Article

A fuzzy mathematics approach in measuring air pollution from motor vehicles

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Abstract
Air pollution from motor vehicle in cities specially in developing countries has been a major source of urban air pollution and hence a cause of concern for the administrators of the cities as well as for researchers in the field of ecology and mathematics. In the present work, the fuzzy membership functions for some of the attributes regarding air pollution from motor vehicle are proposed. To study the index of air pollution in different area of a city, the air pollution index is proposed, which takes into account the membership function for the attributes of pollution. By applying a suitable interpolation formula (in our case Lagrange's interpolation formula) a polynomial curve is obtained, which gives the measure of air pollution at any point of the city under certain assumptions. The effectiveness and suitability of the method is established by taking an example.

Keywords pollution matrix(PM); weighted pollution matrix(WPM); pollution index; weight; air pollution.

1 Introduction

Air quality in any area depends upon many factors (Telesca and Lovallo, 2011; Zhang et al., 2011; Hayati and Sayadi, 2012). Therefore, the researchers as well as the ecologists identify it as a multidimensional problem. Owing to their rapidly increasing numbers and very limited use of emission control technologies, motor vehicles are emerging as the largest source of urban air pollution in developing countries. Air problem has already emerged as a major cause for public health in most cities of the developing world .The pollution level in the cities of industrialized countries are in fact less than that in the megacities of the developing nations. Epidemiological studies show that air pollution in developing countries accounts for many deaths in these cities. The combustion of gasoline and the hydrocarbons fuels in automobiles, trucks and also jet planes produces several primary pollutants; nitrogen oxides, gaseous hydrocarbons, carbon monoxide, as well as large quantities of particulates, mainly lead. In urban areas where transportation is the main cause of air pollution, nitrogen dioxide tints the air, blending with other contaminants and the atmospheric water vapor to produce brown smog. Although the use of catalytic converters has reduced the smog producing compounds in the motor vehicle exhaust emission, recent studies have shown that in doing so the converters produce nitrous oxide, which contaminates substantially to global warming.

In the present work we have proposed a method based on fuzzy mathematics to study the amount of air pollution due to motor vehicles in different areas of a metro city in a developing country and then we have constructed the PM (Mukherjee et al., 2011) and WPM (Mukherjee et al., 2011) the construction of which has been discussed in the problem formulation (Faiz et al, 1996). Finally we have obtained the poverty index which gives the measure of air pollution and depending on that a rank has been allotted to each area of the city under consideration. Sum of membership degrees has been obtained the formula for which has been illustrated in the example taken. These membership grades has been used as the "y " coordinate to obtain the polynomial curve $y=f(x)$, where x values has been taken as the distance of the different areas from the reference area, taking any one as the reference based on the exact location, which has been assigned $x=0$. From this polynomial curve we can predict the pollution levels in any area under the assumption that the area lies close to the points where the exact survey has been done.

2 Methods

2.1 Formulation of problem

There are quite a number of methods for measuring the level of pollution caused by motor vehicles. The problem with these available methods is that almost all of them are based on classical two valued logic i.e. true or false. Moreover in checking the emission of a motor vehicle these methods measure the amount of pollutants coming out of the vehicle under ideal conditions and in doing that discard many real attributes which in fact also contribute largely to the pollution caused by motor vehicle. The problem in this approach is that, the attributes which contribute largely are real and hence are fuzzy in sense. For such type of attribute true or false (1 or 0) (Mukherjee et al., 2011) does not help to solve the problem effectively. So we propose a new approach based on fuzzy mathematics (Mamdani, 1974; Cerioli and Zani, 1990; Tanaka et al., 1992; Ferrarini, 2011a, b) to counter the above mentioned limitations of the existing methods.

Let us consider the city say Jamshedpur (In India). We consider n areas of the city of Jamshedpur, say J_1, J_2, J_3,……, J_n, and let T_1, T_2, T_3,……, T_m be m attributes (parameters) which may be a mixture of crisp and non crisp set. For e.g. consider the case of the attribute set {Having metal roads, Number of vehicles, Tall buildings, Available open space}. Here "Having metal roads" is a crisp set all other are non crisp terms, and hence can be represented in the fuzzy sense. These attributes are now grouped and certain membership degrees are sought from surveyors (Experts) for each group. For e.g. "Tall buildings" in our example can be grouped under the following categories.

i) The top of the building subtending angle of 90^o with the road which is considered to be the horizontal.

ii) The top of the building subtending angle of 45^o with the road which is considered to be the horizontal.

iii) The top of the building subtending angle in between 45^o and 90^o with the road which is considered to be the horizontal.

The surveyor has to find out exactly under which group the area say J_i falls. The experts then assign membership values to this different groups .The membership functions are then assembled and then we deal with there average. So without any loss of generality, we assume T_1, T_2, T_3,……, T_k be k attributes which gives crisp values "0" and "1".On the other hand $T_{(k+1)}$, $T_{(k+2)}$, $T_{(k+3)}$, ……, T_m be grouped under three categories.

If we appoint n experts E_1 E_2, E_3,… …, E_n in n busiest places of a particular area say J_i and then ask them to assign membership grades to these n attributes. Then we take the average of them .While taking the average to make our method more effective we can give more weight age to the value given by that expert who is encountering more number of vehicles.

For e.g. If in J_1 location the number of vehicles encountered by E_1, E_2, E_3 are 1000, 700, 500. Then average number of vehicles is given by the formula $(3\times1000+2\times700+500)/6=816.6\approx817$.

2.2 Measurement of air pollution in fuzzy framework

For measuring the amount of air pollution and then to give a rank to each area being considered here. In their paper Mukherjee et al. (2011) have constructed the poverty matrix and weighted poverty matrix extending the very idea we construct two matrices, the first matrix is known as pollution matrix (PM) and the second one as weighted pollution matrix (WPM) .

2.3 Rule to construct pollution matrix (PM)

The rule to construct pollution matrix is indicated in Tables 1.

Table 1 Pollution Matrix (PM)

	T_0	T_1	T_2	....	T_k	T_{k+1}	T_{k+2}	....	T_m
J_0	Z_{11}	Z_{12}	Z_{13}		Z_{1k}	Z_{1k+1}	Z_{1k+2}		Z_{1m}
J_1	Z_{21}	Z_{22}	Z_{23}		Z_{2k}	Z_{2k+1}	Z_{2k+2}		Z_{2m}
J_2									
J_3									
. . . . .									
J_n	Z_{n1}	Z_{n2}	Z_{n3}		Z_{nk}	Z_{nk+1}	Z_{nk+2}		Z_{nm}

Table 2 Weighted Pollution Matrix (WPM)

	T_0	T_1	T_2	....	T_k	T_{k+1}	T_{k+2}	....	T_m
J_0	Z'_{11}	Z'_{12}	Z'_{13}		Z'_{1k}	Z'_{1k+1}	Z'_{1k+2}		Z'_{1m}
J_1	Z'_{21}	Z'_{22}	Z'_{23}		Z'_{2k}	Z'_{2k+1}	Z'_{2k+2}		Z'_{2m}
J_2									
J_3									
. . . . .									
J_n	Z'_{n1}	Z'_{n2}	Z'_{n3}		Z'_{nk}	Z'_{nk+1}	Z'_{nk+2}		Z'_{nm}

2.4 Rule to construct weighted pollution matrix

To construct the WPM, we consider the weights w_j we want to impose on the attributes T_j, j = 1,2,… …,m and $\Sigma^{m}_{j=1} w_j = 1$. Each column of the PM is multiplied by their corresponding weights and we obtain a new matrix as WPM. In WPM $z'_{ij} = w_j z_{ij}$. We also define $(J_i) = \Sigma^{m}_{j=0} z'_{ji}$.

The rule to construct weighted pollution matrix is indicated in Table 2.

This matrix will help us to identify, which location of the city under consideration is more polluted, this in turn will help the policy makers and city administrators for various purposes like planning of the township, to detect reasons for pollutions etc. Another benefit of this matrix is that, we can construct a polynomial by Lagrange's interpolation or any other suitable interpolation formula and hence can predict the air pollution in some adjoining area of the city, without employing any expert in this new area. The limitation of this approach is that, the accuracy of prediction of pollution depends upon the distance between the adjoining areas. If the new area is close to the existing area which has already been surveyed .through which we have obtained the polynomial by interpolation the accuracy will be more. The method can be described as follows (Table 3):

To employ the Lagrange's interpolation method (Dutta Majumdar et al., 2007), we take any one location as our reference and respect to that measure the distance in k.m.(unit) of other location in the city and these values of the distance are taken as x values and the corresponding y values are taken from the WPM.

Table 3 Weighted Pollution Matrix (WPM)

x	J_i	J_0	J_1	J_2	....	J_n
y	$f(J_i) = f(J_i(T_m))$	z_0	z_1	z_2		z_m

where $0 \leqslant i \leqslant n$ and $0 \leqslant j \leqslant m$.

and $z_i = \Sigma^{m}_{p=1} (z'_{ip}/(m+1))$, ($i$=1,2,3,… …, m+1). Using the Lagrange's interpolation formula we can write,

$f(J_i(T_m)) = w(x) \Sigma^{n}_{r=0} (f(J_r)/((x-J_r)w'(J_r)))$

where,

$w'(x) = (x-J_1)(x-J_2)\ldots\ \ldots\ \ldots(x-J_{n-1})(x-J_n) + (x-J_0)(x-J_2)\ldots\ \ldots\ \ldots(x-J_{n-1})(x-J_n)$

$+(x\text{-}J_0)\ (x\text{-}J_1)\ldots\ldots\ldots(x\text{-}J_{n\text{-}1})\ (x\text{-}J_n)+\ldots\ldots\ldots$

$+(x\text{-}J_0)\ (x\text{-}J_1)\ldots\ldots\ldots(x\text{-}J_{n\text{-}1})$

$w'(J_r) = (J_r\text{-}x_0)\ (J_r\text{-}J_1)\ldots\ldots\ldots(J_r\text{-}J_{r\text{-}1})\ (J_r\text{-}J_{r+1})\ldots\ldots\ldots(J_r\text{-}J_{n\text{-}1})\ (J_r\text{-}J_n)$

Since in the above formula the values of $f(J_r)$ involved are all fuzzy numbers, it will be tedious and difficult to calculate the Lagrange polynomial as well as the value at the interpolating point so for that we can use the computer programming of the above mentioned method which is as follows.

2.5 Algorithm

1. READ
2. FOR (i=1:$i \leqslant n$;)
3. READ $J[i]$, $f[i]$
4. NEXT i
5. FOR (k=1:$k \leqslant n$;) do
6. $L[k]$=1
7. FOR (i=1:$i \leqslant n$;) do
8. If i=k then go to step 10.
9. $L[k]$= $L[k]$ (J-$x[i]$)/($x[k]$-$x[i]$)
10. NEXT i
11. NEXT j
12. f_y = 0
13. FOR (i=1:$i \leqslant n$;)
14. f_y= f_y+ $L[i]$ $f[i]$
15. NEXT i
16. PRINT f_y
17. STOP

2.6 Analysis of error in polynomial interpolation

The error committed in above interpolation is given by,

$R_{n+1} = (x\text{-}J_0)\ (x\text{-}J_1)\ldots\ldots\ldots\ (x\text{-}J_n)\ f^{n+1}(\xi)\ /\ (n+1)!$

where $J_0 < \xi < J_n$

$R_{n+1} = w(x)\ f^{n+1}(\xi)/(n+1)!$

The error will be maximum or minimum at a point in our case at some particular place can be obtained by extremising the above error function according to the rules for extremising a function of one variable.

3 Application

3.1 implementation of the proposed method by an example

We illustrate our problem by an example where we consider a city named as "*J*", we next take four neighboring areas in this city say J_0, J_1, J_2, J_3. We take the following attributes which serves as the parameters to determine the amount of pollution (Faiz et al,, 1996) in air due to motor vehicle.

(1) T_0: Number of vehicles passing this area as recorded by three experts posted in three most busiest area of this location and then taking the average giving more weight age to that expert who records more number of

vehicles and similarly to other experts too, the theory for which has been discussed in the formulation of the problem.

(2) T_1: Surroundings near the road, here we use the concept of fuzzy set. If there is a building close to the road ,top of which subtends an angle of 45^o with the road which is assumed to be straight we assign it "0" membership and if it the top of the building makes an angle of 90^o with the road we assign it a membership "1". Any building subtending angle in between 45^o and 90^o we assign it a membership in between "0" and "1". Now in any place where the experts are posted from practical point of view it is understandable that the surrounding will contain different types of building short (according to our definition the one that subtends 45^o with the road, tall (according tour definition the one that subtends angle of 90^o with road.) and also building with varying sizes. Based on the above assumptions we can categorize the attribute T_1 in following types.

i) Type-I *ii) Type-II* *iii) Type-III*

(3) T_2: Conditions of the road, which is also classified under three types as shown below,

i) Totally broken (TB) *ii) Semi broken (SB)* *iii) Smooth (SM)*

(4) T_3: Availability of open space

i) Type-I (0%-20%) *ii) Type-II* (20%-40%) *iii) Type-III* (40%-60%)

Assuming more than 60% land in a crowded locality in a developing country cannot be open. At the overlapping point i.e. 20 and 40 in the above division the membership is suitably assigned to avoid any erroneous result.

(5) T_4: Condition of the vehicles

i) Type-I (Heavy vehicles) *ii) Type-II (Medium and little vehicles)* *iii) Type-III (All types)*

3.2 Status of pollution as obtained from the surveyor

Status of pollution as obtained from the surveyor is indicated in Table 4.

Table 4 Status of pollution as obtained from the surveyor

	T_0	T_1	T_2	T_3	T_4
J_0	500	*III*	*III*	*II*	*II*
J_1	1100	*II*	*III*	*I*	*II*
J_2	1400	*III*	*II*	*III*	*III*
J_3	800	*III*	*I*	*III*	*III*

3.3 Membership as given by the experts for different attributes and there averages

Membership as given by the experts for different attributes and there averages (Table 5).

Table 5 Membership as given by the experts for different attributes and there averages

Attributes	Types	E_1	E_2	E_3	Average
T_0	500	0.2	0.15	0.3	0.22
	800	0.4	0.2	0.35	0.32
	1100	0.55	0.26	0.42	0.41
	1400	0.7	0.3	0.5	0.5
	1700	0.8	0.4	0.54	0.58
	Weight (w_1)	0.5	0.6	0.4	0.5
T_1	*I*	0.3	0.2	0.4	0.3
	II	0.4	0.25	0.45	0.37

	III	0.45	0.3	0.55	0.43
	Weight (w_2)	0.027	0.015	0.039	0.81
T_2	*I*	0.6	0.7	0.5	0.6
	II	0.5	0.4	0.4	0.433
	III	0.3	0.1	0.1	0.17
	Weight (w_3)	0.033	0.021	0.012	0.1
T_3	*I*	0.4	0.6	0.5	0.5
	II	0.2	0.2	0.3	0.233
	III	0.1	0.1	0.2	0.133
	Weight (w_4)	0.010	0.034	0.022	0.065
T_4	*I*	0.8	0.75	0.6	0.72
	II	0.2	0.1	0.25	0.183
	III	0.4	0.5	0.5	0.47
	Weight (w_5)	0.125	0.083	0.041	0.25

3.4 Pollution matrix (PM)

Table 6 shows the pollution matrix.

Table 6 Pollution matrix

	T_0	T_1	T_2	T_3	T_4
J_0	0.22	0.43	0.17	0.233	0.183
J_1	0.41	0.37	0.17	0.5	0.183
J_2	0.5	0.43	0.433	0.133	0.47
J_3	0.32	0.43	0.6	0.133	0.47

3.5 Weighted pollution matrix (WPM)

Table 7 shows the weighted pollution matrix.

Table 7 Weighted pollution matrix

	T_0	T_1	T_2	T_3	T_4
J_0	0.11	0.348	0.017	0.015	0.0458
J_1	0.205	0.299	0.017	0.0325	0.0458
J_2	0.25	0.348	0.0433	0.0086	0.118
J_3	0.16	0.348	0.06	0.0086	0.118

3.6 Computation of aggregating membership degrees, calculation of pollution index and ranking different places

Results for computation of aggregating membership degrees, calculation of pollution index and ranking different places are indicated in Table 8.

Table 8 Results for computation of aggregating membership degrees, calculation of pollution index and ranking different places

	Sum of membership Degrees	Degree of pollution	Pollution Index	Rank
J_0	0.5358	0.1582	0.347	4
J_1	0.5993	0.1696	0.384	3
J_2	0.7679	0.2057	0.487	1
J_3	0.6946	0.2129	0.454	2

To obtain the polynomial representing the air pollution in between the area discussed above i.e. J_0 to J_3. We set the data's obtained in Table 9.

Table 9 Set data

J:-	0 (J_0)	6 (J_1)	9 (J_2)	15 (J_3)
$y=f(x)$	0.5358	0.5993	0.7679	0.6946

Using the Lagrange's interpolation formula we can write

$$f(x) = (x-J_1)(x-J_2)(x-J_3)/((J_0-J_1)(J_0-J_2)(J_0-J_3))\, f(J_0) + (x-J_0)(x-J_2)(x-J_3)/((J_1-J_0)(J_1-J_2)(J_1-J_3))\, f(J_1)$$
$$+ (x-J_0)(x-J_1)(x-J_3)/((J_2-J_0)(J_2-J_1)(J_2-J_3))\, f(J_2) + (x-J_0)(x-J_1)(x-J_2)/((J_3-J_0)(J_3-J_1)(J_3-J_2))\, f(J_3)$$

After simplification the required final polynomial is as follows,

$$f(J) \approx 0.01003\, x^3 - 0.22275\, x^2 + 1.167\, x - 0.5346.$$

To verify the result we take x=1 and substitute in the above polynomial equation which gives $f(1)$ = 0.41968, which is quite close to the value of $f(0)$ obtained in the table above. The error involved in this can however be obtained from the formula of error already given above.

4 Conclusion

The above proposed method of measuring air pollution through motor vehicles gives us the tools to account for the air pollution caused by those vehicles also which has been certified fit for using. At the same time it deals with some attributes which are real and in general not taken into account for measuring the air pollution made by the motor vehicles by the methods presently available. Finally with the help of a suitable interpolation formula we can obtain a pollution curve which will help the city administrators as well as the policy makers to study different aspect of air pollution in different adjoining areas without employing any experts practically in the place. While obtaining the polynomial curve as well as the level of pollution in any adjoining area where the expert has not been posted, the data used will practically become difficult to handle as we come across membership functions, which lies in between "0" and "1", so we have suggested algorithm which could be effectively used as a computer program and hence will save time and labor as well as increase the efficiency of the proposed method.

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