexists on Limit Cycle
	40	0.001-0.3	Coexists on Limit Cycle
	45	0.001-0.5	Coexists on Limit Cycle
	50	0.001-0.9	Coexists on Limit Cycle
	55-70	0.001-1.0	Coexists on Stable focus and Limit Cycle
(K, c)	K	c	
	10	0.001-0.04	Coexists on Stable focus
	15-25	0.001-0.02	Coexists on Stable focus
		0.03	Coexists on Limit Cycle
	30	0.001-0.02	Coexists on Stable focus
	35-70	0.001-0.02	Coexists on Stable focus
		0.03	Coexists on Limit Cycle
(K, w_3)	K	w_3	
	10	0.6	Coexists on Limit Cycle

		0.7-2.0	Coexists on Stable focus
	15	0.6-0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus
	20	0.6-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus
	25	0.8-1.0	Coexists on Limit Cycle
		1.1-2.0	Coexists on Stable focus and Limit Cycle
	30-35	0.8-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	40-50	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	55-70	0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
(b,c)	b	c	
	0.001-0.1	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
	0.6-0.1	0.001-0.02	Coexists on Stable focus and Limit Cycle
(b,w_3)	b	w_3	
	0.001	0.8-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	0.006-0.06	0.7-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	0.1	0.7-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	0.6	0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	1.0	1.0-2.0	Coexists on Stable focus and Limit Cycle
(c,w_3)	c	w_3	
	0.001	0.04-2.0	Coexists on Stable focus and Limit Cycle
	0.006	0.2-2.0	Coexists on Stable focus and Limit Cycle
	0.01	0.3	Coexists on Limit Cycle
		0.4-2.0	Coexists on Stable focus and Limit Cycle
	0.06	1.7-2.0	Coexists on Stable focus and Limit Cycle

Simulation experiments of model system Eq. (2) with fixed parameter values $w=1.0, D=10, a_2=0.7, w_1=2.0, D_1=10, w_2=0.405, D_2=10, D_3=20$. The parameter values which are common in all the experiments are $a_1=1.93, K=36, b=0.2, c=0.027, w_3=0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60 \le X, Y \le 60; -80 \le Z \le 80$.

Table 5 Crowley–Martin functional response (Extinction table)

Group	Range in which parameter varied		Outcome
(a_1,c)	a_1	c	
	0.01-3.25	0.04	Y becomes extinct
(K,c)	K	c	
	10-70	0.04	Y becomes extinct
(b,c)	b	c	
	0.001-0.1	0.04	Y becomes extinct
(c,w_3)	c	w_3	
	0.001	0.02	Y becomes extinct
	0.01	0.2	Y becomes extinct
		0.4	Y becomes extinct

Simulation experiments of model system Eq. (3) with fixed parameter values $w = 1.0, D = 10, a_2 = 0.7, w_1 = 2.0, D_1 = 10, w_2 = 0.405, d = 0.13, D_3 = 20$. The parameter values which are common in all the experiments are $a_1 = 1.93, K = 36, b = 0.2, c = 0.027, w_3 = 0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60 \le X, Y \le 60; -10 \le Z \le 10$.

Table 6 Crowley–Martin functional response (Coexistence table)

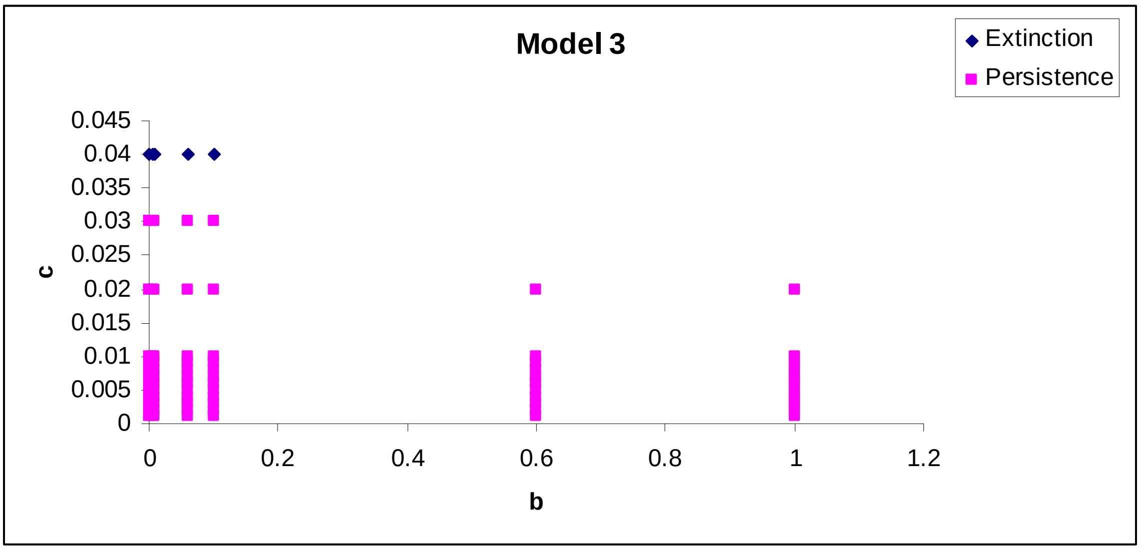
Group	Range in which parameter varied		Species Coexistence Outcome
(a_1, K)	a_1	K	
	0.01-0.75	10-20	Coexists on Stable focus
		21-70	Coexists on Stable focus and Limit Cycle
	1.0	10-23	Coexists on Stable focus
		24-70	Coexists on Stable focus and Limit Cycle
	1.25	10-21	Coexists on Stable focus
		22	Coexists on Limit Cycle
		27-28	Coexists on Limit Cycle
		29-70	Coexists on Stable focus and Limit Cycle
	1.50	10-18	Coexists on Stable focus
		19-22	Coexists on Limit Cycle
		28-33	Coexists on Limit Cycle
		34-70	Coexists on Stable focus and Limit Cycle
	1.75	10-15	Coexists on Stable focus
		17-22	Coexists on Limit Cycle
		30-38	Coexists on Limit Cycle
		39-70	Coexists on Stable focus and Limit Cycle
	2.0	10-13	Coexists on Stable focus
		14-23	Coexists on Limit Cycle
		31-41	Coexists on Limit Cycle
		42-70	Coexists on Stable focus and Limit Cycle
	2.25	10-12	Coexists on Stable focus
		13-22	Coexists on Limit Cycle
		33-47	Coexists on Limit Cycle
		48	Coexists on Stable focus and Limit Cycle
		49	Coexists on Limit Cycle
		50	Coexists on Stable focus and Limit Cycle
		51	Coexists on Limit Cycle
		52-70	Coexists on Stable focus and Limit Cycle
	2.50	10-11	Coexists on Stable focus
		12-20	Coexists on Limit Cycle
		35-55	Coexists on Limit Cycle
		56	Coexists on Stable focus and Limit Cycle
		57	Coexists on Limit Cycle
		58-59	Coexists on Stable focus and Limit Cycle
		60	Coexists on Limit Cycle
		61-70	Coexists on Stable focus and Limit Cycle
	2.75	10	Coexists on Stable focus
		11-19	Coexists on Limit Cycle
		36-56	Coexists on Limit Cycle
		57	Coexists on Stable focus and Limit Cycle
		58-60	Coexists on Limit Cycle
		61-70	Coexists on Stable focus and Limit Cycle
	3.0	10	Coexists on Stable focus
		11-18	Coexists on Limit Cycle
		39-63	Coexists on Limit Cycle
		64	Coexists on Stable focus and Limit Cycle

		65-66	Coexists on Limit Cycle
		67	Coexists on Limit Cycle
		68-69	Coexists on Stable focus and Limit Cycle
		70	Coexists on Stable focus and Limit Cycle
		10-17	Coexists on Limit Cycle
	3.25	39-68	Coexists on Limit Cycle
		69	Coexists on Stable focus and Limit Cycle
		70	Coexists on Limit Cycle
		10-16	Coexists on Limit Cycle
	3.50	43-70	Coexists on Limit Cycle
		10-16	Coexists on Limit Cycle
	3.75	44-70	Coexists on Limit Cycle
		10-15	Coexists on Limit Cycle
	4.0	46-70	Coexists on Limit Cycle
(a_1, b)	a_1	b	
	0.01-1.5	0.001-1.0	Coexists on Stable focus and Limit Cycle
	1.75	0.001-0.006	Coexists on Stable focus and Limit Cycle
		0.007	Coexists on Limit Cycle
		0.008-0.009	Coexists on Stable focus and Limit Cycle
		0.01-0.06	Coexists on Limit Cycle
		0.07-0.09	Coexists on Stable focus and Limit Cycle
		0.1-0.5	Coexists on Limit Cycle
	2.0	0.001-0.4	Coexists on Limit Cycle
	2.25-2.50	0.001-0.3	Coexists on Limit Cycle
	2.75	0.001-0.2	Coexists on Limit Cycle
(a_1, c)	a_1	c	
	0.01-1.50	0.001-0.03	Coexists on Stable focus and Limit Cycle
	1.75-2.25	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
	2.50	0.001-0.02	Coexists on Stable focus and Limit Cycle
	2.75-3.0	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
	3.25-3.75	0.001-0.01	Coexists on Stable focus and Limit Cycle
		0.02	Coexists on Limit Cycle
	4.0	0.001-0.01	Coexists on Stable focus and Limit Cycle
(a_1, w_3)	a_1	w_3	
	0.01-0.25	0.6-2.0	Coexists on Stable focus and Limit Cycle
	0.50-1.0	0.6	Coexists on Limit Cycle
		0.7-2.0	Coexists on Stable focus and Limit Cycle
	1.25	0.6-0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.50	0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
	1.75	0.6-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	2.0	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	2.25	0.7-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	2.50	0.8-1.0	Coexists on Limit Cycle
		1.1-2.0	Coexists on Stable focus and Limit Cycle
	3.0	0.9-1.1	Coexists on Limit Cycle
		1.2-2.0	Coexists on Stable focus and Limit Cycle
	3.25	1.0-1.1	Coexists on Limit Cycle
		1.2-2.0	Coexists on Stable focus and Limit Cycle
	3.50-3.75	1.0-1.2	Coexists on Limit Cycle
			Coexists on Stable focus and Limit Cycle

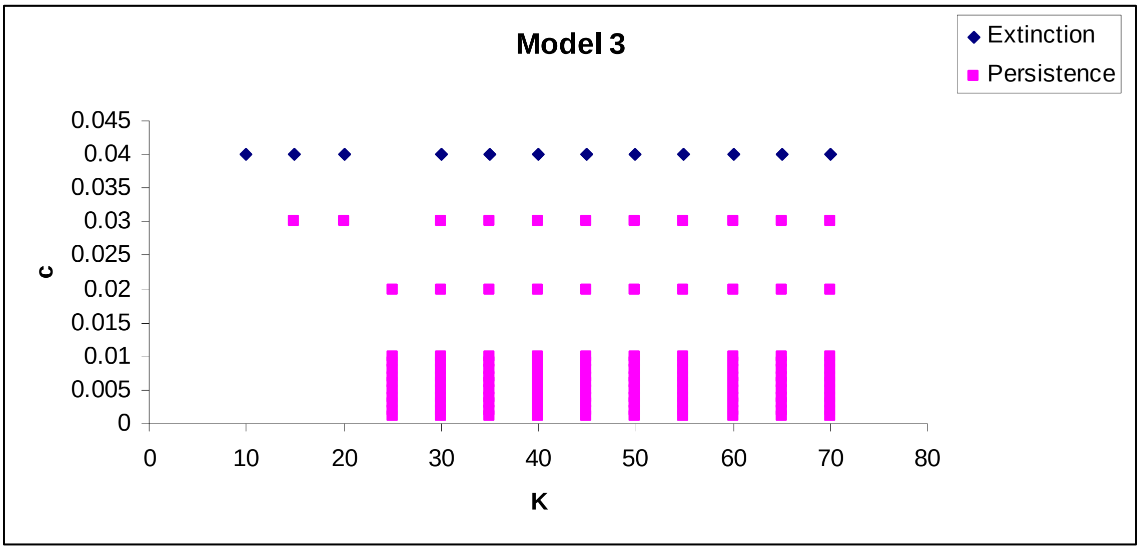
		1.3-2.0	Coexists on Limit Cycle
	4.0	1.1-1.3	Coexists on Stable focus and Limit Cycle
		1.4-2.0	
(K,b)	K	b	
	10	0.001-1.0	Coexists on Stable focus
	15	0.001-0.4	Coexists on Limit Cycle
	20	0.001-0.2	Coexists on Limit Cycle
	35	0.001-0.4	Coexists on Limit Cycle
	40	0.001-0.04	Coexists on Stable focus and Limit Cycle
		0.05-0.6	Coexists on Limit Cycle
	45	0.001-0.2	Coexists on Stable focus and Limit Cycle
		0.3	Coexists on Limit Cycle
		0.4	Coexists on Stable focus and Limit Cycle
		0.5-1.0	Coexists on Limit Cycle
	50	0.001-0.9	Coexists on Stable focus and Limit Cycle
		1.0	Coexists on Limit Cycle
	55-70	0.001-1.0	Coexists on Limit Cycle
(K,c)	K	c	
	10	0.001-0.03	Coexists on Stable focus
	15-20	0.001-0.02	Coexists on Limit Cycle
		0.03	Coexists on Stable focus and Limit Cycle
	25	0.001-0.02	Coexists on Stable focus and Limit Cycle
	30	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.04	Coexists on Stable focus and Limit Cycle
	35-70	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
		0.04	Coexists on Stable focus and Limit Cycle
(K,w_3)	K	w_3	
	10	0.6	Coexists on Limit Cycle
		0.7-2.0	Coexists on Stable focus
	15	0.6-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus
	20	0.6-0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus
	25	1.0	Coexists on Limit Cycle
		1.1-2.0	Coexists on Stable focus and Limit Cycle
	30	0.9	Coexists on Limit Cycle
		1.0-2.0	Coexists on Stable focus and Limit Cycle
	35	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	40-50	0.6-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	55-70	0.6-0.7	Coexists on Limit Cycle
		0.8-2.0	Coexists on Stable focus and Limit Cycle
(b,c)	b	c	
	0.001-0.006	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
	0.01-0.1	0.001-0.02	Coexists on Stable focus and Limit Cycle
		0.03	Coexists on Limit Cycle
		0.04	Coexists on Stable focus and Limit Cycle
	0.6-1.0	0.001-0.02	Coexists on Stable focus and Limit Cycle
(b,w_3)	b	w_3	
	0.001	0.7-0.8	Coexists on Limit Cycle
		0.9-2.0	Coexists on Stable focus and Limit Cycle
	0.006-0.1		Coexists on Limit Cycle

		0.7-0.8	Coexists on Stable focus and Limit Cycle
	0.6-1.0	0.9-2.0	Coexists on Stable focus and Limit Cycle
		0.9-2.0	
(c, w_3)	c	w_3	
	0.001	0.03-2.0	Coexists on Stable focus and Limit Cycle
	0.006	0.2-2.0	Coexists on Stable focus and Limit Cycle
	0.01	0.3	Coexists on Limit Cycle
		0.5-2.0	Coexists on Stable focus and Limit Cycle
	0.06	1.5-1.9	Coexists on Limit Cycle
		2.0	Coexists on Stable focus and Limit Cycle
	0.1	2.0	Coexists on Stable focus and Limit Cycle

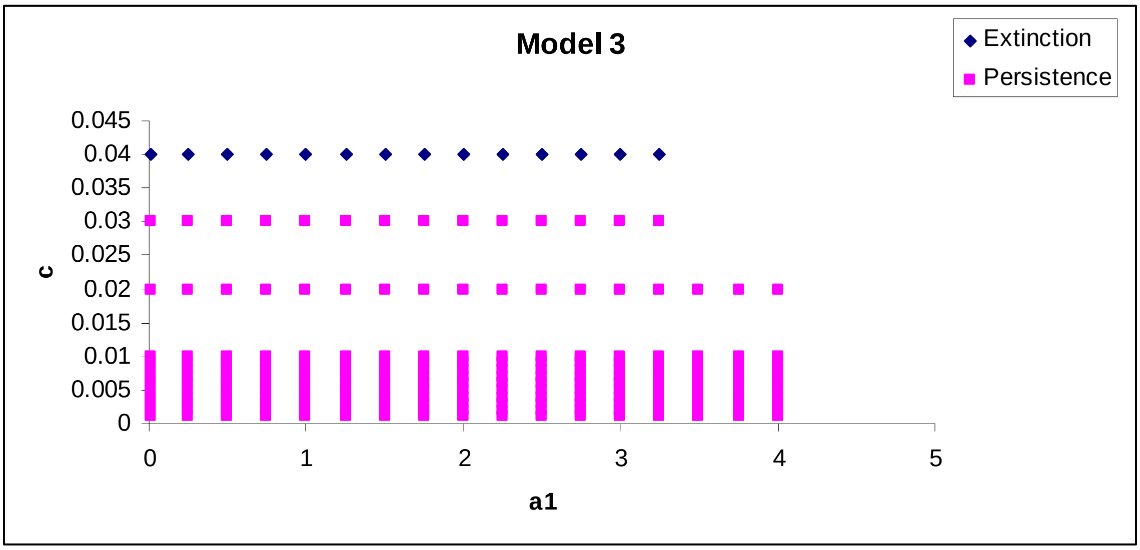
Simulation experiments of model system Eq. (3) with fixed parameter values $w = 1.0, D = 10, a_2 = 0.7, w_1 = 2.0, D_1 = 10, w_2 = 0.405, d = 0.13, D_3 = 20$. The parameter values which are common in all the experiments are $a_1 = 1.93, K = 36, b = 0.2, c = 0.027, w_3 = 0.8$. And initial condition is [0.5, 0.5, 0.5] and domain is $-60 \le X, Y \le 60; -10 \le Z \le 10$.



(a)

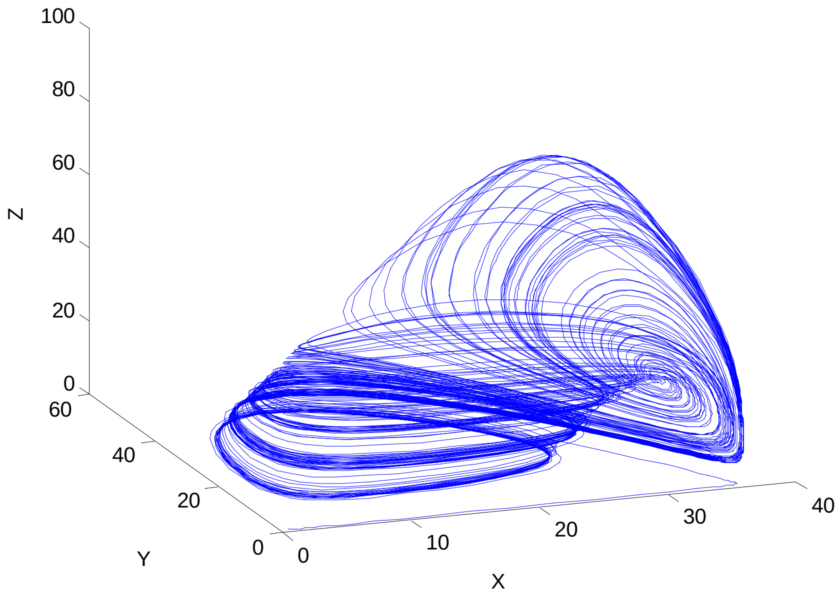


(b)

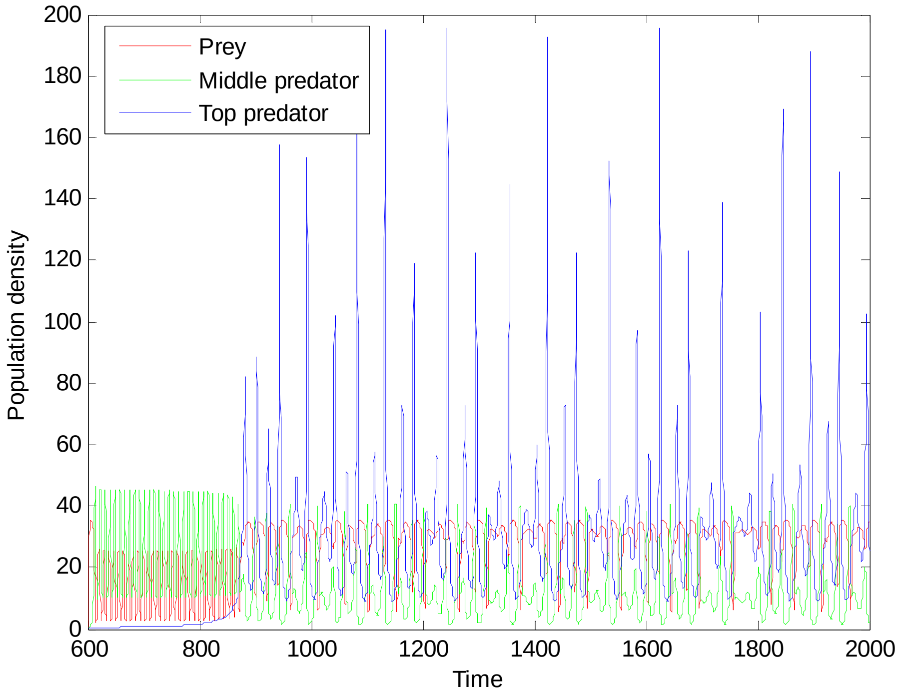


(c)

Fig. 6 Extinction – persistence events for model ecosystem 3.



(a)

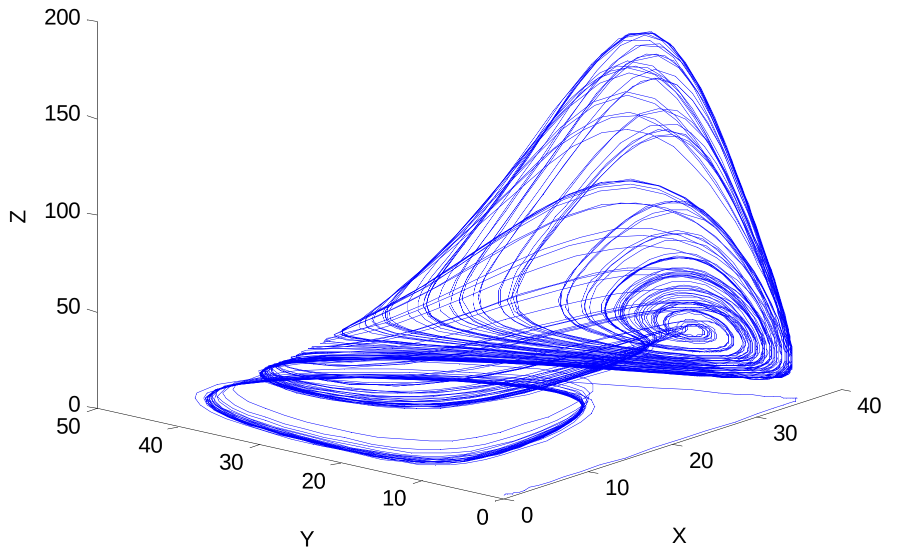


(b)

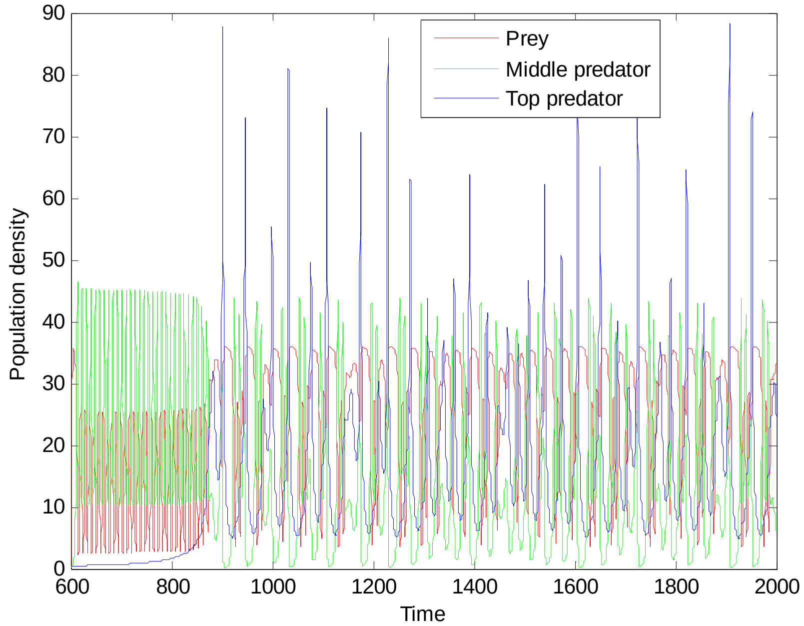
Fig. 7 a) Chaotic attractor b) time –series for model ecosystem 1 designed by using Holling Type II functional response as the link between two subsystems.

$$a_1 = 1.93, K = 36, w = 1.0, D = 10, a_2 = 1.0, w_1 = 2.0, D_1 = 10,$$
$$w_2 = 0.405, D_2 = 10, c = 0.027, w_3 = 0.8, D_3 = 20.$$

Initial condition = [0.5, 0.5, 0.5].



(a)

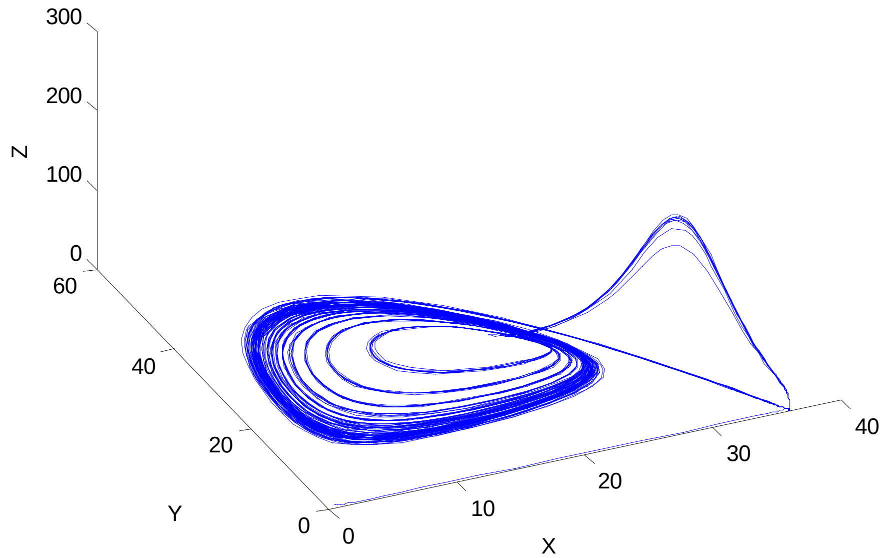


(b)

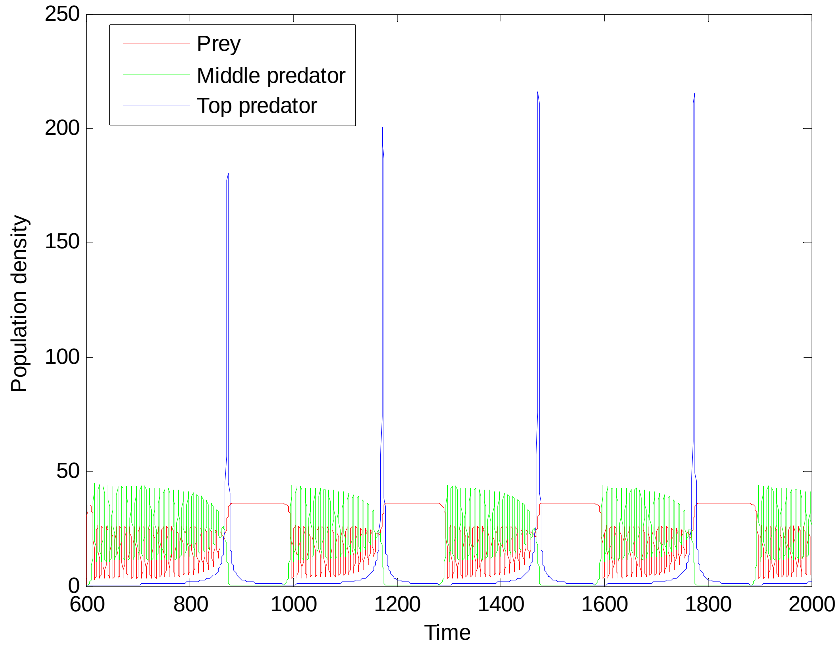
Fig. 8 a) Chaotic attractor b) time – series in the model ecosystem designed by using Beddington – DeAngelis functional response as the link between two subsystems

$$a_1 = 1.93, K = 36, w = 1.0, D = 10, a_2 = 1.0, w_1 = 2.0, D_1 = 10,$$
$$w_2 = 0.405, D_2 = 10, b = 0.2, c = 0.027, w_3 = 0.8, D_3 = 20.$$

Initial condition = [0.5, 0.5, 0.5].



(a)



(b)

Fig. 9 a) Chaotic attractor b) Time – series for the model ecosystem obtained by linking two subsystems through Craweley – Martin functional response. Note the excursions of generalist's population to extinction – sized densities. Parameter values used to generate these figures are as follows

$$a_1 = 1.93, K = 36, w = 1.0, D = 10, a_2 = 1.0, w_1 = 2.0, D_1 = 10,$$
$$w_2 = 0.405, D_2 = 10, b = 0.2, d = 0.13, c = 0.027, w_3 = 0.8, D_3 = 20.$$

Initial condition = [0.5, 0.5, 0.5].

6 Discussion

From Figures (3 – 6), it is clear that persistence events dominate species extinctions. We present typical strange chaotic attractors and corresponding time – series (Figs. 7 – 9) for all three species. We have not computed basin boundary structures which would have provided insight into how these ecosystems would behave when acted upon by environmental perturbations. Existence of chaos might mean extinction for species whose population densities crashes to near zero (note the population densities of the generalist predators in Figures 7 – 9). Therefore, one should be careful while drawing inferences from these studies on these well – mixed mechanistic models.

As we noted earlier, mathematical models are caricatures of reality. In this article we have not analyzed any time – series which represents measurements on a real world ecosystem with prefixed sample rate and precision (we always commit mistakes in estimating the population densities of plants and animals due to various). At this juncture, the relevant question to ask is *What use these mathematical models are for*. The answer is both benign and humble. These simulations serve as the guide for experiments and field observations. The mathematical modeling and computer simulations have paved the way for better time – series on model ecosystems (ecotron) and gathering of data derived from field observations (John Van der Meer, Agroforestry).

We review a few behavioral traits in animals and its relations to nonlinear phenomena (bifurcations and chaos). According to a recent study, Rhesus Macaques, *Macca mulatta* generate highly complex and unpredictable vocalizations without requiring equivalently complex neural control mechanisms. These vocalizations are related to the sexual behavior of female rhesus monkeys.

Van Gemerden (1974) has carried out experiments on uptake of hydrogen sulphide by a bacterium purple in color. He could fit in type IV functional response to the data. Animals which exhibit group defense also endorse this kind of functional response. For example, musk ox are successful in fending wolves when in herds than when alone (Freedman and Wolkowicz, 1986).

Inducible defenses are responses activated through a previous encounter with a consumer or competitor that offers some degree of resistance so subsequent attacks. Although the structural defenses produced by invertebrates to their competitors and predators are not the same, as immune response triggered by parasites; e. g., bacteria, viruses and fungi, two share three common properties 1) Specificity, 2) amplification, and 3) memory. The ecological consequences and evolutionary causes can be found in Bourdeau (2010).

The adaptive response in marine snail (*Nucella lamellosa*) has been investigated to examine if induced thickening of shells leads to an increased structural strength. Results indicate that the response is a by – product of reduced feeding and somatic growth rather than an active physiological response to predation risk.

Experiments on tropical water flea *Daphnia lumholtzi* have suggested that this species becomes dominant in comparison to the native species *Daphnia pulicaria* when challenged by fish predators (Engel, 2009). In the presence of predatory fishes, this invasive species formed an inducible defense against predation risk and becomes dominant.

The other reality which we have not considered is the disease condition in individuals of either plant or animal species. It may be brought to the notice of all concerned that Trichinosis, a disease caused by eating undercooked meat containing cysts of *Trichinella spiralis*, is found in pork, fox, rat, horse, and lion meat. Wild animals especially carnivores (meat eaters) or omnivores (animals that eat both both meat and plants) are considered a possible source of this roundworm disease. The sexually reproducing species in our model ecosystems are animals with low reproduction rates. For suitable choices of parameters, it can represent *homo sapiens* as well. The roundworm tends to invade muscle tissues, including the heart and diaphragm (the breathing muscle under the lungs). They can also affect the lungs and brain.

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