Article

System dynamic approach for management of urban parks: a case study

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Abstract

Public agencies tackling the issues of natural resource management need objective analysis to anticipate the ecosystem consequences of their actions. Mathematical models can be regarded as an effective tool to support decision makers in this context because of their predictive capacity and of their ability to describe complex natural systems. In particular, the aim of this paper is to show how mathematical modelling and simulation can be applied in the field of urban park management. As an example, we consider a case study where a domestic herbivore, the sheep, was introduced as a natural regulator of grass growth. The results show that the introduction of sheep is not currently sustainable and that it should be complemented by other measures in order to tackle both the issue of regulating grass growth and to prevent debarking of the newly planted trees.

Keywords debarking; dynamical systems; ecological models; grazing; green area management; public lawn maintenance.

1 Introduction

Although model formulation often requires a drastic simplification of nature, mathematical models have been successful in describing very complex systems such as biological populations and their dynamics and interactions (Ferrarini, 2011a, b; Ivanchikov and Nedorezov, 2011; Nedorezov, 2011a, b; Rai et al., 2011; Sharma and Raborn, 2011; Yasamis, 2011; Zhang, 2011). Moreover, in recent years their application to tackle management issues has dramatically increased. Forecasting based on ecosystem models is nowadays frequently required for public policy and natural resource management (Clark et al., 2001). The success in decision making in these fields largely depends upon the ability to anticipate the consequences of political actions. Thus, decision makers are referring more and more to scientific models to objectively justify their action and to profit from their predictive capacity (Pielke, 2003). For example, mathematical models and simulation have been developed to tackle issues related to wildlife management (Zhang, 2000), soil loss (Morgan, 2001), timber harvest (Insley and Rollins, 2005), agroforestry (Keesmana et al., 2007) and land use (Olubode-Awosolaa et al., 2008).

In particular, the aim of this paper is to show how mathematical modelling and simulations can be applied within the context of a biomanipulation carried out to manage a urban park in the city of Turin (Piedmont, Italy). Urban parks are of particular interest for public agencies since they provide many environmental and ecological services within anthropized landscapes (Bolunda and Hunhammar, 1999; McFrederic and LeBuhn, 2006) and, as small-scale green areas located close to where people live and work, they are also important for their effects on the quality of life (Chiesura, 2004). Indeed, these parks are viewed and perceived in many different ways by stakeholders. Beside their relevance as habitat for plant and animal species, they may be considered as spaces where a spectrum of recreational and leisure activities can be pursued, as well as remnants of the regional landscape that existed before the actual city development (Gobster, 2001). In our case study, in particular, the choice was made to restore one of the city parks as a reflection of the original, peri-urban agricultural landscape. Integrating tradition with the actual use of the park involved the introduction of sheep (*Ovis aries* L., 1758) as a natural regulator of grass growth. This decision raises the issue of estimating sheep compatibility with the environment of the park, mainly made of pastures and newly planted trees, and of evaluating the sustainability of the management strategy under different scenarios of grazing pressure. In order to reach these goals, modelling and simulations are needed for predicting and optimizing the outcomes of further future political decisions.

2 A Case Study: the Meisino Park

2.1 The park

The Meisino park, a urban park of the city of Turin, has an extension of about 45 ha. It was built in the year 2000 in an area previously occupied by a mix of fields and small production activities requiring large storing spaces, such as marble workshops. The area conserved a rural landscape: large squared lawns with no trees, surrounded by small invasion woods. In spite of the important environmental role of trees in a urban context, the Turin municipality decided to reproduce in the new park the original rural landscape. Thus most of the fields have been kept free from trees. Furthermore, during the building phase, about 100 large trees were cut, while small trees and bushes were planted. More specifically, according to the project, some major purchases of plants were made between 2002 and 2003. Disregarding plants used as anti-noise barriers, these purchases included:

(1) 6000 shrubs, including 3164 willows (*Salix* spp.) and 3010 plants belonging to the following species: *Cornus sanguinea, Corylus avellana, Crataegus monogyna, Euonymus europaeus, Ligustrum vulgare, Prunus spinosa, Rosa canina, Viburno lantana*;

(2) respectively 2446 and 5640 young trees, with a height between 1.0 and 1.5 meters, belonging to the following species: *Acer campestre*, *Carpinus betulus, Fraxinus excelsior*, *Prunus avium, Quercus robur*, *Tilia cordata*, *Ulmus effusa*;

(3) 190 taller trees, with a height between 3.0 and 3.5 meters, belonging to the species of *Populus alba, Populus nigra* and *Quercus robur*.

Costs ranged between 8.86 and 27.80 Euros each for the smallest trees, while the they ranged between 38 and 76 Euros for the tallest ones, not including the planting and mulching costs. The overall costs of the main tree purchases, about 8250 plants, and their planting, reached approximatively 240000 Euros.

To sum up, the Meisino park is characterized by large empty lawns alternating with forested areas, composed mostly by young trees.

2.2 Sheep grazing

Since 2007, the Turin municipality has partially replaced the traditional tools for the maintenance of public lawns by sheep grazing. More in detail, according to information provided by the municipality, 500 “Biellese” meat-sheep have been introduced in the park for a 2 months period, since April 15-th until June 15-th, under the supervision of a shepherd. This decision is consistent with the original project of the Meisino park, which was designed to reproduce a rural landscape in an area on the immediate outskirts of the town, but it also follows a more general policy implemented by the municipality to increase the sustainability of public green areas management. However, the outcome has led to non-reversible damages to the park caused by the sheep. More specifically, they not only grazed the grass in the park, but they also debarked the trees. Among the latter the most affected are in particular the youngest ones with a thin bark.

Even though grass generally represents the preferred resource for domestic herbivores (Pulina and Bencini, 2004), the sheep habit of debarking has often been observed, even in natural ecosystems (Sharrow et al., 1992). In herbivores, this behaviour becomes more likely when the availability of the preferred resource decreases, as for instance in the woods during the winter months. However, since bark is composed by fibrous material, with a high content in lignin and cellulose, it is not easily assimilated. Thus it cannot satisfy the metabolic requirements of herbivores. In fact winter starvation can bring them to death, even in presence of a large amount of available bark. Another relevant fact about this behavior concerns the way of grazing barks: while some herbivores, like for instance deer (de Nahlik, 1959), peel off vertical stripes, without totally interrupting the communication between leaves and roots, sheep instead peel the tree all around the trunk (Anderson et al., 1985), causing its death after a few weeks.

No control in order to asses the effects of the new meadow maintenance method has been performed in the Meisino park after the first year of sheep introduction. The first check, performed only after the second year in 2008, reported 219 damaged plants, most of them in a non-reversible way. In spite of this, the same experiment has been repeated in the following year and, according to statements of the municipality, it will likely be repeated again. In 2009, sheep introduction in the park resulted in a renewed damage of another 102 plants. As mentioned above, most of the damaged trees are the youngest ones, which where planted in the park at relevant costs only few years before. In the meantime these trees have increased in size, thereby increasing their commercial value, so that the damage incurred is even larger.

In addition to the economic and environmental damages just discussed, death of a large number of trees entails also an architectural damage. Indeed, planting trees in parks has the main long term purpose of ensuring a good arboreal canopy. If a large number of them dies, this goal cannot be achieved.

Based on these observations, the introduction of sheep in the park raises questions about the long term sustainability of this management choice. The purpose of this investigation is to undertake a deeper study of the situation. To this end we develop a mathematical model, whose description and analysis are presented in the next Sections. The outcomes of the simulations are discussed in terms of policy implications.

3 The Model

3.1 Mathematical formulation

The Meisino park is modelled via a three-population system consisting of a consumer population, namely the herbivores H, and two resources, grass G and trees T (Tamburino and Venturino, 2010). This resource splitting rises from the following two basic arguments:

(1) the resources are ecologically different: grass is a r-strategist population with a low carrying capacity and a fast growth-rate, while trees are k-strategists growing slowly but reaching high sizes. Tough other ecological differences exist between the two resources — e.g. trees develop deep roots that improve the draining soil capacity, they protect soil from erosion and sunlight, refresh air, keep humidity, are carbon sinks and so on — we do not take into account these properties, assuming them less relevant for our purposes;

(2) herbivores show a different level of preferences for grass and trees: as explained in Subsection 2.2, grass is their preferred resource, so that they switch their attention to trees only occasionally, with a higher probability when grass availability decreases.

In our model the functions $H(t)$, $G(t)$, $T(t)$ represent the biomass of the corresponding populations at a given time *t*. The function that models grass consumption is based on the following Beddington-De Angelis response function (Beddington, 1975; De Angelis et al., 1975), which itself is a generalization of the Michaelis-Menten model (see Murray, 1989)

$$\frac{G\ H}{c+\alpha G+H}$$

where α and c denote positive constants. It satisfies the following properties:

$$\frac{G\ H}{c+\alpha G+H}=G¿ \quad \text{and} \quad \frac{G\ H}{c+\alpha G+H}=\frac{H}{\alpha}¿$$

which state that the growth of *H* entails that the consumption of food cannot go beyond the whole amount of grass in the system. Conversely, in case of unlimited grass availability a single sheep cannot feed itself beyond a maximum level, so that consumption cannot exceed an amount proportional to the sheep population: H/α. From the second property, we infer that $1/\alpha$ represents the sheep per-capita consumption rate, when grass is available without limits. The parameter *c* has a meaning similar to the half saturation constant of the Michaelis-Menten model.

The function representing the consumption of trees is again of Beddington-De Angelis type, and follows also ideas of models incorporating predators' feeding switching behaviour. The latter has been proposed in the classical paper (Tansky, 1978), which has been followed by some other models (Vilcarromero et al., 2001; Khan et al., 2002; Khan et al., 2004 and more recently Ajraldi and Venturino, 2009). Since tree consumption by sheep grows when the availability of grass decreases, we introduce *G* in the denominator as well, obtaining

$$\frac{T\ H}{g+\alpha G+\beta T+H}$$

Grass and tree compete intra-specifically but although in a natural environment they also experience an inter-specific competition, in fact the grass can suffocate the new tree plantulae while on the contrary woods can expand and thus replace fields, in a urban park grass and tree areas are artificially kept separate. Hence, in this context, no competition arises among the two distinct populations for space, possibly only for resources like water in the soil. We can thus exclude inter-specific competition from the model.

The model thus becomes (Tamburino and Venturino, submitted)

$$\dot{H}=-\mu H+e\frac{HG}{c+H+\alpha G}+f\frac{HT}{g+H+\beta T+\alpha G}$$

$$\dot{G}=r_1 G\ \ 1-\frac{G}{K_1+hH}-\frac{HG}{c+H+\alpha G} \tag{1}$$

$$\dot{T}=r_2 G\ \ 1-\frac{T}{K_2+hH}-\frac{HT}{g+H+\beta T+\alpha G}$$

where μ represents the metabolic rate of sheep, *e* and *f* are assimilation coefficients, r_1 and r_2 are the growth rates respectively of grass and trees and K_1 and K_2 represent instead their respective carrying capacities, α and β are weights to measure the respective influence of grass and trees in the Beddington-De Angelis response function. We account also for the sheep fertilization of grass and trees, by means of their wastes, represented by the term *hH*, which needs to be added to the carrying capacity of both the grass and trees equations. Instead, we consider that fertilization does not affect the growth rate of resources, although in fact it could make the resources to grow faster. This increase in growth speed should depend not only on the herbivore biomass in the system, but also on the inverse of the resource biomass: indeed, the larger the area on which a certain amount of fertilizer is spread, the smaller its mass per unit area is, and therefore the lesser its effects are. Accounting for this, however, would lead to a much more complex system, the outcome of which would make the mathematics hardly tractable. For this reason, we prefer to confine the contribution of sheep wastes only to the carrying capacity.

3.2 Considerations on the model parameters

There are some general relationships between the parameters, which we now outline.

Since sheep do not assimilate the whole biomass they consume, but eject a part of it in form of wastes, then the parameter h can be well approximated by the difference between the resource consumption by the sheep (which, as seen above, is represented by $1/\alpha$ when grass availability is unlimited) and the assimilated part, i.e.: $h \approx (1-e)/\alpha$. The assimilated part is used to satisfy the metabolic requirements and only the exceeding part results in an increase of sheep biomass. Grass represents the preferred resource for sheep, which can indeed survive by eating grass alone (Pulina and Bencini, 2004). We can then assume that an available adequate amount of grass can satisfy the metabolic needs of sheep, from which the following relation holds:

$$\frac{e}{\alpha} > \mu \tag{2}$$

On the contrary, trees alone are not able to satisfy the metabolic needs of sheep, therefore we must have:

$$\frac{f}{\beta} < \mu \tag{3}$$

3.3 Parameter values

The time and biomass units used in simulations are, respectively, the day and the mean sheep weight, the latter being approximatively 75-80 Kg for a “Biellese” meat-sheep (AA.VV., 2009), a choice intended to facilitate the reading of the plots.

All parameter values were selected in a biologically meaningful range, identified by values taken from the literature, whenever possible. An empirical method to estimate the quantity of food consumed daily by a ruminant is proposed in Pulina and Bencini (2004). The food ingested can be approximated as a percentage of the weight of the ruminant itself. In case of sheep, this percentage is about 4%-5.5%. Since the parameter α represents the inverse of the grass amount consumed by a single sheep when grass is unlimited, see Section 3, we can take its value to be $\alpha=0.05^{-1}=20$. We set $\mu=0.03$, which means that a sheep, without eating, dies in about 30 days. This is consistent with the starvation time of similar mammals, i.e. mammals not accustomed to lethargic periods and with a weight comparable with the one of the “Biellese” sheep. Recalling that $e<1$ and $e>\alpha\mu$, (2), we then set $e=0.62$ as reference value, varying it within the previous bounds.

An estimation of the tree damage due to the sheep is more difficult. When a sheep switches its attention to a tree rather than to grass, even if it takes just a small piece of bark, it may cause the death of the whole tree. Therefore, it actually removes a much larger quantity of tree biomass from the system than the small bytes grazed. Since the parameter β represents the inverse of the per-capita tree consumption by sheep (similarly as for α), we then set as a reference value $\beta=1$. Of a whole tree a sheep can assimilate at most the piece of bark it really takes, but as already remarked even that is only partially assimilated in view of the high lignin and cellulose content of barks, see Subsection 2.2. Therefore, the assimilation coefficient from trees f must be very small: we set it to be $f=0.001$.

In Fujimori (2001), we can find some estimations about the phytomass and the annual NPP (Net Primary Production) of several natural environments. Basing our considerations on the former, observing that the Meisino park has an extension of about 45 ha, we estimated its grass carrying capacity to be approximately

945000 Kg, which is equivalent to 12600 “Biellese” sheep. But this is an overestimate, since the grass does not cover the whole park extension. Comparing the phytomass with the annual NPP, we calculated the daily growth rate of the grass. To this end, considering that at our latitudes the plant growth period is limited to about five months, we divided the grass biomass produced in a year by 150 days. From these considerations, we set K_1=12600 and r_1=0.004. In a similar way, we calculated the daily growth rate of trees, obtaining r_2=0.0006. We took into account only the young trees, planted in the park during its construction, that grow at a faster rate than the older ones. This, because older trees interact barely with the rest of the system, due to their thicker bark, which makes them resistant to sheep attacks. Note also that the carrying capacity of trees depends only on the biomass attainable by the trees already in place. Indeed, since the Meisino is a urban park, new spontaneous trees are indeed not allowed to grow. Fortunately, information on the area interested by new tree plantation and the tree density are included in the park project, allowing us to estimate their carrying capacity. However the park is not a natural forest, with a developed canopy, and therefore the direct application of the tables in Fujimori (2001) could lead to an over-estimation. In addition, it is possible to assess the tree carrying capacity starting from the average tree weight reported on forestry management tables. But the latter data refer only to the commercial part of trunks, that underestimate the real tree biomass. To compensate, we consequently augmented these estimates, to calculate the real carrying capacity of the trees in the Meisino park. In both ways the results obtained were similar, this agreement being a sign of assurance in the procedure. These findings give a carrying capacity of about 1800 t, equivalent to 24000 sheep. We thus set K_2=24000.

As reported in Section 3, sheep eject in form of wastes the food they consume but do not assimilate. We therefore set $h=\alpha^{-1}(1-e)=0.02$.

In our simulations we let most of these parameters vary, but keeping their values around the ones indicated above. Only when the uncertainty was larger, for instance for some tree-related parameters, like β and K_2, we explored wider ranges.

4 Results

4.1 Biological meaning of the analytic results

We summarize here some basic facts concerning the model, further extensive mathematical details for the interested reader are fully reported in Tamburino and Venturino (2010) and Tamburino and Venturino (submitted).

Under certain natural parameter conditions such as $e<1$ and $f<1$, the system is shown to be bounded. Clearly, this result shows the biological feasibility of the model. In view of environment finite resources, in fact, populations cannot grow unboundedly.

The system admits several possible equilibria, of which a few are unconditionally unstable. Among the ones that allow local asymptotic stability, under suitable parameter restrictions, we find the points $P_1=(0,K_1,K_2)$, $P_2=(H_1,G_2,0)$ and possibly the coexistence equilibrium $P_*=(H_*,G_*,T_*)$.

The condition for which P_1 is stable becomes

$$e\frac{K_1}{c+\alpha K_1}+f\frac{k_2}{g+\alpha K_1+\beta K_2}<\mu \tag{4}$$

For the equilibrium $P_2=(H_1,G_2,0)$ a nonlinear algebraic system must be solved, which reduces to the following quadratic equation for G, whose roots give the sought value G_2,

$$a_2G^2+a_2G+a_0=0 \tag{5}$$

Here

$$a_1 = \frac{K_1 r_1 e}{\mu} - \frac{K_1 e}{\mu} + K_1 \alpha - \frac{hcr_1 e}{\mu} \quad 2\frac{hce}{\mu} - 2hc\alpha$$

$$a_2 = \frac{hr_1 e^2}{\mu^2} - \frac{he^2}{\mu^2} \quad 2\frac{he\alpha}{\mu} - \frac{hr_1 e\alpha}{\mu} - \alpha^2 h - \frac{r_1 e}{\mu}$$

Since $a_0=K_1c>0$, thus for the special case $h=0$, we have $a_2<0$ so that there are two real solutions, one positive and the other one negative. The same occurs if $h\neq0$ and $a_2<0$. Otherwise, if $h\neq0$ and $a_2>0$, to have positive solutions, we must impose $a_1<0$ and $0<\Delta=a_1^2-4a_2a_0$. In addition, for feasibility we need to require $H_2>0$ which gives

$$G_2 \quad \frac{c\mu}{e-\alpha\mu} \tag{6}$$

Local stability of P_2 is difficult to assess, but numerical experiments reveal that the equilibrium can indeed be attained. Similarly, the study of P_* is too hard to be performed analytically, but the simulations show that it exists under certain conditions on the parameters (Tamburino and Venturino 2010, Tamburino and Venturino, submitted).

These results indicate that some subsystems of the model are indeed biologically sustainable, namely the purely vegetation one $(0,K_1,K_2)$, in which the herbivores are absent, and the pure “pasture” one $(H_1,G_1,0)$, in which trees are absent. For the former to be stable, (4) must be satisfied.

Since $e\alpha^{-1}>\mu$, see (2), the second term $e\dfrac{K_1}{c+\alpha K_1}$ will have a value greater than μ or at least very close to μ, except in the case for which c is very large. Hence, the sum of the first two terms has either a positive value or, if negative, its value is very close to zero. Since there is a further, although possibly small, positive term to be added, the final value of the eigenvalue will be positive at least for a wide range of cases. In summary, the stability of this equilibrium depends on the parameters, but in the majority of biologically meaningful cases it will result unstable.

Note that the equilibria that do not exist mathematically, namely those of the forms $(H,0,0)$ and $(H,0,T)$, are also clearly not biologically feasible: the former since in absence of resources any heterotrophic population vanishes, and the latter in view of the fact that an heterotrophic population cannot survive without a resource, from which to draw enough energy to satisfy its metabolic requirements.

The stability of the equilibrium $(H_1,G_1,0)$ appears to be mathematically difficult, as it is the determination of feasibility and stability of possible interior coexistence equilibria, in which all populations survive. For these cases, numerical simulations (Tamburino and Venturino, 2010) supply the positive answer.

4.2 Simulations

Our simulations show that the model is able to reproduce the real case: for instance with 500 sheep in the park, over a 60 days period, the grass decreases, the sheep grow; further, the trees decrease by amounts consistent with the reported damages.

In the long term, the simulations show that the system can attain both the boundary equilibrium, namely the trees-free equilibrium, i.e. for $T=0$, and the coexistence equilibrium. This validates empirically the feasibility issue for these equilibria, left unanswered by the theoretical analysis. As for stability, note that the boundary tree-free equilibrium is only stable if we start the simulation with initially no trees in the model. If initially instead there are some trees, instead, the equilibrium P_2 is unstable in the phase space, i.e. trajectories starting even near it will tend toward the interior coexistence equilibrium P_*. As for the latter, the simulation shows empirically that it can be stabilized, a fact that as mentioned analytically proves very hard to be assessed. In fact the eigenvalues of the full Jacobian are the roots of a polynomial of degree four, with coefficients that depend on all the model parameters. Therefore the Routh-Hurwitz stability conditions, in terms of all the model parameters, in this case are indeed very complicated to study.

Next, we vary a bit some of the system's parameters to give a flavour for their influence on the outcome of the dynamics. More quantitative results with a qualitative comparison of all the weights that each parameter contributes to the final results of the simulations are deferred to the final paragraphs of this Section.

Taking as reference parameter e, i.e. the assimilation coefficient of the grass by sheep, we now study its influence on the system's behavior. In absence of the trees, i.e. for the case T=0, there is only a small range of values for e, between about 0.61 and 0.63, such that the system reaches an equilibrium in presence of the sheep, i.e. for H>0. For values below this range, the sheep cannot survive, while for values above the range the system starts to oscillate. Therefore e=0.63 gives the empirical value for which a Hopf bifurcation arises. Oscillations become more and more pronounced when e grows, leading the minima of both sheep and grass to be very close to zero, see Fig. 1, a possible situation in which due to sudden external perturbations, these populations could collapse. In fact, more precisely, the maxima of the sheep population grow in time while the minima are very close to zero, implying that any environmental disturbance affecting and altering these low numbers might drive the population to extinction. The grass biomass also oscillates, but its maximum settles at the park carrying capacity and generally it is very well below it, on average about half the system's carrying capacity, showing therefore that the sheep grazing can keep under control the grass growth. When T>0, the situation does not change much, but the suitable range for e is slightly shifted to the left: in presence of the second resource given by trees, the sheep can survive even with values of e a bit lower, with respect to the situation without trees, but also the bifurcation occurs earlier. Oscillations are also observed in the trees biomass, remaining quite far from zero, but with an amplitude of about 35% of the maximum value, i.e. the trees' carrying capacity, see Fig. 2.

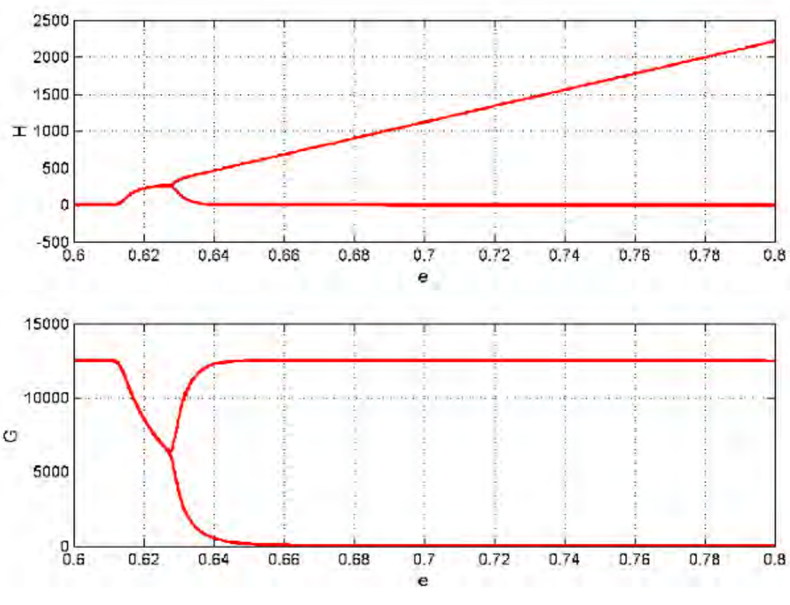

Fig. 1 Hopf bifurcation diagram, with minimum and maximum values of H and G as functions of the bifurcation parameter e, the assimilation coefficient of grass by sheep, in absence of trees, i.e. for the case of T=0. The other parameter values are K_1=12500, r_1=0.004, K_2=25000, r_2=0.0005, α=20, μ=0.03, β=3, f=0.001, h=0, c=5500, g=850000. Note that there is only a small range of values for e such that the system reaches a stable equilibrium with a positive value of sheep, H>0

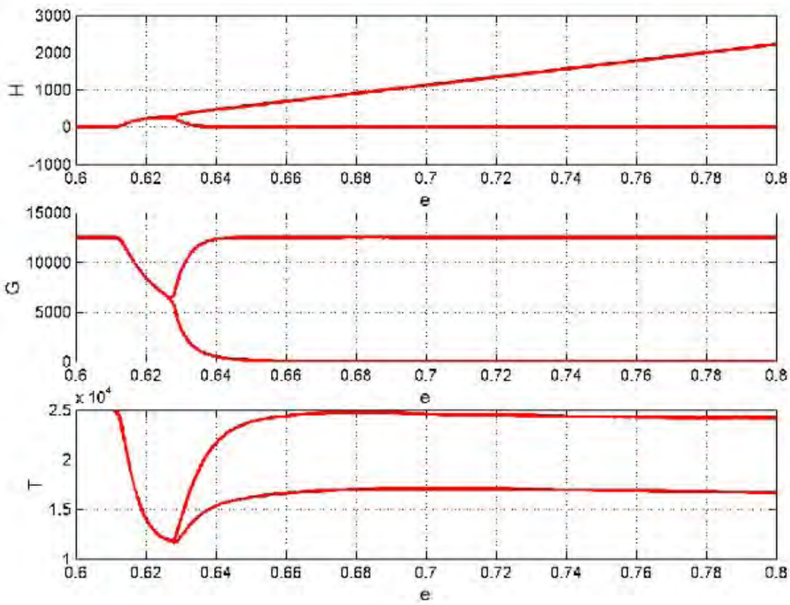

Fig. 2 Hopf bifurcation diagram, with minimum and maximum values of *H*, *G* and *T* as functions of the bifurcation parameter *e*, the assimilation coefficient of grass by sheep. Same parameter values as in Fig. 1. Here too there is only a small range of values for *e* such that the system reaches an equilibrium in presence of sheep. This range is slightly shifted to the left, comparing it with the one for *T*=0

The overall dynamics depends less on perturbations of the tree growth rate given by the parameter r_2. The system is far less influenced by changes in the other parameters, like for instance *f*, i.e. the assimilation coefficient of trees' bark by sheep. In particular *h*, the soil fertilization by the sheep wastes almost does not affect the system behaviour. Taking as bifurcation parameter K_1, i.e. the grass carrying capacity, we find a bifurcation originating at $K_1 \approx 17500$ (picture not reported). Below this threshold, the system settles to an equilibrium. Note however that the bifurcation occurs for a value of K_1 well above the current park's carrying capacity, in fact about 40% of the latter. Since the park is located at the outskirts of the town, it has no means of expansion in the foreseeable future. For this reason limit cycles due to changes in the grass carrying capacity will never occur in reality, and for this reason the corresponding bifurcation diagram has thus been omitted.

In view of the fact that the estimate of K_2 is uncertain, we have varied it in a wide range. With increasing K_2, we discover that herbivores grow and grass decreases. But the main point is that the higher the value of K_2, the larger the tree biomass; however, also the wider the gap becomes between the tree biomass and their carrying capacity.

Fig. 3 shows a comparison between two possible scenarios. The first one, drawn in black, represents the real-like situation: the same number of sheep, 500, as it has been implemented so far by the park management, is introduced for 60 days in the system every 150 days, i.e. every "year", represented by the the vegetative growth period. The red curve represents the second situation, namely the undisturbed park, i.e. the system without sheep. The time span of the simulation is more than 25 years. The results of this policy are evident: sheep grazing helps in reducing yearly the amount of grass, keeping it below the park carrying capacity. But at the same time the tree canopy is reduced by about 40%.

Fig. 4 contains the results of a similar simulation carried out with the introduction of a smaller number of sheep, 80. In this case also, over the same time span, the tree biomass and grass in the system are reduced. Again here grazing keeps the grass below the carrying capacity, by a smaller amount than in when 500 sheep are introduced, but still the grass is controlled. This policy reduces instead the trees' carrying capacity by only about less than 7%, showing its much smaller impact on this resource.

Fig. 5 is the same as Fig. 3, but shown over a shorter time span, to better depict the alternating periods of presence and absence of grazing.

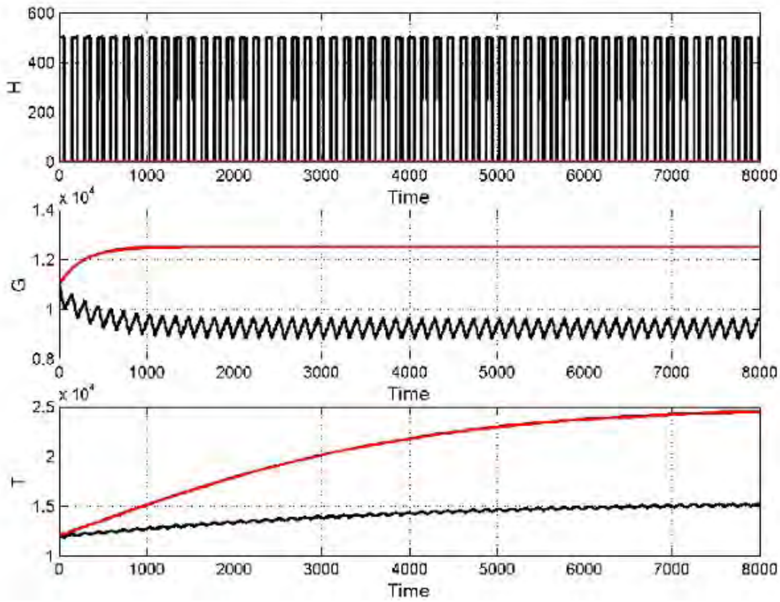

Fig. 3 The black curve represents the case in which 500 sheep are introduced into the system for 60 days every vegetative year of 150 days; the red one the case of the system without sheep. The grazing of sheep helps in reducing yearly the amount of grass, keeping it well below the park carrying capacity. The time span is therefore more than 25 years. Here the canopy is reduced by about 40%. The parameter values are as in Figure 1 with e=0.62

4.3 Parameter sensitivity analysis

We investigated then also the sensitivity of the system to variations in the parameters, studying the ones that influence most the final outcome.

The plot of the absolute sensitivity function for the herbivores shows (Fig. 6 left), that the two parameters that most influence it are μ and r_1. The same holds true for the grass (Fig. 6 right). The next two parameters that influence these populations almost similarly are r_2 and f; finally the parameter e. All the other ones play essentially an insignificant role, in that they do not even appear clearly in the picture.

If we look at the influence of each such parameter, the largest values of μ are seen to affect the peak and the limiting value of the herbivores, for which we conclude quite naturally that the herbivores' mortality influences their ultimate population size. The peak of r_1 is instead located in the region where the herbivores population changes more rapidly, so the grass reproduction rate affects mainly the growth rate of its grazers. The trees reproduction rate and their assimilation by the herbivores affects the peak of their population, while the grass assimilation affects essentially the sheep steady state.

The grass is primarily influenced by the parameter μ, so a high sheep mortality rate determines the amount of grass available. The next three parameters, r_1, r_2, f affect also similarly the grass population, with also little differences in their quantitative values, as well as e.

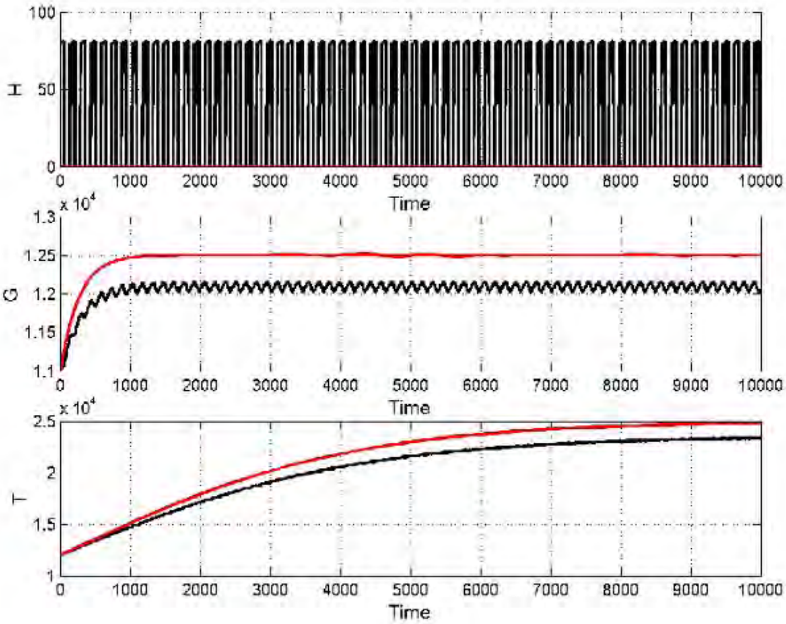

Fig. 4 The black curve represents the case in which 80 sheep are introduced into the system for 60 days every 150 days; the red one the case of the system without sheep. The grazing of sheep help in reducing yearly the amount of grass, keeping it again below the park carrying capacity, at about 95% of its value. Over a time span of more than 25 years, the canopy is reduced in this case only by about 6.6%. The parameter values are the same as in Fig. 3

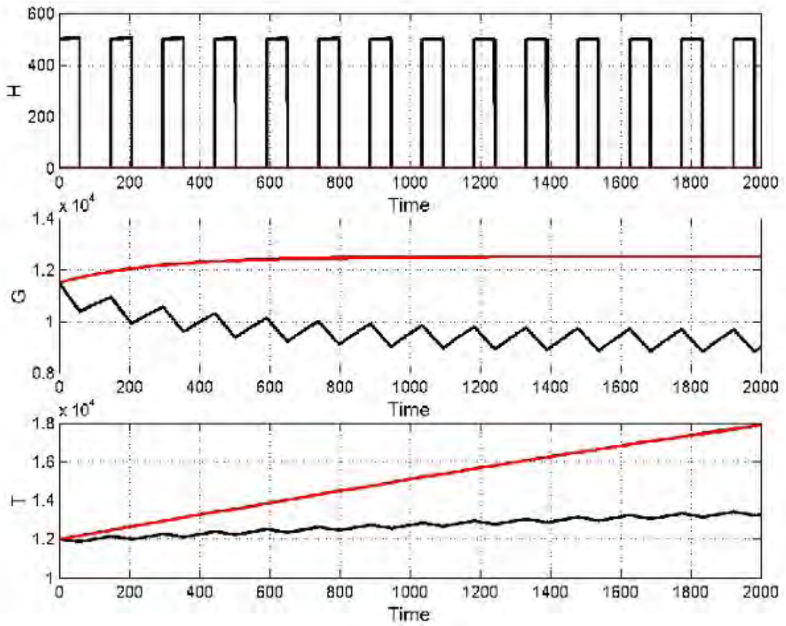

Fig. 5 The black curve represents the situation in which 500 sheep are introduced into the system for 60 days every vegetative growth period of 150 days; the red one the behavior of the system without sheep. The figure is the same as Figure 3 but run only over a shorter time span, to better show the alternating periods of presence and absence of grazing

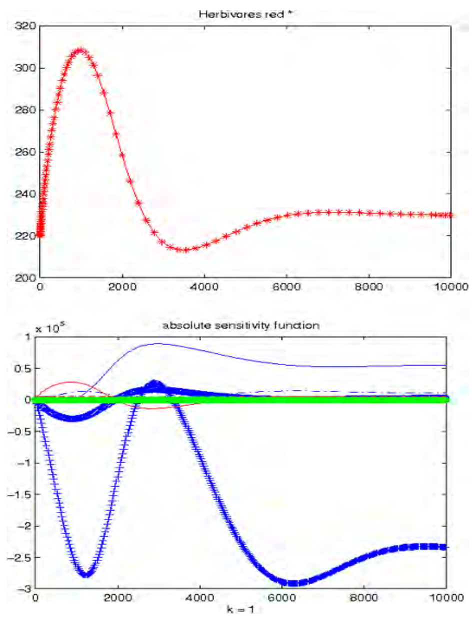

Fig. 6 Absolute sensitivity function for the herbivores, bottom, compared with the herbivores population evolution, top. The blue crossed line represents the parameter μ, the continuous thin blue line instead is the parameter r_1, r_2 is the thick blue line, *e* the discontinuous blue line, *f* the red line

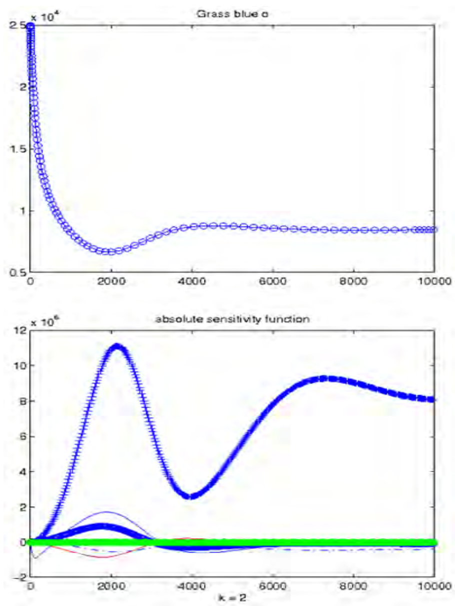

Fig. 7 Absolute sensitivity function for the grass, bottom, compared with the grass population evolution, top. The blue crossed line represents the parameter μ, the continuous thin blue line instead is the parameter r_1, r_2 is the thick blue line, *e* the discontinuous blue line, *f* the red line

For the trees population, the most significant parameter appears to be r_2 (Fig. 8 middle). Comparing with the top figure, their reproduction rate thus is mainly affects clearly their own growth rate. In fact r_2 influences mostly the initial transient phase, in which grass and trees decrease to their asymptotic values and herbivores grow to a peak. The next parameters that play an important role in this context are better seen in the middle picture of Fig. 8. They are the sheep mortality μ, and the grass reproduction rate, r_1, both affecting primarily the trees steady state.

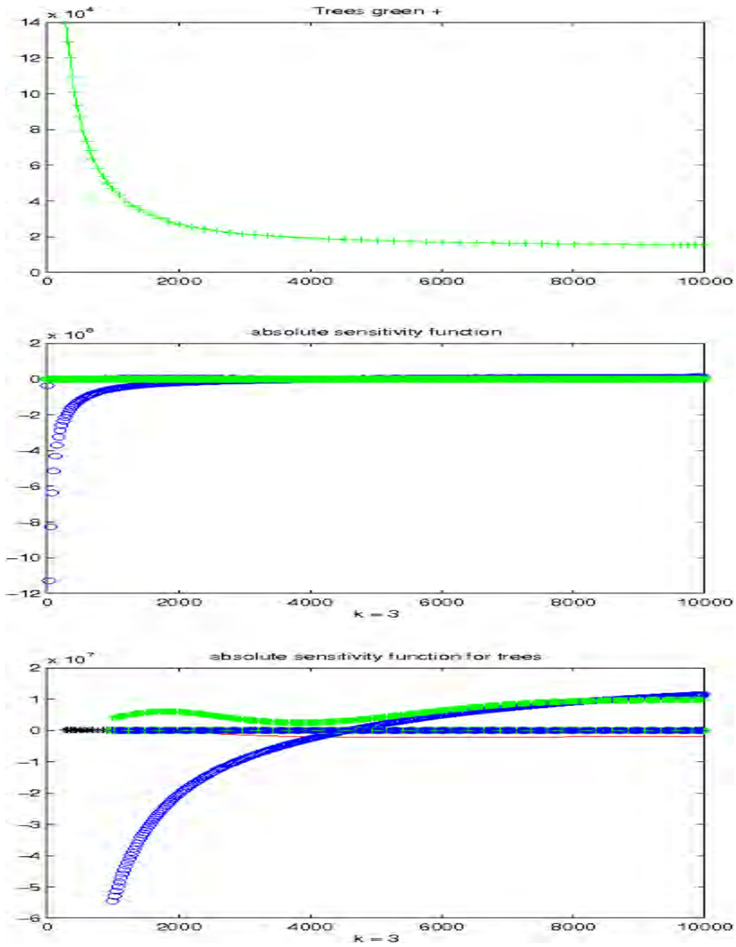

Fig. 8 Absolute sensitivity function for the trees, middle, its blowup, bottom, compared with the trees population evolution, top. Parameter r_2, blue circles; parameter r_1 continuous thin red line, μ thick green line

5 Discussion and Conclusions

In order to present real advantages, the sheep introduction in the system should lead to the following outcomes:

(1) sheep should increase in biomass;

(2) grass should decrease, or, at least, it should keep below a certain level under the carrying capacity; otherwise the sheep would not achieve the meadow maintenance, which is the main purpose of the idea of introducing sheep in the park;

(3) trees biomass should not decrease.

Over a single 60 day period, it seems that it is possible to achieve the first two goals, by using a number of sheep within a certain range, in our simulations the latter being between 70 and 500. However, the trees are always damaged.

In particular this becomes more clear if we simulate the system for a longer period of time. Introducing a bunch of sheep for 60 days every year, the trees cannot be kept at their carrying capacity. Indeed, sheep introduction results always in a loss of tree biomass, even in the case of a very small set of sheep. This is apparent from the simulations comparing the evolution of the unperturbed environment, with those in which the herbivores are allowed to semi-freely graze in the park. If these are carried out for a suitable time period, so that in both cases all the variables settle to values near their carrying capacities, which means about a quarter of century, since the slowest growing variable is represented by the tree canopy, the outcomes are evident.

In the case of sheep introduction, the trees biomass always remains below the carrying capacity value achieved when the sheep are absent. Thus the trees never reach their carrying capacity. This also happens for the grass, which is one of the objectives of this policy. This entails that, due to presence of sheep, the tree canopy expected for the park according to the original project will never be reached, causing a net loss of money invested to plant the trees. An additional cost follows from the loss environmental services provided by trees, which disappear when the trees die.

Sheep introduction thus prevents both grass and trees to reach their natural carrying capacity. Clearly, the larger the number of sheep, the greater is the damage. The first conclusion is within the scope of the undertaken action by the municipality, but the one on trees foreshadows a reduced trees growth, of a certain percentage, depending on the number of sheep introduced. This reduction however occurs over a period of time of more than a quarter of a century. On the one hand, this entails that the economic damages of this policy in the short period can be overlooked, since their costs impact could be thought to be neglected. But they cannot be trifled with, since on the other hand the decision of letting sheep damage the trees may lead to lasting serious consequences in the long term, in view of the slow regrowth ability of the damaged canopy. A small number of sheep instead causes a small reduction in the tree biomass, but on the other hand it does not keep grass at a sufficient low level. Hence a good meadow maintenance is not achieved, see Fig. 4.

With 500 sheep, i.e. the number currently introduced in the park by the Turin municipality, for 60 days every year, the model indicates that the tree biomass will reach at most a level of about 15000 units against the 25000 expected in absence of sheep: a loss of about 40% of the expected biomass amounting to around 100000 euros. To this economic cost the value of environmental services provided by trees, that are lost with them, has to be added. Furthermore this is likely to be an underestimate of the actual loss, since the trees' growth in size, i.e. in value, is not accounted for. The model, in case of a non-repeated damage, shows that trees can anyway reach their carrying capacity, eventually with a delay, but in reality since the Meisino park is a urban park where spontaneous trees are not allowed to grow, the tree carrying capacity depends only on the trees already in place. When they die, unless they are replaced, their carrying capacity actually decreases. Therefore the damage resulting from the model could actually be an underestimate of the real one also for a single such intervention.

By introducing sheep in the Meisino Park, the Turin municipality aimed at controlling grass growth. In order to assess the usefulness and viability of this policy, we developed a mathematical model which makes possible to set a quantitative value for the desired containement. Let us assume that by this intervention we want to keep the grass at say 80% of the natural carrying capacity of the Meisino park. Since here we have estimated K_1=12500, this means that $G \leq 10000$. By experimenting with the numbers of sheep introduced, we find that it must be between 400 and 450. The grass is maintained oscillating around the desired level, thereby satisfying the goal. But one can also see that the damage suffered by the trees is quite large, amounting to about 35% over the timespan of 25 years. With the smaller number of sheep, the goal would not completely be achieved, but the damage to the trees would still be significant.

To sum up, this management choice as it is implemented now does not seem to be sustainable, from several viewpoints. In order to make it viable, it should be complemented by other measures, aimed at tree preservation, like the use of fences around the trunks, to prevent bark grazing, or requiring from the shepherd the fencing out of trees from the grazing grounds.

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