Article

A comparative study of a system of Lotka-Voltera type of PDEs through perturbation methods

H. A. Wahab[1], **M. Shakil**[1], **T. Khan**[1], **S. Bhatti**[2], **M. Naeem**[3]
[1]Department of Mathematics, Hazara University, Manshera, Pakistan
[2]Department of Mathematics, COMSATS Institute of Information Technology, University Road, Abbottabad, Pakistan
[3]Department of Information Technology, Hazara University, Manshera, Pakistan
E-mail: wahabmaths@yahoo.com

Received 9 August 2013; *Accepted 15 September 2013*; *Published online 1 December 2013*

Abstract
In this paper the Adomian Decomposition Method (ADM) is employed in order to solve linear and nonlinear functional equations and the results are then compared with those produced by Homotopy Perturbation Method (HPM) through a system of Lotka Voltera type of PDEs. The result produced by HPM are promising and ADM appears as a special case of HPM for Lotka Voltera type of PDEs.

Keywords Lotka Voltera PDE; Adomian Decomposition Method; Homotopy Perturbation Method.

Computational Ecology and Software
ISSN 2220-721X
URL: http://www.iaees.org/publications/journals/ces/online-version.asp
RSS: http://www.iaees.org/publications/journals/ces/rss.xml
E-mail: ces@iaees.org
Editor-in-Chief: WenJun Zhang
Publisher: International Academy of Ecology and Environmental Sciences

1 Introduction

The Lotka Voltera equations are a pair of first order non linear differential equations these are also known as the predator prey equations. The Lotka Voltera type problems were originally introduced by Lotka in 1920 (Lotka, 1920) as a model for undumped oscillating chemical reactions and after that these were applied by Voltera (Voltera, 1926) to predator prey interactions, consist of a pair of first order autonomous ordinary differential equations. Since that time the Lotka Voltera model has been applied to problems in chemical kinetics, population biology, epidemiology and neural networks. These equations also model the dynamic behavior of an arbitrary number of competitors.

Finding the solution of Lotka Voltera equations may become a difficult task either if the equations are fractionalized, namely they become a non-local one. Recently some new methods such as Tanh function method (Fan, 2000; Wazwaz, 2005) extended Jacobi elliptic function expansion method (Fu et al., 2001) and the simplest equation method (Kudryashov, 2005) have been used in literature to find exact solutions for both partial differential equation and system of partial differential equations. However, it is difficult to obtain closed form solutions for nonlinear problems. In most cases, only approximated solutions either analytical ones or numerical ones are founded. For analytical solutions of non linear problems Perturbation method is one of the well-known methods. Perturbation method is based on the existence of small or large parameters, so-called perturbation quantities (Hinch, 1991; Fernandez-Villaverde, 2011; Nayfeh, 1993).

2 Application Problem

Consider the following system of non linear *Lotka-Voltera Type* of PDEs (Cherniha and King, 2005),

$$w_t = \left(ww_x\right)_x + w\left(a + ew\right) + p + g\varpi, \tag{1}$$

$$\varpi_t = \left(\varpi\varpi_x\right)_x + \varpi\left(b + f\varpi\right) + q + kw, \tag{2}$$

with initial approximations as,

$$w\left(x,0\right) = \Psi\left(x\right), \quad \varpi\left(x,0\right) = \psi\left(x\right). \tag{3}$$

where a,b,e,f,g,k,p,q are arbitrary constants such that, $ef \neq 0, gk \neq 0.$

3 Solutions

3.1 General solution of the problem

In the given problem the system of equations are coupled equations and quadratic non linearity terms are also contained in it.

An exact periodic solution of this system was presented in (Alabdullatif et al., 2007) as,

$$w = \left(\theta_0(t) \pm \left(\theta_0\left(t\right) + \frac{2g}{c}\right)\cos\left(\sqrt{\frac{c}{2}}x \pm |g|t - \beta_0\right)\right), \tag{4}$$

$$\varpi = \left(\theta_0(t) + \frac{4g}{c} \pm \frac{|g|}{g}\left(\theta_0(t) + \frac{2g}{c}\right)\cos\left(\sqrt{\frac{c}{2}}x \pm |g|t - \beta_0\right)\right), \tag{5}$$

where,

$$\theta_0(t) = \begin{cases} \dfrac{2}{s-t} - 6g, a = 3g \\ |3g-a|\tanh\left(\dfrac{|3g-a|}{2}(s-t)\right) - a - 3g, a \neq 3g \end{cases}, \tag{6}$$

Here $e, f, g, k, p, q, \beta_0, s$ are arbitrary constants such that,

$$c = e = f > 0, \quad b = a - 6g, k = -cg, p = \frac{(2ag - 6g^2)}{c}, q = p + \frac{4g}{c}\left(3g - a\right).$$

3.2 Analysis of the problem by using the Adomian Decomposition Method

The Adomian decomposition method (Adomian, 1994; Bildik and Bayramoglu, 2005) defines a linear operator of the form,

$$\Im_t = \frac{\partial}{\partial t}.$$

Then system (1) can be written as,

$$\Im_t w = (ww_x)_x + w(a + ew) + p + g\varpi, \tag{7}$$

$$\Im_t \varpi = (\varpi \varpi_x)_x + \varpi(b + f\varpi) + q + kw, \tag{8}$$

Applying the inverse operator to equations (7) and (8) we will have that,

$$w(x,t) = \Psi(x) + \Im t^{-1}[aw + g\varpi + p + F(w)], \tag{9}$$

$$\varpi(x,t) = \psi(x) + \Im t^{-1}[b\varpi + kw + q + G(\varpi)], \tag{10}$$

where the non linear terms are, $F(w) = (ww_x)_x + ew^2, G(\varpi) = (\varpi\varpi_x)_x + f\varpi^2.$

According to the decomposition method (Bildik and Bayramoglu, 2005) it is assumed that the unknown functions $w(x,t)$, $\varpi(x,t)$ are represented as, $w(x,t) = \sum_{n=0}^{\infty} w_n(x,t)$, $\varpi(x,t) = \sum_{n=0}^{\infty} \varpi_n(x,t)$. $F(w), G(\varpi)$ are the non linear and these non linear terms can be splited into an infinite series of polynomials as, $F(x,t) = \sum_{n=0}^{\infty} A_n$, $G(x,t) = \sum_{n=0}^{\infty} B_n$. Here the components $w(x,t), \varpi(x,t)$ can be founded by use of recursive relations and A_n's, B_n's are the Adomian polynomials of w_n's, ϖ_n's respectively. w_n, ϖ_n For $n \geq 0$ are represented by the following recursive relations.

$$w_0 = w(x,0) = \Psi(x), \tag{11}$$

$$\varpi_0 = \varpi(x,0) = \psi(x), \tag{12}$$

The other components are as follow,

$$w_1 = \Im t^{-1}[aw_0 + g\varpi_0 + p + A_0], \tag{13}$$

$$\varpi_1 = \Im t^{-1}[b\varpi_0 + kw_0 + q + B_0], \tag{14}$$

The result can be generalized for n-terms as,

$$w_{n+1} = \Im t^{-1}[aw_n + g\varpi_n + p + A_n], n \geq 0,$$

$$\varpi_{n+1} = \Im t^{-1}[b\varpi_n + kw_n + q + B_n], n \geq 0,$$

By using the recursive relations for $w(x,t), \varpi(x,t)$ will be determined as,

$$w(x,t) = \lim_{x \to \infty} \varphi_n(x,t), \varpi(x,t) = \lim_{x \to \infty} \phi_n(x,t),$$

$$\varphi_n(x,t) = \sum_{i=0}^{n-1} w_i(x,t), \phi_n(x,t) = \sum_{i=0}^{n-1} \varpi_i(x,t).$$

Case #1

Consider $a = 3g$, $\theta_0(t) = \frac{1}{3c}\left(\frac{2}{s-t} - 6g\right)$, $b = a - 6g$, $p = q = 0, e = f = c, g = 1, k = -1.$ The obtained solutions are as follows

$$w_0 = \frac{1}{3c}\left(\frac{2}{s}-6\right)+\left(\frac{1}{3c}\left(\frac{2}{s}-6\right)+\frac{2}{c}\right)\cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right), \quad (15)$$

$$\varpi_0 = \frac{1}{3c}\left(\frac{2}{s}-6\right)+\left(\frac{1}{3c}\left(\frac{2}{s}-6\right)+\frac{2}{c}\right)\sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right), \quad (16)$$

$$w_1 = \begin{pmatrix} ct\left(\frac{2}{s}-6g\right)+(\frac{2}{s}\left(\frac{2}{c}-6\right)g\cos^2\left(\sqrt{\frac{c}{2}}x-\beta_0\right)-\frac{1}{cs}(\frac{t}{2}(2sg+B(2-6sg))) \\ \cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right)(c(2-6sg))+(2sg+c(2sg)\cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right)))+\frac{ct}{2}(\frac{2}{s}+ \\ (\frac{2}{c}-6)g)^2\sin^2\left(\sqrt{\frac{c}{2}}x-\beta_0\right)+tg(\frac{2}{s}-6g+\frac{4g}{c}+\frac{1}{g}((\frac{2}{s}+(\frac{2}{c}-6)g\left|g\right| \\ \sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right)))-t(\frac{2}{s}-6g+(\frac{2}{s}+(\frac{2}{c}-6)g)\cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right))a+tp) \end{pmatrix}. \quad (17)$$

$$\varpi_1 = \begin{pmatrix} \frac{t}{2}(\frac{1}{g^2}(c(\frac{2}{s}+(\frac{2}{c}-6)g)^2\left|g\right|+\cos^2\left(\sqrt{\frac{c}{2}}x-\beta_0\right))-2g(\frac{2}{s}-6g+(\frac{2}{s}+(\frac{2}{c}-6)g) \\ \cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right)+2c(\frac{2}{s}-6g-\frac{4g}{c}+\frac{1}{g}((\frac{2}{s}+(\frac{2}{c}-6)g)\left|g\right|\sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right))))^2 \\ -\frac{1}{cs^2g^2}((2sg-c(2-6sg))g+\left|g\right|\sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right)(g(4sg-c(2-6sg))+2sg \\ +c(2-6sg))\left|g\right|\sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right)))+2(\frac{2}{s}-6g-\frac{4g}{c}-\frac{1}{g}((\frac{2}{s}+(\frac{2}{c}-6)g)\left|g\right| \\ \sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right)))b+2q) \end{pmatrix}. \quad (18)$$

Case #2

For $a \neq 3g$, $\theta_0(t) = \left(\left|3g-a\right|\tanh\left(\frac{\left|3g-a\right|}{2}(s-t)\right)-a-3g\right)$, $a=3, p=1, g=1.2, c=3, k=-1.2.$,

the following results are obtained,

$$w_0 = \left(-0.6975+0.1024\cos\left(3-\sqrt{\frac{3}{2}}x\right)\right), \quad (19)$$

$$\varpi_0 = \left(0.9024-0.1024\sin\left(3-\sqrt{\frac{3}{2}}x\right)\right), \quad (20)$$

$$w_1 = \left(t \left(\begin{array}{l} 1.4500 - 0.0142\cos\left(-1.22x+3\right) + 0.0157\cos^2\left(-1.22x+3\right) \\ -0.1229\sin\left(-1.22x+3\right) + 0.0157\sin^2\left(-1.22x+3\right) \end{array} \right) \right). \tag{21}$$

$$\varpi_1 = \left(t \left(\begin{array}{l} 1.4500 - 0.1229\cos\left(-1.22x+3\right) + 0.0157\cos^2\left(-1.22x+3\right) \\ +0.1423\sin\left(-1.22x+3\right) + 0.0157\sin^2\left(-1.22x+3\right) \end{array} \right) \right). \tag{22}$$

Now in the next subsection, the analysis of this problem is presented by using Homotopy Perturbation Method (He, 1999; He, 2000).

3.3 Analysis of the problem by using the Homotopy Perturbation Method

Constructed Homotopy for the given system of equations (1) and (2) respectively will be as follows,

$$w_t - y_{0t} + h[y_{0t} - (ww_x)_x - aw - ew^2 - p - g\varpi] = 0, \tag{23}$$

$$\varpi_t - z_{0t} + h[z_{0t} - (\varpi\varpi_x)_x - b\varpi - f\varpi^2 - q - kw] = 0, \tag{24}$$

The initial approximations are choosed as,

$$w_0\left(x,t\right) = y_0\left(x,t\right) = w\left(x,0\right) = \Psi\left(x\right), \tag{25}$$

$$\varpi_0\left(x,t\right) = z_0\left(x,t\right) = \varpi\left(x,0\right) = \psi\left(x\right). \tag{26}$$

Let us assume the solution for the above system of equations respectively as follow,

$$w\left(x,t:h\right) = w_0 + hw_1 + h^2 w_2 ..., \tag{27}$$

$$\varpi\left(x,t;h\right) = \varpi_0 + h\varpi_1 + h^2\varpi_2 ..., \tag{28}$$

Putting equations (27) and (28) in equations (23) and (24) and comparing the same powers of h,

$$h^0 : w_{0t} - y_{0t} = 0, w_0\left(x,t\right) = w\left(x,0\right) = \Psi\left(x\right), \tag{29}$$

$$h^0 : \varpi_{0t} - z_{0t} = 0, \varpi_0\left(x,t\right) = \varpi\left(x,0\right) = \psi(x), \tag{30}$$

$$h^1 : w_{1t} + y_{0t} - w_0 w_{0xx} + w_{0x}{}^2 - aw_0 - ew_0{}^2 - p - g\varpi_0{}^2 = 0, w_1\left(x,0\right) = 0, \tag{31}$$

$$h^1 : \varpi_{1t} + z_{0t} - \varpi_0\varpi_{0xx} + \varpi_{0x}{}^2 - b\varpi_0 - e\varpi_0{}^2 - q - kw_0 = 0, \varpi_1\left(x,0\right) = 0, \tag{32}$$

$$h^2 : w_{2t} + 2w_{0x}w_{1x} + w_0 w_{1xx} + w_1 w_{0xx} - aw_1 - 2ew_0 w_1 - 2\varpi_0\varpi_1 = 0. \tag{33}$$

$$h^2 : \varpi_{2t} + 2\varpi_{0x}\varpi_{1x} + \varpi_0\varpi_{1xx} + \varpi_1\varpi_{0xx} - a\varpi_1 - 2e\varpi_0\varpi_1 - 2w_0 w_1 = 0. \tag{34}$$

Case # 1

Now consider the first case i.e. $a = 3g$.

$$\theta_0(0) = \left(\frac{2}{3cs} - \frac{2g}{c} \right). \tag{35}$$

And by taking $a = 3, p = 1, g = 1.2, c = 3, k = -1.2.$, we find the following relations,

$$w_0(x,t)=w(x,0)=\left(\frac{2}{3cs}-\frac{2}{c}\right)+\left(\frac{2}{3cs}\right)\cos\left(\sqrt{\frac{c}{2}}x-\beta_0\right)=\Psi(x). \tag{36}$$

$$\varpi_0(x,t)=\varpi(x,0)=\left(\frac{2}{3cs}-\frac{2}{c}\right)+\left(\frac{2}{3cs}\right)\sin\left(\sqrt{\frac{c}{2}}x-\beta_0\right)=\psi(x). \tag{37}$$

$$w_1=\left(\begin{array}{l} -\left(\frac{1}{3}\right)\frac{\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\cos\left(\frac{-1}{2}\sqrt{2c}x+B\right)}{cs}\right)}{s}\cos\left(-\frac{1}{2}\sqrt{2c}x+B\right)t-\frac{2}{9}\frac{\sin\left(-\frac{1}{2}\sqrt{2c}x+B\right)^2 t}{cs^2}+ \\ a\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)t+e\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)^2 t+pt+ \\ g\left(\frac{2}{3cs}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)^2 t \end{array}\right)$$

$$\varpi_1=\left(\begin{array}{l} -\left(\frac{1}{3}\right)\frac{\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\sin\left(\frac{-1}{2}\sqrt{2c}x+B\right)}{cs}\right)}{s}\sin\left(-\frac{1}{2}\sqrt{2c}x+B\right)t-\frac{2}{9}\frac{\cos\left(-\frac{1}{2}\sqrt{2c}x+B\right)^2 t}{cs^2}+ \\ a\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)t+e\left(\frac{2}{3cs}-\frac{2}{c}+\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)^2 t+pt+ \\ g\left(\frac{2}{3cs}-\frac{2}{c}-\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2c}x+B\right)}{cs}\right)^2 t. \end{array}\right)$$

Simplification and comparison shows that these are the same components as were calculated by using Adomian Decomposition Method.

Now from equation (33) and (34) we have that,

$$w_2(x,t) = \frac{1}{3}\left(-\frac{1}{3}\,\frac{1}{\sqrt{c}\,s}\left(\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x\right.\right.\right.$$

$$\left.+B\right)\sqrt{2}\left(\frac{1}{9}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)\sqrt{2}\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{\sqrt{c}\,s^2}\right.$$

$$-\frac{1}{6}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)\sqrt{2}\sqrt{c}}{s}$$

$$+\frac{1}{3}\,\frac{a\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)\sqrt{2}}{\sqrt{c}\,s}$$

$$\left.\left.+\frac{2}{3}\,\frac{e\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)\sqrt{2}}{\sqrt{c}\,s}\right)\right)$$

$$-\frac{1}{2}\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\left(-\frac{1}{9}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{s^2}\right.$$

$$+\frac{1}{6}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)c}{s}$$

$$-\frac{1}{3}\,\frac{a\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}+\frac{4}{9}\,\frac{e\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}$$

$$-\frac{2}{3}\,\frac{e\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{s}\Bigg)$$

$$+\frac{1}{6}\,\frac{1}{s}\Bigg(\Bigg(\Bigg(-\frac{1}{3}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{s}$$

$$-\frac{2}{9}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)^{2}}{c\,s^{2}}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)^{2}+p+g\,w^{2}\Bigg)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)$$

$$+\frac{1}{2}\,a\Bigg(-\frac{1}{3}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{s}$$

$$-\frac{2}{9}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)^{2}}{c\,s^{2}}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)^{2}+p+g\,w^{2}\Bigg)+\Bigg(\frac{2}{3\,c\,s}-\frac{2}{c}$$

$$-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\Bigg)$$

$$\Bigg(\frac{1}{3}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{s}$$

$$-\frac{2}{9}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)^{2}}{c\,s^{2}}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)^{2}+p+g\Bigg(\frac{2}{3\,c\,s}-\frac{2}{c}$$

$$+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\Bigg)^{2}\Bigg)\Bigg)\Bigg)\,t^{3}$$

$$\varpi_2(x,t) = \Bigg(-\frac{1}{3}\,\frac{1}{\sqrt{c}\,s}\Bigg(\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}\,\Bigg($$

$$-\frac{1}{9}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}\,\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{\sqrt{c}\,s^2}$$

$$-\frac{1}{6}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}\,\sqrt{c}}{s}$$

$$+\frac{1}{3}\,\frac{a\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}}{\sqrt{c}\,s}$$

$$+\frac{2}{3}\,\frac{e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}}{\sqrt{c}\,s}$$

$$+\frac{2}{3}\,\frac{g\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\,\frac{\cos\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)\sqrt{2}}{\sqrt{c}\,s}\Bigg)\Bigg)$$

$$-\frac{1}{2}\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\Bigg(-\frac{1}{9}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)^2}{s^2}$$

$$-\frac{1}{6}\,\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\,\frac{\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\,\sqrt{c}\,x+B\right)c}{s}$$

$$+\frac{1}{3}\frac{a\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}+\frac{4}{9}\frac{e\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}$$

$$+\frac{2}{3}\frac{e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}$$

$$+\frac{4}{9}\frac{g\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}$$

$$-\frac{2}{3}\frac{g\left(\frac{2}{3\,c\,s}-\frac{2}{c}+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}\Bigg)$$

$$-\frac{1}{6}\frac{1}{s}\Bigg(\Bigg(\Bigg(\frac{1}{3}\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}$$

$$-\frac{2}{9}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)^2+p+g\left(\frac{2}{3\,c\,s}-\frac{2}{c}\right.$$

$$\left.+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)^2\Bigg)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)\Bigg)$$

$$+\frac{1}{2}a\left(\frac{1}{3}\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}\right.$$

$$-\frac{2}{9}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)^2+p+g\bigg(\frac{2}{3\,c\,s}-\frac{2}{c}$$

$$+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\bigg)^2\bigg)\bigg)+\bigg(\frac{2}{3\,c\,s}-\frac{2}{c}$$

$$-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\bigg)$$

$$\Bigg(\frac{1}{3}\frac{\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{s}$$

$$-\frac{2}{9}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)^2}{c\,s^2}+a\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)$$

$$+e\left(\frac{2}{3\,c\,s}-\frac{2}{c}-\frac{2}{3}\frac{\sin\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\right)^2+p+g\bigg(\frac{2}{3\,c\,s}-\frac{2}{c}$$

$$+\frac{2}{3}\frac{\cos\left(-\frac{1}{2}\sqrt{2}\sqrt{c}\,x+B\right)}{c\,s}\bigg)^2\bigg)\bigg)\Bigg)t^3$$

Case # 2

Now consider the second case i.e. $a \neq 3g$, and by taking, $a=3, p=1, g=1.2, c=3, k=-1.2, s=4.$,

then for this case we find that, $\theta_0(0)=-0.6966.$

Using these values the following results are calculated,

$$w_0(x,t)=w(x,0)=\left(-0.6966\pm(0.1022)\cos\left(\sqrt{\frac{3}{2}}x-3\right)\right), \quad (38)$$

$$\varpi_0\left(x,t\right)=\varpi\left(x,0\right)=\left(0.9020-\left(0.1022\right)\sin\left(\sqrt{\frac{3}{2}}x-3\right)\right).$$

(39)

$$w_1=\begin{pmatrix}-(0.1536000000(0.9024000000+0.1024000000sin(-3+1.224744871x)))\\ sin(-3+1.224744871x)t-0.1572864000e-cos(-3+1.224744871x)^2t+\\ 3.707200000t+0.3072000000sin(-3+1.224744871x)t+3(0.9024000000\\ +0.1024000000sin(-3+1.224744871x))^2t-1.200000000(-0.6975000000\\ +0.1024000000cos(-3+1.224744871x))^2t.\end{pmatrix}$$

$$\varpi_1=\begin{pmatrix}-(0.1536000000(-0.6975000000+0.1024000000cos(-3+1.224744871x)))\\ cos(-3+1.224744871x)t-0.1572864000e-sin(-3.+1.224744871x)^2t-\\ 1.092500000t+0.3072000000cos(-3.+1.224744871x)t+3(-0.6975000000\\ +0.1024000000cos(-3+1.224744871x))^2t+1.2200000000(0.9024000000+\\ 0.1024000000\sin(-3+1.224744871x))^2t.\end{pmatrix}$$

Simplification and comparison shows that these are the same components as were calculated by using Adomian Decomposition Method.

Now from equation (33) and (34) we have that,

$$
\begin{aligned}
w_2(x,t) =& -0.4608000000\,(0.9024000000+0.1024000000\sin(-3.+1.224744871\,x))\sin(-3. \\
&+1.224744871\,x)\,\frac{t^{2}}{2}-0.04718592000\cos(-3.+1.224744871\,x)^{2}\,t+11.12160000\,t \\
&+0.9216000000\sin(-3.+1.224744871\,x)\,\frac{t^{2}}{2}+9.\,(0.9024000000+0.1024000000\sin(-3. \\
&+1.224744871\,x))^{2}\,\frac{t^{2}}{2}-3.600000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^{2}\,\frac{t^{2}}{2}+6\left(-0.6975+0.1024\cos\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\right)\Bigg(\\
&-0.1536000000\,(0.9024000000+0.1024000000\sin(-3.+1.224744871\,x))\sin(-3. \\
&+1.224744871\,x)\,\frac{t^{2}}{2}-0.01572864000\cos(-3.+1.224744871\,x)^{2}\,\frac{t^{2}}{2}+3.707200000\,t \\
&+0.3072000000\sin(-3.+1.224744871\,x)\,\frac{t^{2}}{2}+3.\,(0.9024000000+0.1024000000\sin(-3. \\
&+1.224744871\,x))^{2}\,\frac{t^{2}}{2}-1.200000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^{2}\,\frac{t^{2}}{2}\Bigg)+2\left(0.9024+0.1024\sin\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\right)\Bigg(-0.1536000000\,(\\
&-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3.+1.224744871\,x)\,t \\
&-0.01572864000\sin(-3.+1.224744871\,x)^{2}\,\frac{t^{2}}{2}-1.092500000\,\frac{t^{2}}{2}+0.3072000000\cos(-3. \\
&+1.224744871\,x)\,\frac{t^{2}}{2}+4.200000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^{2}\,\frac{t^{2}}{2}\Bigg)+0.1024000000\sin\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\Bigg(0.01926357117\cos(-3. \\
&+1.224744871\,x)\sin(-3.+1.224744871\,x)\,\frac{t^{2}}{2}+0.5643624366\,(0.9024000000 \\
&+0.1024000000\sin(-3.+1.224744871\,x))\cos(-3.+1.224744871\,x)\,t+0.3762416244\cos(\\
&-3.+1.224744871\,x)\,t+0.3009932995\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))\,t\sin(-3.+1.224744871\,x)\Bigg)\sqrt{6}-\Bigg(-0.6975+0.1024\cos\Bigg(-3 \\
&+\frac{1}{2}\sqrt{6}\,x\Bigg)\Bigg)\Bigg(-0.06134169597\sin(-3.+1.224744871\,x)^{2}\,\frac{t^{2}}{2}+0.09437183996\cos(-3. \\
&+1.224744871\,x)^{2}\,\frac{t^{2}}{2}-0.6911999996\,(0.9024000000+0.1024000000\sin(-3. \\
&+1.224744871\,x))\sin(-3.+1.224744871\,x)\,\frac{t^{2}}{2}-0.4607999997\sin(-3.+1.224744871\,x)\,\frac{t^{2}}{2} \\
&+0.3686399998\,(-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3. \\
&+1.224744871\,x)\,\frac{t^{2}}{2}\Bigg)+0.1536000000\Bigg(-0.1536000000\,(0.9024000000+0.1024000000\sin(
\end{aligned}
$$

$$-3. + 1.224744871\, x))\, \sin(-3. + 1.224744871\, x)\, \frac{t}{2}^{2} - 0.01572864000 \cos(-3.$$
$$+ 1.224744871\, x)^{2}\, \frac{t}{2}^{2} + 3.707200000\, \frac{t}{2}^{2} + 0.3072000000 \sin(-3. + 1.224744871\, x)\, \frac{t}{2}^{2}$$
$$+ 3.\, (0.9024000000 + 0.1024000000 \sin(-3. + 1.224744871\, x))^{2}\, \frac{t}{2}^{2} - 1.200000000\, ($$
$$-0.6975000000 + 0.1024000000 \cos(-3. + 1.224744871\, x))^{2}\, \frac{t}{2}^{2}\Big) \cos\Big(-3 + \frac{1}{2}\sqrt{6}\, x\Big)$$

$$
\begin{aligned}
\varpi_2(x,t) =& -0.1024000000\cos\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\Big(-0.01926357117\sin(-3.+1.224744871\,x)\cos(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}-0.8653557358\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))\sin(-3.+1.224744871\,x)\,\frac{t^2}{2}-0.3762416244\sin(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}\Big)\sqrt{6}-\left(0.9024+0.1024\sin\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\right)\Big(\\
&-0.02359295999\cos(-3.+1.224744871\,x)^2\,\frac{t^2}{2}+0.1321205759\sin(-3.+1.224744871\,x)^2\,\frac{t^2}{2} \\
&-1.059839999\,(-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}-0.4607999997\cos(-3.+1.224744871\,x)\,\frac{t^2}{2}\Big)+0.1536000000\Big(\\
&-0.1536000000\,(-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}\,t-0.01572864000\sin(-3.+1.224744871\,x)^2\,\frac{t^2}{2}-1.092500000\,t \\
&+0.3072000000\cos(-3.+1.224744871\,x)\,\frac{t^2}{2}+4.200000000\,(-0.6975000000 \\
&+0.1024000000\cos(-3.+1.224744871\,x))^2\,\frac{t^2}{2}\Big)\sin\left(-3+\frac{1}{2}\sqrt{6}\,x\right)-0.4608000000\,(\\
&-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3.+1.224744871\,x)\,\frac{t^2}{2} \\
&-0.04718592000\sin(-3.+1.224744871\,x)^2\,\frac{t^2}{2}-3.277500000\,t+0.9216000000\cos(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}+12.60000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^2\,\frac{t^2}{2}+6\left(0.9024+0.1024\sin\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\right)\Big(-0.1536000000\,(\\
&-0.6975000000+0.1024000000\cos(-3.+1.224744871\,x))\cos(-3.+1.224744871\,x)\,\frac{t^2}{2} \\
&-0.01572864000\sin(-3.+1.224744871\,x)^2\,\frac{t^2}{2}-1.092500000\,t+0.3072000000\cos(-3. \\
&+1.224744871\,x)\,t+4.200000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^2\,\frac{t^2}{2}\Big)+2\left(-0.6975+0.1024\cos\left(-3+\frac{1}{2}\sqrt{6}\,x\right)\right)\Big(\\
&-0.1536000000\,(0.9024000000+0.1024000000\sin(-3.+1.224744871\,x))\sin(-3. \\
&+1.224744871\,x)\,\frac{t^2}{2}-0.01572864000\cos(-3.+1.224744871\,x)^2\,\frac{t^2}{2}+3.707200000\,t \\
&+0.3072000000\sin(-3.+1.224744871\,x)\,\frac{t^2}{2}+3.\,(0.9024000000+0.1024000000\sin(-3. \\
&+1.224744871\,x))^2\,\frac{t^2}{2}-1.200000000\,(-0.6975000000+0.1024000000\cos(-3. \\
&+1.224744871\,x))^2\,\frac{t^2}{2}\Big).
\end{aligned}
$$

4 Concluding Remarks

In this study, approximated solutions for some non linear problems are calculated by Adomian Decomposition Method (ADM) and Homotopy Perturbation Method (HPM) and then the numerical results are compared.

It is analyzed that in Adomian Decomposition Method first Adomian polynomials are calculated which is a bit difficult and time wasting process and the fact that HPM solves nonlinear problems without using Adomian's polynomials is a clear advantage of this technique over ADM.

The comparative study between these two methods shows that the results obtained by using HPM with a special convex constructed Homotopy is almost equivalent to the results obtained by using ADM for these types of non linear problems.

References

Adomian G. 1994. Solving Frontier Problems of Physics-The Decomposition Method. Kluwer Academic, Boston, MA, USA

Alabdullatif M, Abdusalam HA, Fahmy ES. 2007. Adomian Decomposition Method for Nonlinear Reaction Diffusion System of Lotka-Voltera Type. International Mathematical Forum (2) No. 2, 87-96

Bildik N, Bayramoglu H. 2005. The solution of two dimensional non linear differential equation by Adomian Decomposition Method. Applied Mathematics and Computation, 163: 519-524

Cherniha R, King JR. 2005. Non-linear reaction-diffusion systems with variable diffusivities: Lie symmetries, ansatz and exact solutions. Journal of Mathematical Analysis and Applications, 308(1): 11-35

Fan E. 2000. Extended Tanh-Function Method and its applications to nonlinear equations. Physics Letters A, 277: 212-218

Fernandez-Villaverde J. 2011. Perturbation Methods. University of Pennsylvania, USA

Fu ZT, Liu SK, Zhao Q. 2001. New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equations. Physics Letters. A, 290(1-2): 72-76

He JH. 1999. Homotopy perturbation technique. Computer Methods in Applied Mechanics and Engineering, 178(3-4): 257-262

He JH. 2000. A coupling method of a homotopy technique and a perturbation technique for nonlinear problems. International Journal of Non-Linear Mechanics, 35(1): 37-43

Hinch EJ. 1991. Perturbation Methods. Cambridge University Press, UK

Kudryashov NA. 2005. Simplest equation method to look for exact solutions of nonlinear differential quations. Chaos, Solitons and Fractals, 24: 1217-1231

Lotka AJ. 1920. Undamped oscillations derived from the las of mass action. Journal of the American Chemical Society, 42(8): 1595-1599

Nayfeh AH. 1993. Introduction to Perturbation Techniques. Wiley Classics Library Edition, USA

Voltera V. 1926. Variations and Fluctuations of the Number of the Individuals in Animal Species Living together. In: Animal Ecology (Chapman RN, ed). McGraw-Hill, New York, USA

Wazwaz AM. 2005. The tan h method: solitons and periodic solutions for the Dodd–Bullough–Mikhailov and the Tzitzeica–Dodd–Bullough equations. Chaos, Solitons and Fractals, 25: 55-63