

Article

## Computing the uncertainty associated with the control of ecological and biological systems

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Received 20 May 2013; Accepted 23 June 2013; Published online 1 September 2013



### Abstract

Recently, I showed that ecological and biological networks can be controlled by coupling their dynamics to evolutionary modelling. This provides numerous solutions to the goal of guiding a system's behaviour towards the desired result. In this paper, I face another important question: how reliable is the achieved solution? In other words, which is the degree of uncertainty about getting the desired result if values of edges and nodes were a bit different from optimized ones? This is a pivotal question, because it's not assured that while managing a certain system we are able to impose to nodes and edges exactly the optimized values we would need in order to achieve the desired results. In order to face this topic, I have formulated here a 3-parts framework (network dynamics - genetic optimization - stochastic simulations) and, using an illustrative example, I have been able to detect the most reliable solution to the goal of network control. The proposed framework could be used to: a) counteract damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage systems at the least possible cost, and d) plan optimized bio-manipulations.

**Keywords** genetic algorithms; network control; stochastic simulations; uncertainty.

**Computational Ecology and Software**  
ISSN 2220-721X  
URL: <http://www.iaees.org/publications/journals/ces/online-version.asp>  
RSS: <http://www.iaees.org/publications/journals/ces/rss.xml>  
E-mail: [ces@iaees.org](mailto:ces@iaees.org)  
Editor-in-Chief: WenJun Zhang  
Publisher: International Academy of Ecology and Environmental Sciences

### 1 Introduction

Recently, I showed that ecological and biological networks can be efficiently controlled by coupling their dynamics to evolutionary modelling (Ferrarini, 2013). This provides numerous solutions to the goal of guiding a system's behaviour towards the desired state. The proposed solution is able to drive networks towards the desired state without the need of a permanent control on nodes values as, instead, it was proposed by Liu *et al.* (2011). The application of genetic algorithms to network dynamics allows to act on the highest number of switches, while maintaining the computational effort to tractable levels. In addition, they allow to control multiple nodes and edges at the same time in a reasonably computational effort (Ferrarini, 2011).

In this paper, I face another pivotal topic, i.e. how reliable is the achieved solution? In other words, which is the degree of uncertainty about getting the desired result if values of edges and nodes were a bit different

from optimized ones? This is a pivotal question, because it's not assured that while managing a certain system we are able to impose exactly the optimized values to nodes and edges we would need in order to achieve the desired result. To this aim, here I have coupled system dynamics and evolutionary modelling to stochastic simulations and, using an illustrative example, I was able to detect the most reliable solution to the goal of network control.

## 2 Mathematical Conceptualization

Most real systems' dynamics can be modelled and simulated using a system of canonical, linear equations (Liu et al., 2011; Slotine and Li, 1991), as follows:

$$\left\{ \begin{array}{l} \frac{dS_1}{dt} = a_{11}S_1 + \dots + a_{1n}S_n + I_1 + O_1 \\ \dots \\ \frac{dS_n}{dt} = a_{n1}S_1 + \dots + a_{nn}S_n + I_n + O_n \end{array} \right. \quad (1)$$

where  $S_i$  is the number of individuals (or the total biomass) of the generic  $i$ -th species, while  $I$  and  $O$  represent inputs and outputs from outer universe. The previous system has initial values

$$\vec{S}_0 = \langle S_1(0), S_2(0) \dots S_n(0) \rangle \quad (2)$$

and co-domain limits

$$\left\{ \begin{array}{l} S_{1min} \leq S_1(t) \leq S_{1max} \\ \dots \\ S_{nmin} \leq S_n(t) \leq S_{nmax} \end{array} \right. \quad (3)$$

Under genetic optimization (Holland, 1975; Goldberg, 1989, Parolo et al., 2009; Ferrarini, 2012a), equation (1) becomes (Ferrarini, 2013):

$$\left\{ \begin{array}{l} \left( \frac{dS_1}{dt} \right)_{OPT} = a_{11*}S_1^* + \dots + a_{1n*}S_n^* + I_{1*} + O_{1*} \\ \dots \\ \left( \frac{dS_n}{dt} \right)_{OPT} = a_{n1*}S_1^* + \dots + a_{nn*}S_n^* + I_{n*} + O_{n*} \end{array} \right. \quad (4)$$

where asterisks stand for the optimization of edges (i.e., coefficients of interaction among variables) or nodes (i.e., initial stocks), that is the modification of their values at the beginning of the network dynamics in order to get a certain goal (e.g., maximization of the final value of a certain variable).

After optimization is reached, one could wonder how reliable is the achieved result, i.e. which is the degree of uncertainty about getting the desired result if optimized values of edges and nodes change of a small amount? This is a pivotal question, because it's not assured that while managing a certain system we are able to impose exactly the optimized values to nodes and edges. To answer this question, I changed eq. (4) into:

$$\left\{ \begin{array}{l} \left( \frac{dS_1}{dt} \right)_{OPT*} = \underline{a_{11*}}S_1^* + \dots + \underline{a_{1n*}}S_n^* + \underline{I_{1*}} + \underline{O_{1*}} \\ \dots \\ \left( \frac{dS_n}{dt} \right)_{OPT*} = \underline{a_{n1*}}S_1^* + \dots + \underline{a_{nn*}}S_n^* + \underline{I_{n*}} + \underline{O_{n*}} \end{array} \right. \quad (5)$$

where:

$$\begin{cases} 0.95 * a_{ij}^* \leq \underline{a}_{ij} \leq 1.05 * a_{ij}^* \\ 0.95 * S_j^* \leq \underline{S}_j \leq 1.05 * S_j^* \end{cases} \quad (6)$$

or alternatively:

$$\begin{cases} 0.9 * a_{ij}^* \leq \underline{a}_{ij} \leq 1.1 * a_{ij}^* \\ 0.9 * S_j^* \leq \underline{S}_j \leq 1.1 * S_j^* \end{cases} \quad (7)$$

In other words,  $\underline{a}_{ij}$  represents a 5% (or 10%) uncertainty about  $a_{ij}^*$ , while  $\underline{S}_j$  represents a 5% (or 10%) uncertainty about  $S_j^*$ . If, after genetic optimization, we stochastically vary  $n$  times (e.g. 10,000 times)  $a_{ij}^*$  and  $S_j^*$ , we are able to compute how many times such uncertainty make the optimization procedure useless. Hence, uncertainty about network control can be computed as:

$$U_{\%} = 100 * \frac{k}{n} \quad (8)$$

where  $k$  is the number of stochastic simulations acting upon optimized parameters that make the optimization procedure useless (i.e. the goal of optimization is not reached).

### 3 An Applicative Example

Fig. 1 depicts an ecological network borrowed with modifications from Ferrarini (2012b). Greenish nodes represent positive actors or events for the goal of the network control, i.e. the increase of individuals of the target species (centre of the network). Reddish nodes represent ecological actors or events with negative impact on the target species. Blueish nodes represent resources needed by the target species. The goal is to preserve target species' occurrence in the study area. Stocks stand for the actual amounts of individuals or biomass. Updates stand for yearly internal dynamics (i.e., intraspecific gains due to births and/or immigration rates minus losses due to deaths and/or emigration rates). Minimum and maximum values stand for lowest and highest values of stock values. For the sake of simplicity, the maximum possible value for each actor (in italic hereafter) has been set to 100. The percent value associated to links represents the percentage of the receiver that is yearly consumed by the transmitter (average values of the last 5 years) at the beginning of the network simulation. Road mortality and re-introductions accounts for 18 and 10 individuals per year respectively (average values of the last 5 years).

Since data are yearly-based, I expressed equation (1) using a system of difference recurrent equations, instead of differential ones. The previous ecological network has the following inertial dynamics (Fig. 3), with the *target species* (green line) going extinct after 7 years.

### 4 Solutions to the Goal of Network Control

As latterly depicted (Ferrarini, 2013), I have found several solutions in order to drive the above network to the desired dynamics (i.e. maximization of *target species*' stock). Table 1 summarizes some of the detected solutions.

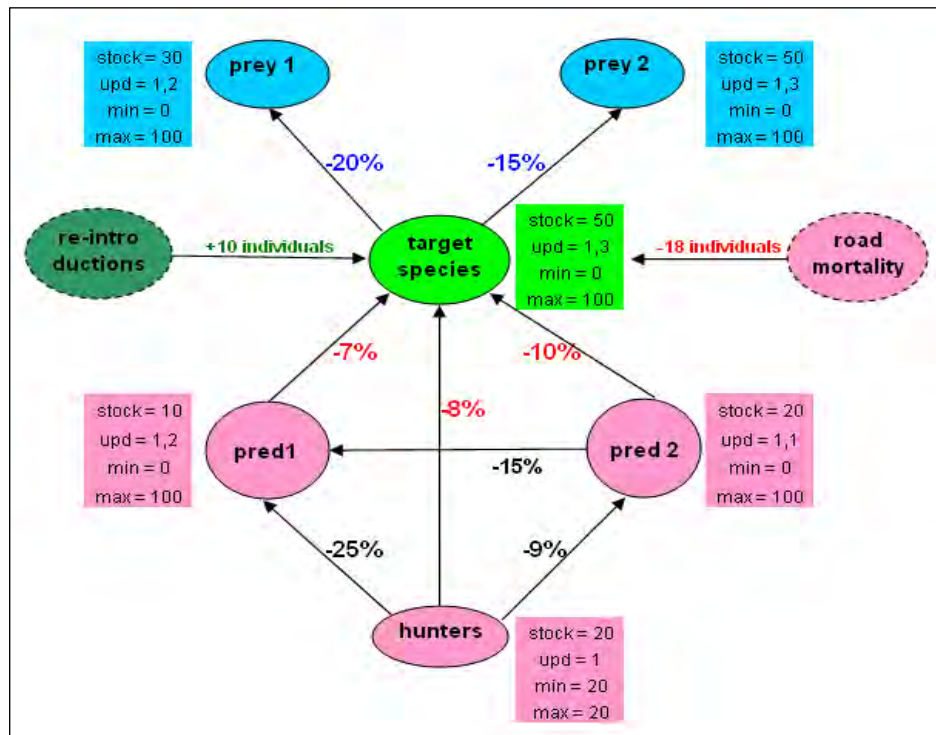


Fig. 1 The ecological network on which evolutionary control has been applied.

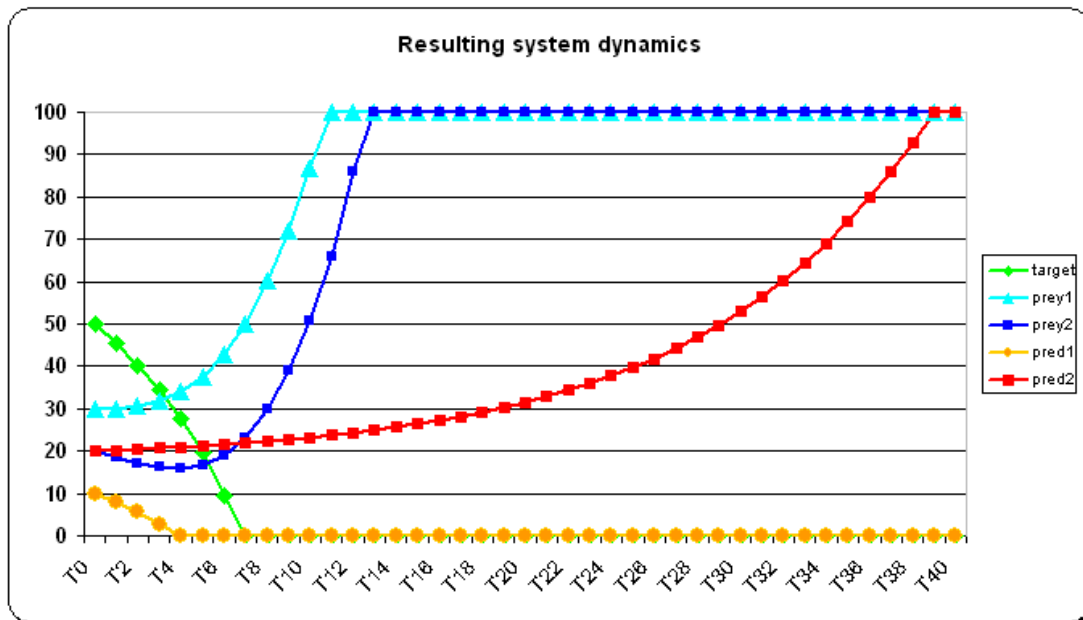


Fig. 2 Resulting dynamics for the network of Fig. 1. X-axis measures time in years. Dynamics were calculated using the software Quant-Lab (Ferrarini, in preparation).

**Table 1** Actions that can drive system's dynamics to the goal of maximizing *target species*' stock.

Solution	Description
1	imposing <i>pred2</i> 's initial stock to be <8
2	increasing <i>target species</i> ' intraspecific coefficient above the value 1.36
3	imposing input-output ( <i>reintroductions</i> minus <i>road mortality</i> ) balance to be > -6
4	yearly subtraction of at least 5 individuals for both <i>pred1</i> and <i>pred2</i>

### 5 Computing Associated Uncertainty

For each of the previous solutions, I computed 1%, 5% and 10% stochastic simulations on all the network parameters simultaneously as depicted in equation (5), then I computed  $U\%$  as expressed in equation (8). Each time, I repeated 10,000 simulations using Quant-Lab (Ferrarini, in preparation).

**Table 2** Control uncertainty (U%) associated to 1% uncertainty on the optimized network parameters. The lower  $U\%$ , the more reliable the action purposed to network control. In bold, solutions with null uncertainty associated. Actions 10, 14 and 19 resulted safe from uncertainty.

ID	Actions	Simulations	U%
1	<i>pred2</i> 's initial stock = 7	10,000	35.07
2	<i>pred2</i> 's initial stock = 6	10,000	28.38
3	<i>pred2</i> 's initial stock = 5	10,000	21.25
4	<i>pred2</i> 's initial stock = 4	10,000	17.05
5	<i>pred2</i> 's initial stock = 3	10,000	15.78
6	<i>pred2</i> 's initial stock = 1	10,000	14.27
7	<i>pred2</i> 's initial stock = 0	10,000	12.58
8	<i>target species</i> ' intraspecific coefficient = 1.361	10,000	34.00
9	<i>target species</i> ' intraspecific coefficient = 1.38	10,000	2.29
<b>10</b>	<b><i>target species</i>' intraspecific coefficient = 1.39</b>	<b>10,000</b>	<b>0</b>
11	<i>input-output</i> balance = -5	10,000	30.88
12	<i>input-output</i> balance = -4	10,000	3.82
13	<i>input-output</i> balance = -3	10,000	3.82
<b>14</b>	<b><i>input-output</i> balance = -2</b>	<b>10,000</b>	<b>0</b>
15	yearly subtraction of 5 individuals for both <i>pred1</i> and <i>pred2</i>	10,000	25.75
16	yearly subtraction of 6 individuals for both <i>pred1</i> and <i>pred2</i>	10,000	8.75
17	yearly subtraction of 7 individuals for both <i>pred1</i> and <i>pred2</i>	10,000	4.12
18	yearly subtraction of 8 individuals for both <i>pred1</i> and <i>pred2</i>	10,000	1.10
<b>19</b>	<b>yearly subtraction of 9 individuals for both <i>pred1</i> and <i>pred2</i></b>	<b>10,000</b>	<b>0</b>

With 1% uncertainty on the optimized network parameters (Table 2), it's clear that lowering *pred2*'s initial stock is not enough. In fact, even with *pred2*'s initial stock=0 there's no certainty about getting the maximization of *target species* ( $U\%=12.58\%$ ). Instead, the other 3 actions can achieve null uncertainty (actions in bold).

**Table 3** Control uncertainty (U%) associated to 5% uncertainty on the optimized network parameters. The lower  $U_{\%}$ , the more reliable the action purposed to network control. In bold, solutions with null uncertainty associated. Actions 4 and 9 resulted safe from uncertainty.

ID	Description	Simulations	U%
1	<i>target species</i> ' intraspecific coefficient = 1.39	10,000	28.15
2	<i>target species</i> ' intraspecific coefficient = 1.42	10,000	7.19
3	<i>target species</i> ' intraspecific coefficient = 1.45	10,000	1.61
<b>4</b>	<b><i>target species</i>' intraspecific coefficient = 1.46</b>	<b>10,000</b>	<b>0</b>
5	input-output balance = -2	10,000	24.71
6	input-output balance = +8	10,000	1.94
7	input-output balance = +12	10,000	1.70
8	input-output balance = +15	10,000	0.25
<b>9</b>	<b>input-output balance = +16</b>	<b>10,000</b>	<b>0</b>
10	yearly subtraction of 9 individuals for both <i>pred1</i> and <i>pred2</i>	10,000	36.25

With 5% uncertainty on the optimized network parameters (Table 3), the yearly subtraction of 9 individuals for both *pred1* and *pred2* is not enough ( $U_{\%} = 36.25\%$ ). Instead, the other 2 actions can achieve null uncertainty (actions in bold).

With 10% uncertainty on the optimized network parameters (Table 4), two actions are still effective (actions in bold). In this case, *target species*' intraspecific coefficient should be set to 1.57 or above, or *input-output* balance should be set to +21 or above.

**Table 4** Control uncertainty (U%) associated to 10% uncertainty on the optimized network parameters. The lower  $U_{\%}$ , the more reliable the action purposed to network control. In bold, solutions with null uncertainty associated. Actions 7 and 15 resulted safe from uncertainty.

ID	Description	Simulations	U%
1	<i>target species</i> ' intraspecific coefficient = 1.39	10,000	43.41
2	<i>target species</i> ' intraspecific coefficient = 1.42	10,000	30.29
3	<i>target species</i> ' intraspecific coefficient = 1.45	10,000	18.64
4	<i>target species</i> ' intraspecific coefficient = 1.50	10,000	7.09
5	<i>target species</i> ' intraspecific coefficient = 1.54	10,000	4.29
6	<i>target species</i> ' intraspecific coefficient = 1.56	10,000	1.88
<b>7</b>	<b><i>target species</i>' intraspecific coefficient = 1.57</b>	<b>10,000</b>	<b>0</b>
8	input-output balance = -2	10,000	33.30
9	input-output balance = 0	10,000	19.31
10	input-output balance = 2	10,000	9.82
11	input-output balance = 4	10,000	9.70
12	input-output balance = 8	10,000	5.29
13	input-output balance = 12	10,000	3.09
14	input-output balance = 18	10,000	1.65
<b>15</b>	<b>input-output balance = +21</b>	<b>10,000</b>	<b>0</b>

It's clear that the proposed 3-component framework (network dynamics - genetic optimization - stochastic simulations) is able to find several further solutions, for instance by acting simultaneously on *target species'* intraspecific coefficient, *input-output* balance, *pred2's* initial stock etc. In addition, using the proposed framework one could simulate uncertainty just on a subset of the network parameters (e.g., actors' stocks), or could also seek multiple goals (e.g. *target species'* maximization + *pred2's* minimization).

## 5 Conclusions

Ecological and biological networks' control is a pivotal issue. It could be used to a) counteract damages to ecological and biological networks, b) safeguard rare and endangered species, and c) manage ecological systems at the least possible cost. Here, I have proposed a feasible solution to this topic by coupling system dynamics and evolutionary modelling to stochastic simulations.

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