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An ecological type nonlinear model for the removal of carbon dioxide from the atmosphere by introducing liquid species

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Abstract

The average temperature of our planet is increasing in past several decades due to emission of global warming gases such as CO_2, CH_4, etc. in the atmosphere leading to undesirable environmental consequences. Therefore, it is necessary to find a mechanism by which a global warming gas can be removed from the regional atmosphere. In this paper, therefore, we proposed an ecological type nonlinear mathematical model for the removal of a global warming gas CO_2 from the regional atmosphere by externally introduced liquid species, which may react with this gas and removed it by gravity. The model consists of three dependent variables namely; the concentration of carbon dioxide, the concentration of externally introduced liquid species and the concentration of particulate matters (secondary species) formed due to interaction of carbon dioxide with liquid species. The local and global stability conditions are discussed using Routh-Hurwitz criteria and suitable Lyapunove's function respectively. It is shown, analytically and numerically, that the concentration of carbon dioxide decreases as the rate of introduction of externally introduced liquid species increases.

Keywords carbon dioxide; liquid species; particulate matters; stability; mathematical model.

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1 Introduction

During past several years scientists and ecologists have found that the temperature of the environment is slowly increasing due to emission of global warming gases such as CO_2, CH_4, etc., leading adverse effects on human being and its environment. Carbon dioxide is the main contributor to the global warming (Zhang and Liu, 2012). Global warming may lead several undesirable consequences such as, poor air quality, rise in see levels, melting of glaciers, decrease in rainfall, draught, heat waves, effect on human health, etc. (Cazenave, 2006; IPCC, 2007; Moench, 2007; Robinson et al., 2007; Ramanathan and Feng, 2009; Zhang and Wang, 2011; Gao et al., 2012; Wu and Zhang, 2012; Wu et al., 2012; Zhang and Zhang, 2012). According to fourth assessment report (AR4) on climate change in 2007, the Intergovernmental Panel on Climate Change (IPCC), have observed the rise in global temperature by 1.3^0F over the past centaury and this rise in temperature may 2^0F to 11.5^0F by the year 2100. Zhang and Wang (2011) have shown that solar radiation in very broad spectral wavebands can be absorbed by Black carbon (BC) aerosols leading a significant change in the earthatmosphere system with considerable affect on global or regional climate.

Some experiments have been performed to improve ambient air quality and to control global warming gas (Lackner, 2009; Jiang, 2011; Wang and Hao, 2012; Chadwick et al., 2013). Lackner (2009) have performed an experiment to capture carbon dioxide from ambient air using carbon dioxide capture devices and found considerable decrease in its concentration. Chadwick et al. (2013) described two idealized general circulation model experiments where CO_2 concentration is steadily increased then decreased along the same path.

In recent years some modeling studies have been conducted for the removal of gaseous pollutant and particulate matters from the atmosphere of a city by precipitation but this concept has not been used for the removal of global warming gases from the atmosphere (Pandis and Seinfeld, 1990; Shukla et al., 2008; Sundar et al., 2009; Sundar and Naresh, 2012; Shukla et al., 2013). For example, Sundar et al. (2009) proposed a nonlinear mathematical model to study the interactions of hot gases with cloud droplets as well as with raindrops and their removal by rain from the atmosphere. They have shown that the cumulative concentration of gaseous pollutant decreases as the growth rate of raindrops increases. Sundar and Naresh (2012) have proposed and analyzed a nonlinear mathematical model to study the role of vapours and cloud droplets on removal of primary pollutants forming secondary species from the atmosphere and found considerable decrease in the concentration of the pollutants.

In view of the above, therefore, in this paper, an ecological type nonlinear mathematical model is proposed and analyzed for the removal of a global warming gas CO_2 from the regional atmosphere by externally introduced liquid species. As global warming gas interacts with externally introduced liquid droplets, secondary species (such as particulate matters) is formed and removed from the atmosphere due to gravity reducing the concentration of global warming gas CO_2 from the regional atmosphere.

2 Mathematical Model

Let $C(t)$ be the cumulative concentration of a global warming gas CO_2, $A(t)$ be the concentration externally introduced liquid species used for the removal of global warming gas CO_2, and $C_p(t)$ be the concentration of particulate matters formed due to interaction of global warming gase with liquid species. The following assumptions have been made in the modeling process,

1. The rate of discharge of CO_2 from different sources such as chimneys of power plants, industries, etc. is assumed to be constant.
2. The rate of introduction of liquid species is directly proportional to the concentration of global warming gas CO_2.
3. The decrease in the concentration of global warming gas is assumed to be in the direct proportion of cumulative concentration of this global warming gas as well as the concentration of liquid species.
4. Secondary species (such as particulate matter) is formed due to interaction of global warming gas and externally introduced liquid species and removed due to gravity.

In view of the above, let Q is the rate of introduction of global warming gas CO_2 into the regional atmosphere. When global warming gas interacts with externally introduced liquid species, secondary species (particulate matters) is formed and get removed by gravity. The removal of CO_2 is assumed to be directly proportional to the cumulative concentration of CO_2 as well as the concentration of liquid species (i.e. δCA), δ is the interaction rate coefficient. The rate of introduction of liquid species is assumed to be directly proportional to the concentrations of global warming gas CO_2 (i.e. λC). The constant λ_1 is the depletion rate coefficient of liquid species due to interaction with global warming gas. The natural depletion rates of global warming gas as well as externally introduced liquid species are assumed to be directly proportional to their respective concentrations. The constants δ_0 and λ_0 are natural depletion rate coefficients of global warming gas and liquid species respectively. Let θ be the rate by which particulate matter is formed and θ_0 is it's natural depletion rate coefficient. All the constant taken here are positive.

Thus, the dynamics of the system is governed by the following three-dimensional nonlinear mathematical model,

$$\frac{dC(t)}{dt} = Q - \delta_0 C(t) - \delta C(t)A(t) \quad (1)$$

$$\frac{dA(t)}{dt} = \lambda C(t) - \lambda_0 A(t) - \lambda_1 C(t)A(t) \quad (2)$$

$$\frac{dC_P(t)}{dt} = \theta\,\delta\, C(t)A(t) - \theta_0 C_P(t) \quad (3)$$

with $C(0) \geq 0$, $A(0) \geq 0$, $C_p(0) \geq 0$.

It is noted here that, for very large value of δ, $\frac{dC}{dt}$ may become negative. This implies that, if the interaction rate coefficient δ is very large, the global warming gas CO_2 would be removed almost completely from the regional atmosphere.

Lemma 2.1 The region of attraction (Freedman and So, 1985) for all solution of model system (1) – (3) initiating in the positive octant is given by the set,

$$\Omega = \left\{ (C, A, C_P) \in R_+^3 : 0 \leq C \leq \frac{Q}{\delta_0}, 0 \leq A \leq \frac{\lambda}{\lambda_0}\frac{Q}{\delta_0},\ 0 \leq C_P \leq \frac{\theta\delta}{\theta_0}\frac{\lambda}{\lambda_0}\left(\frac{Q}{\delta_0}\right)^2 \right\} \quad (4)$$

3 Equilibrium and Stability Analysis

The model system (1) – (3) has only one non-trivial equilibrium $E^*(C^*, A^*, C_p{}^*)$. In the following, we prove the existence of $E^*(C^*, A^*, C_p{}^*)$, where, C^*, A^* and $C_P{}^*$ are positive solutions of the following system of algebraic equations,

$$Q - \delta_0 C - \delta CA = 0 \quad (5)$$

$$A = \frac{\lambda C}{\lambda_0 + \lambda_1 C} = f(C) \quad (6)$$

$$C_P = \frac{\theta\delta CA}{\theta_0} \quad (7)$$

From equations (5), let

$$F(C) = Q - \delta_0 C - \delta C f(C) = 0 \quad (8)$$

From equation (8), we note that,

$$F(0) = Q > 0$$

$$F\left(\frac{Q}{\delta_0}\right) = -\delta\left(\frac{Q}{\delta_0}\right) f\left(\frac{Q}{\delta_0}\right) < 0$$

This implies that there exist a root of $F(C) = 0$ (say C^*) in $0 \le C \le \frac{Q}{\delta_0}$.

For the uniqueness of the root, we have,

$$F'(C) = -\delta_0 - \delta C f'(C) - \delta f(C) < 0$$

where $f'(C) = \frac{\lambda\lambda_0}{(\lambda_0 + \lambda_1 C)^2} > 0$

This guarantees the uniqueness of the root in $0 \le C \le \frac{Q}{\delta_0}$.

In the following, we check the variations of dependent variables with respect to relevant parameters, From equations (5) and (6), we have

$$(\lambda_1\delta_0 + \lambda\delta)C^2 - (Q\lambda_1 - \delta_0\lambda_0)C - Q\lambda_0 = 0 \tag{9}$$

3.1 Variation of C with Q

Differentiating (9) with respect to 'Q', we get,

$$\frac{dC}{dQ} = \frac{C(\lambda_1 C + \lambda_0)}{Q\lambda_0 + (\lambda_1\delta_0 + \lambda\delta)C^2} > 0$$

This implies that C increases as Q increases.

3.2 Variation of A with Q

Differentiating (6) with respect to 'C', we get,

$$\frac{dA}{dC} = \frac{\lambda\lambda_0}{(\lambda_0 + \lambda_1 C)^2} > 0$$

Since $\frac{dC}{dQ} > 0$, therefore, from $\frac{dA}{dQ} = \frac{dA}{dC}\frac{dC}{dQ}$ we have $\frac{dA}{dQ} > 0$

This implies that, if the rate of emission of global warming gas CO_2 increases, the requirement of externally introduced liquid species also increases.

3.3 Variation of C with λ

Differentiating (9) with respect to 'λ', we get,

$$\frac{dC}{d\lambda} = -\frac{\lambda_1 C^3}{Q\lambda_0 + (\lambda_1\delta_0 + \lambda\delta)C^2} < 0$$

This implies that the concentration of carbon dioxide decreases as the rate of introduction of external liquid species increases.

Similarly, we can show that $\frac{dA}{d\lambda} > 0$, $\frac{dC_P}{d\lambda} > 0$ and $\frac{dC_P}{d\theta_0} < 0$

From the above analysis, it is shown that, if the rate of emission of global warming gas CO_2 increases in the regional atmosphere, more amount of liquid species is required for its removal. Further, as the rate of externally introduced liquid species increases, the concentration of carbon dioxide decreases.

To study the stability behavior of $E^*(C^*, A^*, C_p^*)$, we state the theorems for local and global stability of the non-trivial equilibrium of model (1) – (3).

Theorem 1 The nontrivial equilibrium $E^*(C^*, A^*, C_p^*)$ is locally asymptotically stable, without any condition (see appendix A for proof).

Theorem 2 The nontrivial equilibrium $E^*(C^*, A^*, C_p^*)$ is globally asymptotically stable without any condition with respect to all solutions initiating in the interior of the positive octant (see appendix B for proof).

The above theorems imply that the concentration of global warming gas can be controlled by introducing liquid species in the regional atmosphere.

4 Numerical Simulation and Discussion

In this section, we analyze the model (1) – (3) numerically with respect to $E^*(C^*, A^*, C_P^*)$ for the validation of analytical results and to study the behavior of the model system. The model system (1) – (3) is integrated numerically with the help of MAPLE 7 by considering the following set of parameter values,

$$Q = 1, \delta_0 = 0.2, \delta = 0.6, \lambda = 0.4, \lambda_1 = 0.3, \lambda_0 = 0.1, \theta = 0.8, \theta_0 = 0.7$$

The equilibrium E^* is calculated as,

$$C^* = 1.209035,\ A^* = 1.045176,\ C_p^* = 0.866506$$

The eigenvalues of the variational matrix corresponding to the equilibrium E^* are -0.70, $-0.644908 + 0.171798i$ and $-0.644908 - 0.171798i$. Since, all the eigenvalues are either negative or have negative real part. Therefore, the equilibrium E^* is locally asymptotically stable.

The global stability behavior in $C - A$ and $C - C_p$ plane has been shown in Fig. 1 and 2 respectively with following different initial starts. From these figures, we note that all the trajectories started at any point always reaches to its equilibrium.

1. $C(0) = 0.2,\ A(0) = 0.8,\ C_p(0) = 0.9$
2. $C(0) = 0.5,\ A(0) = 0.1,\ C_p(0) = 0.2$
3. $C(0) = 2.4,\ A(0) = 0.9,\ C_p(0) = 0.8$
4. $C(0) = 2.4,\ A(0) = 1.4,\ C_p(0) = 1.2$

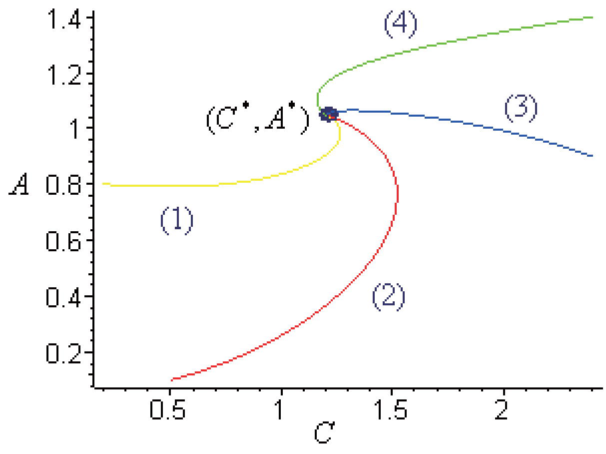

Fig.1 Nonlinear stability in $C - A$ plane

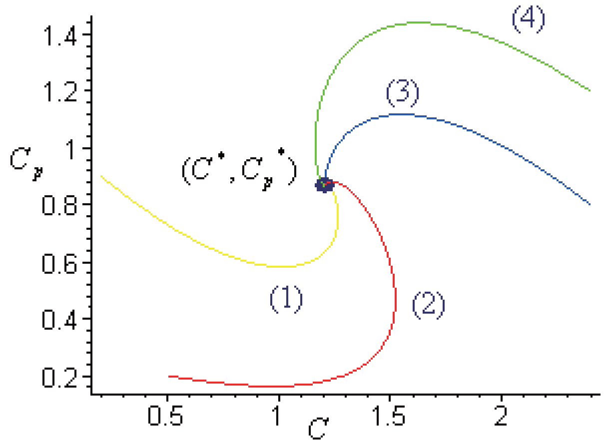

Fig. 2 Nonlinear stability in $C - C_p$ plane

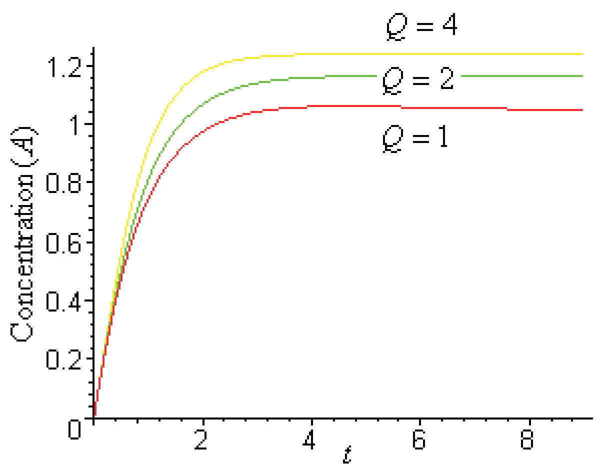

Fig. 3 Variation of A with 't' for different values of Q

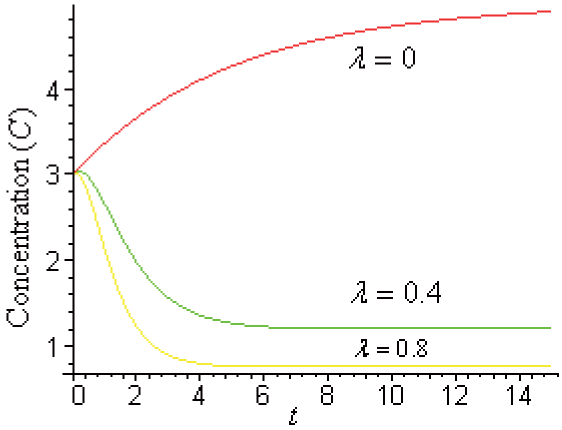

Fig. 4 Variation of C with '*t*' for different values of λ

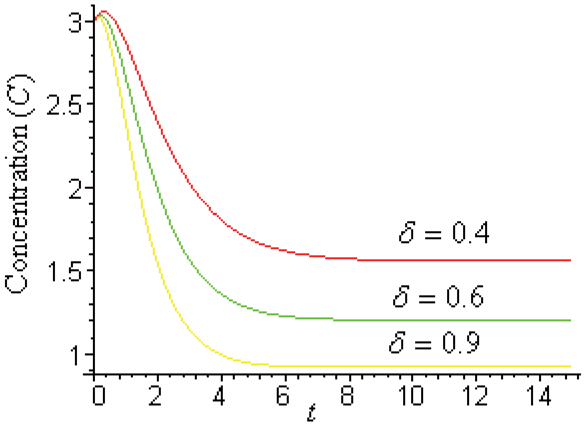

Fig. 5 Variation of C with '*t*' for different values of δ

The variation of concentration of externally introduced liquid species (A) with time $'t'$ for different values of rate of emission of carbon dioxide i.e. at $Q = 1, 2, 4$ is shown in figures 3. From this figure, it is seen that as the rate of emission of carbon dioxide increases, the requirement of the externally introduced liquid species increases. It has been found that the introduction of external liquid species and it's interaction with CO_2 has considerable effect of equilibrium concentration of CO_2 in the regional atmosphere. These results are presented in figures 4 – 6. The variation of concentration of carbon dioxide with time $'t'$ for different values of rate of introduction of liquid species i.e. at $\lambda = 0, 0.4, 0.8$ respectively is shown in figure 4. From this figure, we note that at $\lambda = 0$, when there is no introduction of liquid species in the regional atmosphere, the concentration of global warming gas CO_2 continue to increase and reaches to its maximum equilibrium. But the concentration of CO_2 decreases as the rate of introduction of liquid species increases. In figure 5, the variation of concentration of carbon dioxide with time $'t'$ is shown for different values of δ i.e. at $\delta = 0.4,\ 0.6,\ 0.9$. From figure 5, we depict that the concentration of global warming gas CO_2 decreases as the rate of interaction of it with externally introduced liquid species increases.

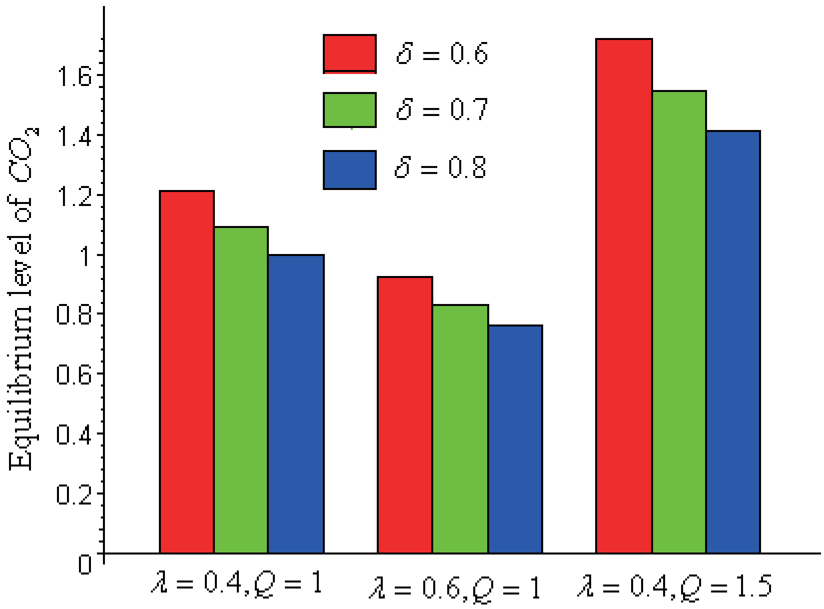

Fig. 6 Effect of increasing interaction rate coefficient on equilibrium of CO_2 at different values of λ and Q

Fig. 6 illustrates that equilibrium concentration of carbon dioxide decreases as the interaction rate coefficient increases (at $\lambda = 0.4$, $Q = 1$) and it further decreases as the rate of introduction of external species increases when other parameters are same. If rate of emission of carbon dioxide i.e. Q is taken to be 1.5, then for same value of $\lambda (= 0.4)$, an increase in equilibrium concentration of CO_2 is obtained. This implies that, if the discharge rate of CO_2 into the regional atmosphere increases, external liquid species is used with enhanced rate of introduction to reduce the concentration of CO_2.

5 Conclusion

In this paper, we have modeled the phenomenon of removal of a global warming gas CO_2 by externally introduced liquid species. It is assumed that the model system consists of three nonlinearly dependent variables namely, the concentration of global warming gas CO_2, the concentration externally introduced liquid species and the concentration of particulate matters formed due to interaction of carbon dioxide with liquid species. The proposed model is analyzed using stability theory of ordinary differential equations and numerical simulations. It is shown, analytically and numerically, that the concentration of global warming gas CO_2 decreases as the concentration of externally introduced liquid species increases and if the rate of introduction of liquid species is very high, the carbon dioxide would be removed completely from the atmosphere. It has also been shown that the concentration of CO_2 decreases as the interaction rate coefficient of CO_2 with liquid species increases. The analyses suggest that the equilibrium level of global warming gas can be reduced from the atmosphere by introducing suitable external species.

Appendix A

Proof of the theorem 1

To establish the local stability of $E^*(C^*, A^*, C_p{}^*)$, we compute the eigenvalues of the following Jacobian matrix,

$$J(E^*) = \begin{bmatrix} -(\delta_0 + \delta A^*) & -\delta C^* & 0 \\ (\lambda - \lambda_1 A^*) & -(\lambda_0 + \lambda_1 C^*) & 0 \\ \theta \delta A^* & \theta \delta C^* & -\theta_0 \end{bmatrix}$$

One eigenvalue is of $J(E^*)$ is $-\theta_0$, which is negative and the remaining two are given by the following characteristic equation,

$$F(x) = x^2 + ax + b = 0 \qquad \text{(A1)}$$

Here, $a = \delta_0 + \delta A^* + \lambda_0 + \lambda_1 C^* > 0$

$$b = (\delta_0 + \delta A^*)(\lambda_0 + \lambda_1 A^*) + \delta C^* (\lambda - \lambda_1 A^*) > 0$$

Since $\lambda - \lambda_1 A^* > 0$, it can easily be seen that, $a > 0,\ b > 0$. Hence, from Routh-Hurwitz criteria, the roots of (A1) are either negative or have negative real part. Thus, all the eigenvalues of the variational matrix are either negative or have negative real part. Hence, $E^*(C^*, A^*, C_p{}^*)$ is locally asymptotically stable without any condition.

Appendix B

Proof of the theorem 2

To prove this theorem, we consider the following positive definite function

$$V = \frac{1}{2} m_1 (C - C^*)^2 + \frac{1}{2} m_2 (A - A^*)^2 + \frac{1}{2} m_3 (C_P - C_P{}^*)^2 \qquad \text{(B1)}$$

where $m_i\ (i = 1,\ 2,\ 3)$ are positive constants, to be chosen appropriately.

Differentiating (B1) with respect to *t*, we get

$$\dot{V} = m_1 (C - C^*)\dot{C} + m_2 (A - A^*)\dot{A} + m_3 (C_P - C_P{}^*)\dot{C}_p$$

Putting the values of derivatives and simplifying, we get,

$$\frac{dV}{dt} = -m_1(\delta_0 + \delta A)(C - C^*)^2 - m_2(\lambda_0 + \lambda_1 C)(A - A^*)^2 - m_3\theta_0 (C_P - C_P{}^*)^2$$

$$+ [-m_1 \delta C^* + m_2 (\lambda - \lambda_1 A^*)](C - C^*)(A - A^*)$$

$$+ m_3 \theta \delta A^* (C - C^*)(C_P - C_P{}^*) + m_3 \theta \delta C (A - A^*)(C_P - C_P{}^*)$$

After some algebraic manipulations and by choosing $m_1 = 1$, $m_2 = \dfrac{\delta C^*}{\lambda - \lambda_1 A^*}$, $\lambda - \lambda_1 A^* > 0$ and

$$m_3 < \frac{\theta_0}{\theta^2} \min \left\{ \frac{\delta_0}{(\delta A^*)^2},\ \frac{\lambda_0 \delta C^*}{\left(\dfrac{\delta Q}{\delta_0}\right)^2 (\lambda - \lambda_1 A^*)} \right\}$$

$\dot{V}$ will be negative definite without any condition inside the region of attraction Ω and hence E^* is globally asymptotically stable without any condition.

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