Article

The role of aerosols to increase rainfall in the regions with less intensity rain: A modeling study

Shyam Sundar, Rajan K. Sharma

Department of Mathematics, P.S. Institute of Technology, Kalpi Road, Bhauti, Kanpur-208020, India

E-mail: ssmishra15@gmail.com

Received 28 September 2012; *Accepted 2 November 2012*; *Published online 1 March 2013*

IAEES

Abstract

In this paper, we established an ecological type three-dimensional nonlinear mathematical model to study the effect of aerosol particles in increasing rainfall in the regions of less intensity rain. The phenomenon of nonlinearity is based on the concepts of ecology related to growth rate, death rate and interaction process (Smith, 1974). It is assumed that clouds are formed in the atmosphere but are not able to develop uninterrupted rainfall. The rainfall can be enhanced by introducing aerosol particles conducive to raindrops formation from cloud droplets. It is shown that the intensity of rainfall increases as the concentration of externally introduced aerosols and the density of cloud droplets increases. The numerical simulation has also been performed to support analytical results.

Keywords mathematical model; cloud droplets; rainfall; conducive aerosol; simulation.

1 Introduction

Climate change and global warming (Zhang and Liu, 2012) have adverse effect on monsoon causing decrease in rainfall in several regions in India and elsewhere. It is noted here that, in regions of less intensity rain, there are clouds in the atmosphere but are not able to produce sufficient rain which has been an important source for, power plants to generate electricity, drinking water, irrigation, atmospheric contaminants removal, etc (Sayadi and Sayyed, 2011; Sayyed and Wagh, 2011). In the regions of less intensity rain, the rainfall can be enhanced by introducing conducive aerosol particles into the atmosphere. For this, we can use small aircrafts to spray aerosols, conducive to rainfall, such as dry ice, common salt and silver iodide particles into the upper part of clouds to stimulate the condensation process leading to rainfall (Xinhua News Agency, 2006; Manjari Mishra, 2008; Zee News, 2009; Michael, 2011). In this regard, China generated artificial heavy rainfall to relieve drought and remove air pollutants (Xinhua News Agency, 2006). Scientists of Indian Institute of Tropical Meteorology (IITM), Pune, set a three phase project and started the experiments to create artificial rain with a national cloud seeding programme (Zee News, 2009).

Some studies have been conducted related to clouds and precipitation (Saxena and Grovenstein, 1994; Flossmann, 1998; Hegg, 2001; Shukla et al., 2008, 2010; Pruppacher and Klett, 2010). For example, an investigation has been made to study the enhancement of cloud condensation nuclei (CCN) by assuming three possible mechanisms namely, cloud formation from a mixture of air masses, sulfate production within clouds themselves and formations of new aerosol particles by the process of homogeneous, hetero molecular nucleation (Saxena and Grovenstein, 1994). A nonlinear mathematical model has been developed and analyzed to study the effect of cloud density on the removal of gaseous pollutants and particulate matters from the atmosphere of a city by rain (Shukla et al., 2008). The artificial rain production by introducing two kinds of aerosols conducive to cloud condensation and raindrops formation has been studied using a nonlinear mathematical model (Shukla et al., 2010). In the modeling process, they assumed that clouds are formed due to the interaction of first kind of aerosol with water vapour.

In our study, we assume that clouds are formed naturally at a constant rate leading to less intensity rain. The main purpose of this work is to study the enhancement of rainfall from cloud droplets by introducing aerosols conducive to rain formation using mathematical model.

Therefore, in this paper, we propose and analyze a mathematical model to study the feasibility of enhancement of rainfall by introducing aerosol particles in the atmosphere. We assume that the atmosphere, under consideration, consists of three nonlinearly interacting phases namely, cloud droplets phase, raindrops phase and the phase of externally introduced aerosols conducive to rainfall.

2 Mathematical Model

To model the phenomena, we assume that the cloud droplets are naturally formed in the atmosphere at a constant rate. The formation of raindrops takes place due to interaction of conducive aerosols with cloud droplets. The aerosol is introduced in the atmosphere with a constant rate.

Let $C_d(t)$ and $C_r(t)$ be the densities of cloud droplets and raindrops in the atmosphere respectively. Let $C(t)$ be the concentration of externally introduced aerosol particles conducive to rain formation. It is assumed that the cloud droplets are formed in the atmosphere at a constant rate q. The natural depletion of cloud droplets and raindrops is assumed to in the direct proportion of their respective densities. The rate of introduction of aerosol conducive to rainfall is Q with natural depletion $\delta_0 C$. It is assumed that, in the absence of aerosol particles with concentration C, the growth rate of raindrops is proportional to number density of cloud droplets (i.e. λC_d). In the presence of aerosol particles, the rate of decrease of density of cloud droplets is assumed to be in the direct proportion of the density of cloud droplets as well as the concentration of conducive aerosol particles i.e. $\lambda_1 C_d C$. The number density of raindrops is assumed to be enhanced with the same amount. The depletion of aerosol due to interaction with cloud droplets is assumed to be in the direct proportion of the concentration of aerosol as well as the number density of cloud droplets (i.e., $\delta_1 C_d C$). All the constants considered here are taken to be positive.

In view of the above, the dynamics of the system is governed by the following system of nonlinear differential equations,

$$\frac{dC_d}{dt} = q - \lambda_0 C_d - \lambda_1 C_d C \tag{1}$$

$$\frac{dC}{dt} = Q - \delta_0 C - \delta_1 C_d C \tag{2}$$

$$\frac{dC_r}{dt} = \lambda C_d + \lambda_1 C_d C - r_0 C_r \tag{3}$$

with $C_d(0) \geq 0, C(0) \geq 0, C_r(0) \geq 0$

The positive constants λ_0, r_0 and δ_0 represent the natural depletion rate coefficients of cloud droplets, raindrops and aerosols respectively.

To describe the bounds of dependent variables, we state the region of attraction of the model (1) – (3) in the following lemma (without proof),

Lemma 2.1 The set

$$\Omega=\left\{(C_d, C, C_r): 0 \le C_d \le \frac{q}{\lambda_0},\ 0 \le C \le \frac{Q}{\delta_0}, 0 \le C_r \le \frac{q}{r_0\lambda_0}\left(\lambda+\frac{\lambda_1 Q}{\delta_0}\right)\right\} \tag{4}$$

attracts all solutions initiating in the interior of the positive octant.

3 Stability Analysis

The model (1) – (3) has only one equilibrium namely $E^*(C_d{}^*, C^*, C_r{}^*)$, where $C_d{}^*, C^*$ and $C_r{}^*$ are positive solutions of the following equations,

$$q-\lambda_0 C_d-\lambda_1 C_d C=0 \tag{5}$$

$$C=\frac{Q}{(\delta_0+\delta_1 C_d)} \tag{6}$$

$$C_r=\frac{\lambda C_d+\lambda_1 C_d C}{r_0} \tag{7}$$

Putting the value of C from (6) in (5), we get

$$\lambda_0\delta_1 C_d{}^2+(\lambda_0\delta_0+\lambda_1 Q-\delta_1 q)C_d-q\,\delta_0=0$$

It gives,

$$C_d=\frac{-(\lambda_0\delta_0+\lambda_1 Q-\delta_1 q)+\sqrt{(\lambda_0\delta_0+\lambda_1 Q-\delta_1 q)^2+4\lambda_0\delta_1 q\,\delta_0}}{2\lambda_0\delta_1}=C_d{}^*\ \text{(let)} \tag{8}$$

From equation (8), we note that, when $Q=0,\ \delta_1=0$; $C_d=\dfrac{q}{\lambda_0}$, therefore we consider plus sign in equation (8) before the root symbol.

Hence, there exists a unique root $C_d{}^*$ in $0 \le C_d \le \dfrac{q}{\lambda_0}$ without any condition, which is always positive.

Using the value of $C_d{}^*$ we can calculate C^* and $C_r{}^*$ from equations (6) and (7) respectively.

From equation (7), we note that the number density of raindrops increases as the concentration of externally introduced aerosol (C) increases, showing the importance of aerosols to enhance rainfall.

In the following, we check the characteristics of various phases with respect to relevant parameters analytically.

Variation of C_d with Q

From equation (8), we note that,

$$\frac{dC_d}{dQ} = -\frac{\lambda_1}{2\lambda_0\delta_1}\left(1 - \frac{(\lambda_0\delta_0 + \lambda_1 Q - \delta_1 q)}{\sqrt{(\lambda_0\delta_0 + \lambda_1 Q - \delta_1 q)^2 + 4\lambda_0\delta_1 q\,\delta_0}}\right) < 0$$

Thus, C_d decreases as Q increases.

Variation of C_r with Q

From equations (5) – (8), we note that, $\frac{dC}{dQ} > 0$ and $\frac{dC_r}{dC} > 0$, this implies, $\frac{dC_r}{dQ} > 0$.

Thus, C_r increases as Q increases. It means that, the rainfall increases as the rate of introduction of aerosol in the atmosphere increases. It shows the importance of introduction of aerosols in increasing rainfall.

Variation of C_r with q

Again from equations (5) – (8), we note that, $\frac{dC_d}{dq} > 0$ and $\frac{dC_r}{dC_d} > 0$, this implies, $\frac{dC_r}{dq} > 0$. Thus, C_r increases as q increases i.e. rainfall increases as the rate of formation of cloud droplets in the atmosphere increases.

To see the stability behavior of $E^*(C_d^{\,*}, C^*, C_r^{\,*})$, we state the following theorems.

Theorem 1 The equilibrium $E^*(C_d^{\,*}, C^*, C_r^{\,*})$ is locally stable without any condition, (See appendix A for proof).

Theorem 2 The equilibrium $E^*(C_d^{\,*}, C^*, C_r^{\,*})$ is nonlinearly stable inside the region of attraction Ω, if the following condition is satisfied, (See appendix B for proof),

$$\left(\lambda_1 \frac{q}{\lambda_0} + \delta_1 \frac{Q}{\delta_0}\right)^2 < (\lambda_0 + \lambda_1 C^*)(\delta_0 + \delta_1 C_d^{\,*}) \tag{9}$$

The above theorems imply that under certain conditions, the rainfall increases as the number density of cloud droplets and the concentration of externally introduced aerosols increases.

Remark

If $Q = 0$ and $\lambda_1 = 0$, then the condition (9) satisfied automatically. This implies that the interaction of cloud droplets with aerosols is destabilizing in nature.

4 Numerical Simulation

In this section, we conduct numerical simulation of the model system to see the effect of various removal parameters on the system dynamics. For this, the system (1) – (3) is integrated numerically using software MAPLE 7 by considering the following set of parameter values,

$$q = 1, \lambda = 0.01, \lambda_0 = 0.85, \lambda_1 = 0.6, r_0 = 0.15, Q = 0.6, \delta_0 = 0.8, \delta_1 = 0.75$$

The equilibrium $E^*(C_d{}^*, C^*, C_r{}^*)$ is calculated as,

$$C_d{}^* = 0.9156,\ C^* = 0.4035,\ C_r{}^* = 1.5391$$

Eigen values corresponding to E^* are obtained as,

$$-0.15, -1.8189, -0.6633$$

Since all the eigenvalues corresponding to E^* are negative, therefore E^* is locally asymptotically stable. The global stability behavior of E^* in $C_d - C$ plane is shown in the figure1. It has also been checked that the global stability condition is satisfied by the set of parameter values. In figure 2, the variation of density of raindrops with time $'t'$ is shown for different values of rate of formation of cloud droplets (i.e., at $q = 0, 1, 2$). From this figure, it is shown that at $q = 0$, when there is no cloud formation, the equilibrium value of raindrops tend to zero. Further, the density of raindrops increases as the rate of formation of cloud droplets increases. In figures 3-4, the variation of densities of cloud droplets and raindrops with time $'t'$ is shown for different values of rate of introductions of conducive aerosol (i.e., at $Q = 0, 0.6, 1.2$). From these figures, it is shown that at $Q = 0$, the number density of raindrops tend to its minimum equilibrium and this equilibrium is due to condensation of clouds naturally leading to less intensity rain. Further, the density of raindrops increases as the rate of introduction of conducive aerosol increases.

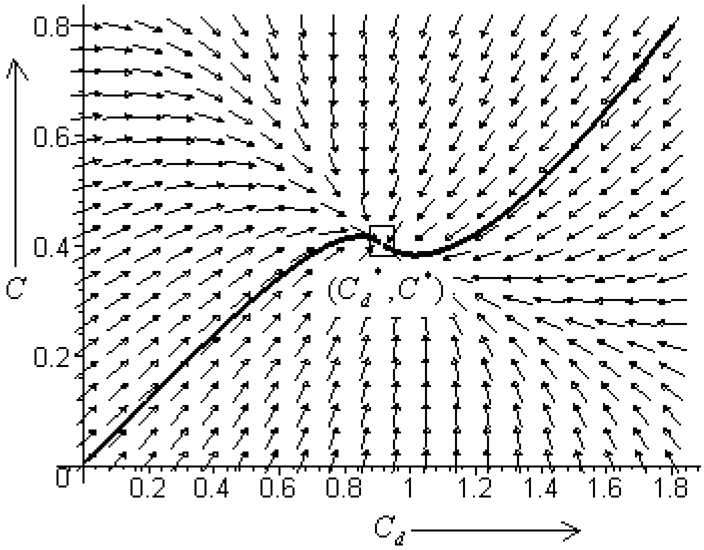

Fig. 1 Global stability in $C_d - C$ plane

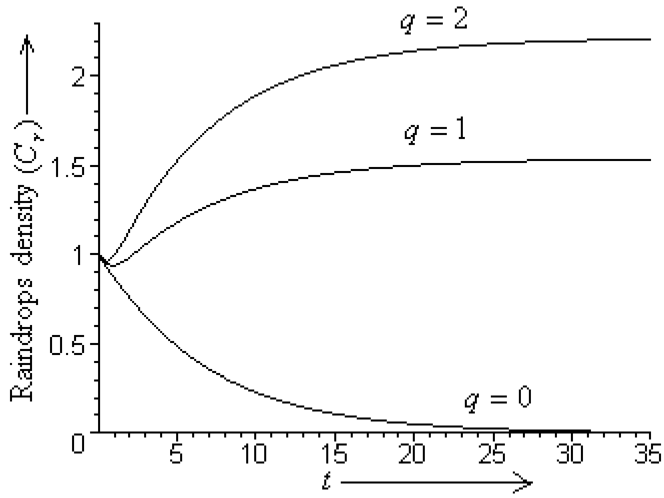

Fig. 2 Variation of C_r with time 't' for different values of q

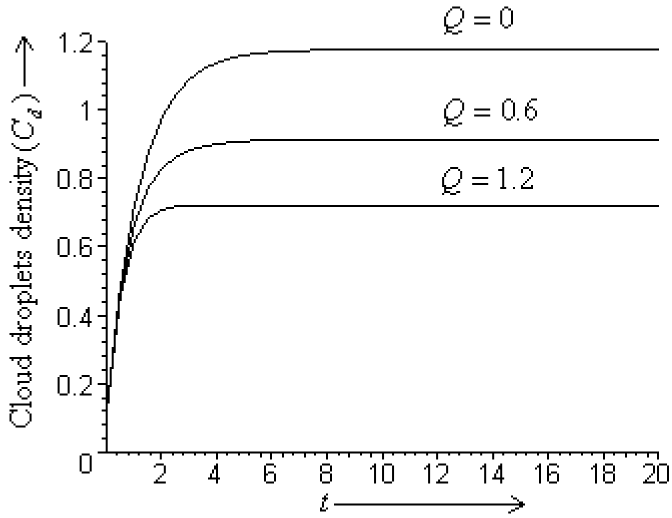

Fig. 3 Variation of C_d with time 't' for different values of Q

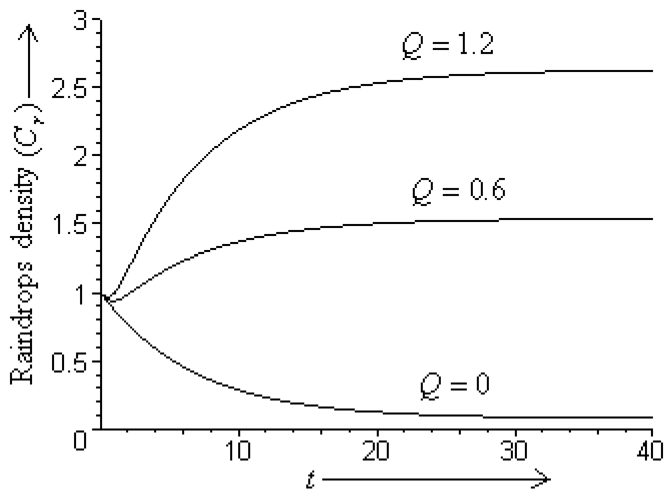

Fig. 4 Variation of C_r with time 't' for different values of Q

5 Conclusion

An ecological type nonlinear mathematical model for enhancement of rainfall, in the regions of less intensity rain, from cloud droplets naturally formed in the atmosphere, is established by taking in to account aerosol particles conducive to rainfall. The proposed model is analyzed using stability theory of differential equations. The model analysis, supported by numerical simulation, shows that the cloud droplets must be formed continuously in the atmosphere for uninterrupted rainfall. It has been shown, analytically and numerically, that rainfall increases as the density of cloud droplets and the concentration of conducive aerosol increase.

Appendix A

Proof of the theorem 1

To establish the local stability of $E^*(C_d^*, C^*, C_r^*)$, we compute the eigenvalues of the following variational matrix,

$$J(E^*) = \begin{bmatrix} -(\lambda_0 + \lambda_1 C^*) & -\lambda_1 C_d^* & 0 \\ -\delta_1 C^* & -(\delta_0 + \delta_1 C_d^*) & 0 \\ \lambda + \lambda_1 C^* & \lambda_1 C_d^* & -r_0 \end{bmatrix}$$

One eigenvalue of $J(E^*)$ is $-r_0$, which are negative. The remaining two eigenvalues are given by the following characteristic equation,

$$\mu^2 + A\mu + B = 0 \tag{A1}$$

Here, $A = (\lambda_0 + \lambda_1 C^*) + (\delta_0 + \delta_1 C_d^*) > 0$ and $B = \lambda_0\delta_0 + \lambda_0\delta_1 C_d^* + \lambda_1\delta_0 C^* > 0$

Since, $A > 0$ and $B > 0$, hence both the roots of (A1) are either negative or have negative real part. Thus, all the eigenvalues of the variational matrix are either negative or have negative real parts. Hence, $E^*(C_d^*, C^*, C_r^*)$ is locally asymptotically stable.

Appendix B

Proof of the theorem 2

Using the following positive definite function,

$$U = \frac{1}{2}[m_1(C_d - C_d^*)^2 + m_2(C - C^*)^2 + m_3(C_r - C_r^*)^2] \tag{B1}$$

Differentiating with respect to'*t*' and using the linearized version of model (1) – (3), we get,

$$\dot{U} = -m_1(\lambda_0 + \lambda_1 C^*)(C_d - C_d^*)^2 - m_2(\delta_0 + \delta_1 C_d^*)(C - C^*)^2 - m_3 r_0 (C_r - C_r^*)^2$$

$$-(m_1\lambda_1 C_d + m_2\delta_1 C)(C_d - C_d^*)(C - C^*) - m_3(\lambda + \lambda_1 C)(C_d - C_d^*)(C_r - C_r^*)$$

$$+ m_3\lambda_1 C_d^*(C_r - C_r^*)(C - C^*)$$

Now $\dot{U}$ will be negative definite under the following conditions,

$$(m_1\lambda_1 C_d + m_2\delta_1 C)^2 < m_1 m_2(\lambda_0 + \lambda_1 C^*)(\delta_0 + \delta_1 C_d^*) \tag{B2}$$

$$m_3(\lambda + \lambda_1 C)^2 < m_1 r_0(\lambda_0 + \lambda_1 C^*) \tag{B3}$$

$$m_3(\lambda_1 C_d^{*})^2 < m_2 r_0(\delta_0 + \delta_1 C_d^{*}) \tag{B4}$$

Maximizing LHS, minimizing RHS, choosing $m_1 = m_2 = 1$ and

$$m_3 < r_0 \min\left\{ \frac{(\lambda_0 + \lambda_1 C^*)}{\left(\lambda + \lambda_1 \dfrac{Q}{\delta_0}\right)^2}, \frac{(\delta_0 + \delta_1 C_d^{*})}{(\lambda_1 C_d^{*})^2} \right\}$$

$\dot{U}$ will be negative definite provided the condition (9) is satisfied inside the region of attraction Ω. It shows that U is a Liapunov's function and hence the equilibrium $E^*(C_d^{*}, C^*, C_r^{*})$ is globally stable.

References

Flossmann AI. 1998. Clouds and Pollution. Pure and Applied Chemistry, 70(7): 1345-1352

Hegg DA. 2001. The impact of clouds on aerosol populations. IGActivities, 23

Manjari Mishra. 2008. Now, artificial rain for Bundelkhand. The Times of India, May 6. http://articles.timesofindia.indiatimes.com/keyword/artificial-rain

Michael LP. 2011. Cloud-making: a new effect of human activity on climate. New Scientist, 211(2827): 12

Pruppacher HR, Klett JD. 2010. Microphysics of Clouds and Precipitation (2nd edition). Springer

Saxena VK, Grovenstein JD. 1994. The role of clouds in the enhancement of cloud condensation nuclei concentrations. Atmospheric Research, 31: 71-89

Sayadi MH, Sayyed MRG. 2011. Variations in the heavy metal accumulations within the surface soils from the Chitgar industrial area of Tehran. Proceedings of the International Academy of Ecology and Environmental Sciences, 1(1): 36-46

Sayyed MRG, Wagh GS. 2011. An assessment of groundwater quality for agricultural use: a case study from solid waste disposal site SE of Pune, India. Proceedings of the International Academy of Ecology and Environmental Sciences, 1(3-4): 195-201

Shukla JB, Sundar S, Misra AK, Naresh R. 2008. Modelling the removal of gaseous pollutants and particulate matters from the atmosphere of a city by rain: Effect of cloud density. Environ. Model. Assess., 13: 255–263

Shukla JB, Misra AK, Naresh R, Chandra P. 2010. How artificial rain can be produced? A mathematical model. Nonlinear Analysis: RWA, 11: 2659-2668

Smith JM. 1974. Models in Ecology. Cambridge University Press

Xinhua News Agency. 2006. China makes artificial rain for Beijing. http://www.chinadaily.com.cn/china/2006-05/06/content_583222.htm

Zee News. 2009. Indian scientists set to create artificial rain. http://zeenews.india.com/news/sci-tech/indian-scientists-set-to-create-artificial-rain_542734.html

Zhang WJ, Liu CH. 2012. Some thoughts on global climate change: will it get warmer and warmer? Environmental Skeptics and Critics, 1(1): 1-7