Article

Catastrophic behavior of aphid population dynamics: An analysis of swallowtail model

Mkdk Piyaratne[1,2], **Huiyan Zhao**[1], **Zuqing Hu**[1], **Xiangshun Hu**[1], **Bailing Wang**[3]
[1]State Key Laboratory of Crop Stress Biology in Arid Areas, College of Plant Protection, Northwest A&F University, Yangling, Shaanxi, 712100, China
[2]Computer Unit, Faculty of Agriculture, University of Ruhuna, Sri Lanka
[3]Meteorological Bureau in Yangling, Shaanxi, 712100, China
E-mail: zhaohy@nwsuaf.edu.cn

Received 29 March 2014; *Accepted 7 May 2014*; *Published online 1 September 2014*

Abstract
Catastrophe phenomena are frequent in insect ecology, especially in aphid populations. Complexity of this phenomenon urges different modeling frameworks other than traditional methodologies to understand the trajectories of their behavior. Situations like this can be best handled using catastrophe theory. A few numbers of experiments have been conducted to develop catastrophe models in insect ecology, especially for aphids, and most of them are based on cusp catastrophe theory which is a lower dimensional model. However few attempts using higher dimensional models such as swallowtail or butterfly theory to analyze aphid population dynamics are also exist. In this paper we tried to analyze a recently developed higher dimensional catastrophe theory model (APHIDSim) in order to identify catastrophe regions, and used independent data to identify if catastrophic behavior is observed in the data and consequently to further verify the model. Here we found that identifying catastrophe regions is possible using catastrophe theory model, and it can be used to analyze catastrophes in insect ecology by graphically interpreting the simulated results. Increasing of insect population is intrinsically catastrophic and catastrophes (jumps) occur between states even if the driving variables still change smoothly. The results further verified the previously developed model, and we suggest that insect management program developers should consider this phenomenon when they design the management strategies for insect controlling.

Keywords catastrophe theory; cusp model, swallowtail model; aphid ecology; population dynamics; ecological software.

Computational Ecology and Software
ISSN 2220-721X
URL: http://www.iaees.org/publications/journals/ces/online-version.asp
RSS: http://www.iaees.org/publications/journals/ces/rss.xml
E-mail: ces@iaees.org
Editor-in-Chief: WenJun Zhang
Publisher: International Academy of Ecology and Environmental Sciences

1 Introduction

Catastrophe movements are frequent in insect ecology, especially in aphid population dynamics. Numerous attempts have been undertaken to understand the population dynamics of insects but the prediction is still hard because of the complexity of this scenario (Schowalter, 2011; Price et al., 2011; Hufbauer, 2002; Karley et al., 2004; Jarošík et al., 2003). There are many control parameters which influence the growth of aphids, and they have complex interactions. Complexity of this phenomenon urges different modeling frameworks other than traditional methodologies to understand the trajectories of their behavior. Situations like this can be best handled using catastrophe theory (Ouimet and Legendre, 1988; Recknagel, 1985). Catastrophe theory can be described as a modeling strategy, and generally it is used in modeling and analysis of complex nonlinear systems (Arnold, 1984; Glimore, 1981). It describes that a continuous and smooth change of control variables in a dynamical system would result a sudden change (catastrophe) of the considered state variable. However some other research reports (Zhang, 2013) have tried to include the catastrophe theory in the self-organization theory. In the view of mathematics, the self-organization theory explains the changes of states of a system as discussed in the catastrophe theory (Zhang, 2013); a system may change towards a steady or unsteady state from the original steady state. Self-organization of insects has also been discussed in several reports (Bonabeaua et al., 1997; Kummel et al., 2013; Bonabeau, 1998; Petrovskii et al., 2014). Especially in insect population ecology, it is considered as a key concept. The self-organization of insects such as aphids may induce by inter-specific interactions (Zhang, 2013). Predator-prey interaction between aphids and ladybirds is considered as long established self-organizing process. High abundance of prey stimulates to have large number of predators in the region and could be migrated to the regions of low prey abundance. Consequently two equilibrium states (high density and low density) in different locations could be established.

However, there are inherent assumptions of catastrophe theory. Firstly, the considered system should be under control of no more than four control variables. Then, according to the catastrophe theory explanations, seven elementary catastrophes are possible to occur under first condition (Zeeman, 1976; Deakin, 1998). This implies that the variables which have most significant effects on the target state variables should be considered as control variables of the system. At present, most catastrophe models are descriptive in worldwide in terms of getting some understanding of relations between control factors and state variables and explaining of some biological phenomena. Cusp is the most widely used model in catastrophe theory applications in insect ecology among the well known seven elementary catastrophe models (Zeeman, 1976; Deakin, 1998). Using cusp model it is possible to explain a dynamical system if it is controlled by two variables and the behavior is depicted by one state variable. Some examples of using cusp model in insect ecology can be found in Jones (1977), Ludwig et al. (1978) and Casti (1982). Casti conducted a qualitative analysis using budworm density as state variable and plant quality and quantity as control variables based on Ludwig-Jones-Holling (LJH) model (Ludwig et al., 1978) and used it to explain and analyze the phenomenon in spruce budworm outbreak, prevention, and control. Further, Ma and Bechinski (2009), Lockwood and Lockwood (1989, 1991, 1993) and Holt and Cheke (1996) have also used cusp model in different ways in controlling insects. Scientists are involved in application of catastrophe theory in insect ecology having different perspectives; explaining catastrophe features which could be characterized by insect population dynamics, constructing of the equilibrium surface equation for a ecological system in order to determine the catastrophe region and deriving a standard catastrophe model for a certain process of that system by fitting the equilibrium equation into the catastrophe model. Some attempts for constructing of the equilibrium surface equation of a catastrophe model have been undertaken by Zhao (1991), Zhao et al. (1993), Zhao and Wang (1993a, 1993b), Zhao et al. (2005). They have provided a scientific base for prediction and controlling of pest insects analyzing their dynamic protection threshold using cusp catastrophe model.

Another major objective for applying catastrophe theory to study problems of insect ecology is to find the critical points of control variables for higher dimensional models when catastrophic movement occurred in a considered system. However, this objective is hard to achieve because of the complexity of the higher dimensional models. The lack of previous work on higher dimensional models further reveals the difficulty of using the theory to achieve said objective. These higher dimensional models are described by minimum of three control variables and one or two state variables. However, some previous attempts which used higher dimensional models can be found in Wei et al. (2009), Wei (2009), Li Zhen et al. (2012), Li Jianfeng et al. (2012) and Piyaratne et al. (2013). Wei et al. (2009) established a swallowtail catastrophe model which uses one state variable and three control variables to analyze pest population dynamics in farmland ecosystem. They have used influential factors such as crop condition (carrying capacity), climate (weather effect) and natural enemy (predation) as control variables while using pest population as state variable. From their study, they have derived a standard swallowtail catastrophe model and analyzed the catastrophic phenomena caused by the pest population dynamics with having a concrete analysis of the swallowtail catastrophe pattern. Further they studied the equilibrium points and system potential functions of each control region generated by the bifurcation set of the swallowtail catastrophe. It was concluded that their results provide a theoretical basis for practical applications of the theory. Later, Piyaratne et al. (2013) practically applied their findings by developing a computer simulated program (APHIDSim) of the swallowtail catastrophe model integrating the modified logistic equation of aphid population dynamics. In this paper we tried to analyze more about aphid population dynamics using APHIDSim software with independent data through identifying the catastrophe regions. In Piyaratne et al. (2013), they have verified the APHIDSim using only one data set and explained the critical points located in catastrophic regions using three dimensional control space diagrams. Here we further analyzed another four individual data sets to identify if catastrophic behavior is observed in the data and presented three dimensional control space diagrams of critical points located in the catastrophe regions.

2 Materials and Methods

2.1 APHIDSim: A software application for population dynamics analysis

The APHIDSim is a simulation software application (Piyaratne et al., 2013) which uses swallowtail catastrophe theory (Glimore, 1981; Zeeman, 1976) with modified logistic growth equation (Wei et al., 2009) to simulate the population dynamics of wheat aphids. As it uses the swallowtail model, the growth of aphid population can be modeled as a function of three controlling variables. The model considers weather factor, crop condition and predator effect as controlling variables and the aphid population as the behavior variable. It explains swallowtail behavior of aphid population dynamics using equilibrium points and plotting them in a three dimensional control space. It uses the discriminant curve (bifurcation set) of swallowtail model and considers five basic regions (Fig. 1) in the three dimensional space which could be used to explain catastrophic behavior (sudden jumps) of populations dynamics of aphids.

2.2 The logistic model

The modified logistic equation adopted from Wei et al. (2009) and Wei (2009) was used to develop APHIDSim program integrating weather factor (e), crop condition or carrying capacity (K) and predator effect (k) as controlling variables. The modified logistic model can be given as

$$\frac{dN}{dt} = rN\left(1 - \frac{eN^2}{K}\right) - \frac{Pk(N-N_m)}{(N-N_m)+d} \quad \forall N > N_m \tag{1}$$

where, N and P are the pest and predator population densities respectively while r and k are the rate of population increase and the rate of predation respectively. K is the carrying capacity which is affected by crop condition. The weather factor is depicted by e. There are two constants in the model, N_m and d, and they represent the minimum pest population where the predation is possible and the half saturated prey consumption by predator respectively.

(a) (b)

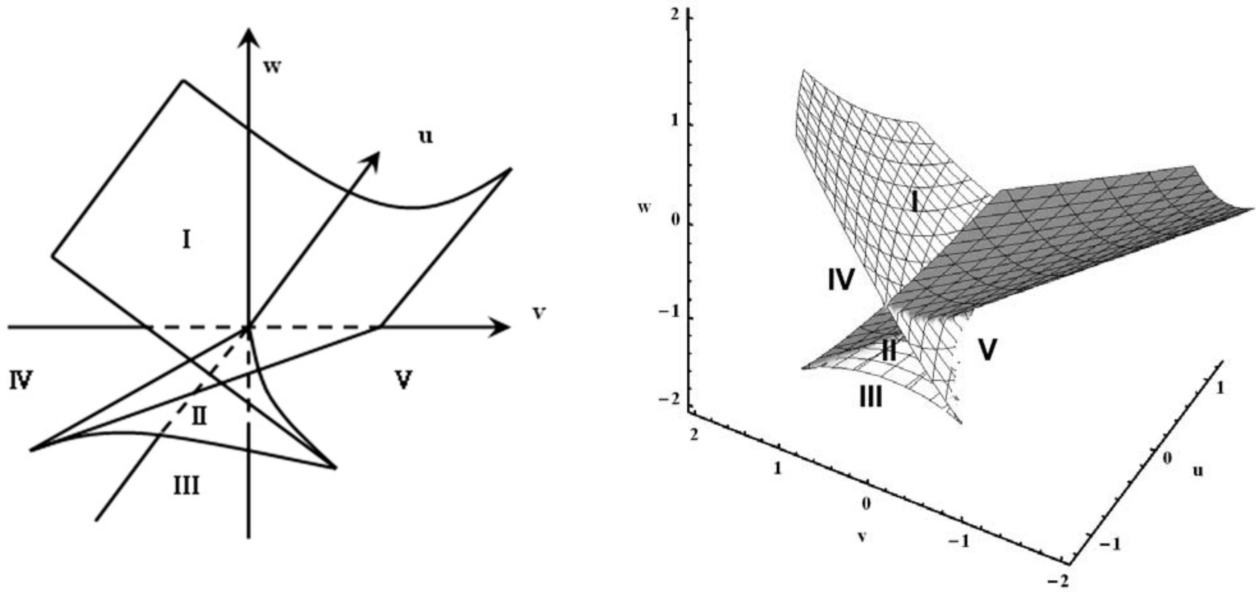

Fig. 1 Basic catastrophe regions (I-V) of the swallowtail catastrophe model: (a) The schematic diagram of the bifurcation set (b) Three dimensional image of the bifurcation set (Piyaratne et al., 2013).

2.3 The swallowtail model

The swallowtail model which is used in the APHIDSim was constructed from the population density of aphids and the weather factor, crop condition (carrying capacity) and predator effect are used as coefficients of the potential function. According to the Glimore (1981) and Zeeman (1976) the potential function with three unfolding parameters can be given by

$$f(x:u,v,w)=x^5+ux^3+vx^2+wx \quad (2)$$

In Eq. (2), x represents the behavior or state variable and u, v and w represent the control variables. The bifurcation theory explains that the critical points can be existed in the catastrophe manifold at the equilibrium state. The equilibrium surface M can be given by

$$f'(x:u,v,w)=5x^4+3ux^2+2vx+w \quad (3)$$

The singularity set S which is all singular points, is the subset of equilibrium surface M can be derived by the second derivative of the potential function of the swallowtail model, and is given by

$$f''(x:u,v,w)=20x^3+6ux+2v \quad (4)$$

The bifurcation set B (Eq. (5) of the swallowtail model is derived by eliminating x from Eq. (3) and (4), and plotted in a three dimensional space having three control variables (u, v and w) (Fig. 1 (B)). The bifurcation set or the discriminant curve can be given by

$$u(81u^3 + 540v^2)w - 360u^2w^2+400w^3=v^2(27u^3+135v^2) \quad (5)$$

Then the APHIDSim fits the logistic growth equation into the swallowtail model deriving a series of equations (Piyaratne et al., 2013). Unknown parameters are also estimated using an algorithm based on the grey estimation theory (Sifeng and Yi, 2006). The grey estimation theory is generally used to describe the dynamic behavior of a certain system, and it has been successfully adopted in various fields for estimating model parameters (Tien, 2009; Kayacan et al., 2010; Wu and Wang, 2011; Trivedi and Singh, 2005). The theory underneath of the model fitting and parameter estimation is not explained here as it is reported elsewhere (Piyaratne et al., 2013) and the major objective of this paper is to analyze independent data using the APHIDSim software and to further verify the software model.

2.4 Data analysis

In this paper, we analyzed four data sets collected by Northwest A & F University in 1984, 1985, 1986 and 2007. Three control variables used in the model were weather factor, natural enemy and crop factor (carrying capacity). Aphid population was used as the state variable for analysis as explained in APHIDSim software. The behavior of aphid population growth is featured on different catastrophie regions which are defined by analyzing three-dimensional control space of the bifurcation set *B* of the swallowtail model (Piyaratne et al., 2013). The three dimensional curved surface of the bifurcation set with five labeled regions (I - V) is shown in Fig. 1. Here we considered five catastrophe regions to analyze the changes of equilibrium points as explained in Wei et al. (2009), Wei (2009) and Piyaratne et al. (2013). The schematic diagram of the control space is up-scaled for clearance, and plotted the *u*, *v* and *w* values calculated by the APHIDSim in order to find the catastrophe regions related to each data set. As explained in previous studies (Wei et al., 2009; Wei, 2009; Piyaratne et al., 2013) the controlling variable, *u*, of the model is collectively affected by crop condition (*K*) and weather (*e*) while *v* and *w* are collectively affected by the effect of natural enemies (*P*) as well in addition to *e* and *K*.

3 Results and Discussion

According to Wei (2009) and Wei et al. (2009), occurring of a catastrophic change depends on the nature of an equilibrium point when the position crosses the curved surface of the bifurcation section. Based on that, they have made a concrete analysis of swallowtail catastrophe pattern, and explained a theory to decide whether the change is catastrophic. According to their explanation, a change of the equilibrium state is considered in two ways; one is that if the point changes from the region IV or V to I, and second is that if the point changes from the region II to I, III, IV or V (Fig 1). As a result of that movement, the equilibrium state of the system could be changed to a different state or to an unstable region. It is considered that the situation on this change would be a catastrophe. Similarly, a catastrophic change could not be happened if the point changes from the region I to II, IV or V or from the region III, IV or V to II. In order to predict the catastrophic behavior of the aphid population growth for different cases of *u*, we analyzed the three dimensional control space for stability of data points of *u, v* and *w* values. The *u, v* and *w* values simulated by the APHIDSim using the data sets collected by the Northwest A&F University are furnished in Table 1 to 4 with aphid population data and the relevant catastrophe regions (I - V) where the equilibrium points are located in the control space. We found that there are some different patterns of behavior of population growth compared to the results published in Piyaratne et al. (2013) while the major catastrophic movement, sudden jump, is still remain in some data in different years. The catastrophic region relevant to each data point is determined by plotting the discriminant curve points (*u*, *v* and *w* values) in the three dimensional control space and shown in Fig. 2 to 5 for four data sets respectively.

Table 1 *u*, *v* and *w* values for the year 1984 calculated by APHIDSim with relevant catastrophic regions.

No.	Day	Aphid Population /100 stems	u	v	w	Region in the control space
1	1	1	$5.23x10^{6}$	$-5.55x10^{9}$	$2.77x10^{11}$	V
2	6	3	$-1.40x10^{6}$	$1.99x10^{9}$	$-1.00x10^{11}$	IV
3	11	5	$-1.61x10^{7}$	$4.03x1^{10}$	$-2.05x10^{12}$	IV
4	16	19	$2.40x10^{6}$	$-1.47x10^{9}$	$7.17x10^{10}$	V
5	21	4	$-1.70x10^{6}$	$3.06x10^{9}$	$-1.55x10^{11}$	IV
6	26	91.7	$-5.19x10^{6}$	$1.39x10^{10}$	$-7.08x10^{11}$	IV
7	31	150	$-2.28x10^{6}$	$-8.87x10^{7}$	$8.26x10^{9}$	IV/V
8	36	205	$1.64x10^{6}$	$-7.50x10^{9}$	$3.84x10^{11}$	V
9	41	687	$-2.25x10^{6}$	$3.93x10^{9}$	$-1.99x10^{11}$	IV
10	46	1065	$-1.70x10^{7}$	$3.83x10^{10}$	$-1.95x10^{12}$	IV
11	51	1073	$-1.01x10^{6}$	$4.92x10^{8}$	$-2.38x10^{10}$	IV/V
12	56	707.7	$1.23x10^{7}$	$9.22x10^{8}$	$-6.85x10^{10}$	IV/V
13	61	251	$7.46x10^{7}$	$5.54x10^{9}$	$-4.12x10^{11}$	V

(a) (b)

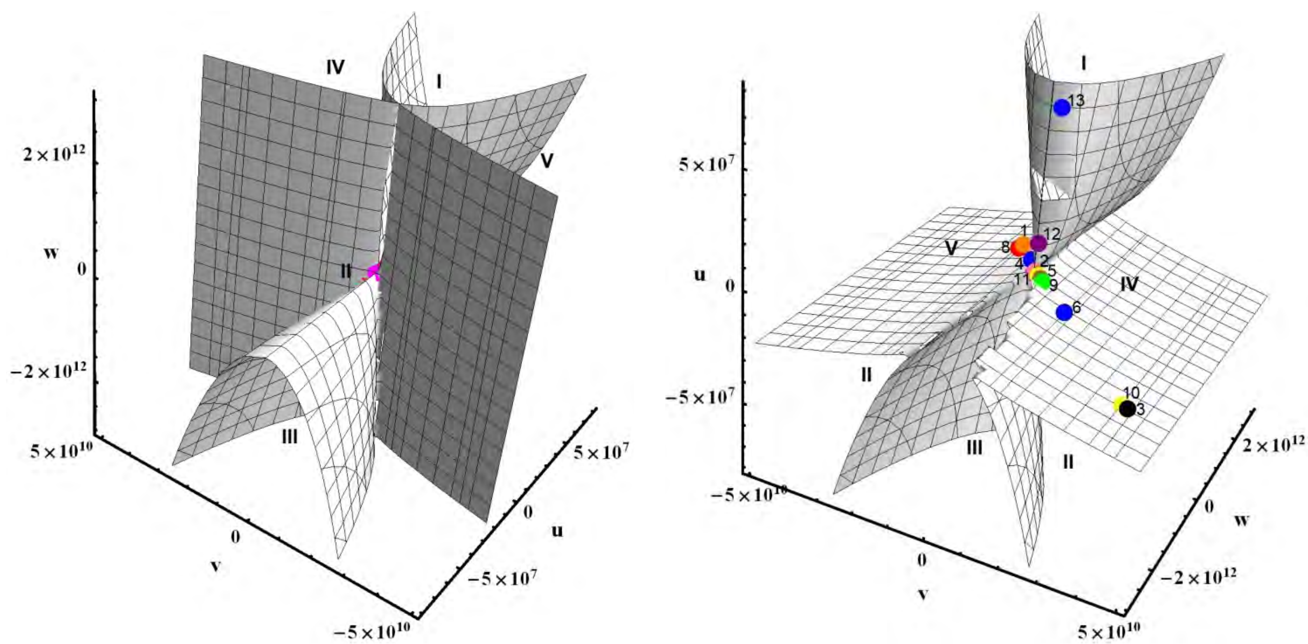

Fig. 2 Three dimensional control space with plotted data points for the year 1984. (a) None of the points can be seen in the region II; (b) all points are located in the region IV and V; points are labeled with the sample number, point colors are used to identify each point separately and therefore no any meaning related with colors used for points. It is noted that only visible points are labeled.

Results for the year 1985, 1986 and 2007 (Tables 2 - 4 and Figs 3 - 5) show that some data points are located in the region of II so that it could be expected catastrophe movements from region II to region I, III, IV or V. In the year 1984 (Table 1 and Fig. 2), all points at $u>0$ and $u<0$ are located in the regions of IV and V and therefore it could not be expected rapid changes in the aphid population growth according to the analysis of Wei (2009). However, it is clear that, since all equilibrium points are located in the regions of IV and V in 1984, it could be changed to the region I at any time (Fig. 2) and therefore, a potential is still there to have sudden change of the aphid population without a rapid change of the control factors.

Table 2 *u, v* and *w* values for the year 1985 calculated by APHIDSim with relevant catastrophic regions.

No.	Day	Aphid Population /100 stems	u	v	w	Region in the control space
1	1	1.7	-3.99×10^5	-2.56×10^7	1.84×10^9	IV/V
2	6	1.3	-4.61×10^5	-3.17×10^7	2.25×10^9	IV/V
3	11	1.3	-4.41×10^5	-2.96×10^7	2.11×10^9	IV/V
4	16	4	-8.97×10^5	-7.69×10^7	5.32×10^9	IV/V
5	21	10.7	-8.83×10^5	-8.73×10^7	5.83×10^9	II/IV
6	26	17.3	5.59×10^5	7.45×10^7	-4.94×10^9	IV/V
7	31	13.7	1.63×10^6	1.39×10^8	-1.01×10^{10}	IV/V
8	36	14	6.73×10^5	1.22×10^8	-7.59×10^9	IV/V
9	41	78	-4.43×10^5	-7.30×10^7	4.35×10^9	IV/V
10	46	73.3	-1.32×10^7	-1.65×10^9	1.07×10^{11}	II
11	51	620	6.49×10^5	5.81×10^7	-4.25×10^9	IV/V
12	56	323	1.41×10^5	1.82×10^7	-1.33×10^9	IV/V
13	61	414	3.29×10^5	3.21×10^7	-2.37×10^9	IV/V
14	66	250	3.26×10^5	3.19×10^7	-2.35×10^9	IV/V

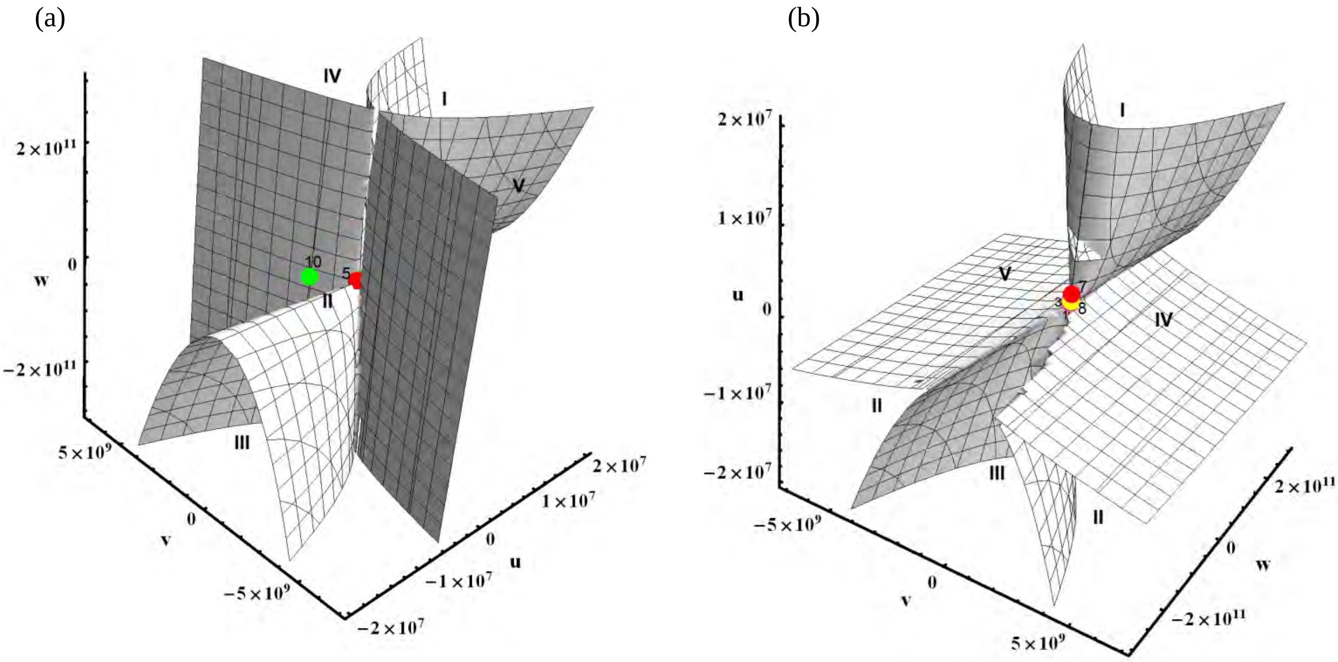

Fig. 3 Three dimensional control space with plotted data points for the year 1985. (a) Only one point (No. 10) can clearly be seen in the region II, also, No. 5 can slightly be seen; (b) points located in the region IV and V, but only visible points are labeled; point colors are used to identify each point separately and therefore no any meaning related with colors used for points.

Table 3 *u*, *v* and *w* values for the year 1986 calculated by APHIDSim with relevant catastrophic regions.

No.	**Day**	**Aphid Population /100 stems**	**u**	**v**	**w**	**Region in the control space**
1	1	2.30	-6.18×10^{5}	-5.29×10^{7}	3.61×10^{9}	IV
2	6	1.70	-8.13×10^{5}	-5.49×10^{7}	4.04×10^{9}	IV
3	11	3.30	-7.39×10^{5}	-5.12×10^{7}	3.73×10^{9}	IV
4	16	3.50	-3.31×10^{7}	-5.56×10^{9}	3.42×10^{11}	II
5	21	34.00	5.25×10^{5}	7.61×10^{7}	-4.97×10^{9}	IV/V
6	26	43.30	-6.15×10^{6}	-1.93×10^{9}	1.10×10^{11}	II
7	31	75.30	-5.34×10^{6}	-3.57×10^{8}	2.05×10^{10}	II
8	36	163.70	4.69×10^{5}	5.16×10^{7}	-3.61×10^{9}	IV/V
9	41	275.30	9.07×10^{5}	4.86×10^{8}	-2.67×10^{10}	IV
10	46	306.00	5.07×10^{5}	1.75×10^{8}	-1.00×10^{10}	IV
11	51	217.00	-1.44×10^{5}	-1.75×10^{8}	9.95×10^{9}	V
12	56	570.00	1.21×10^{6}	1.43×10^{8}	-9.56×10^{9}	IV/V
13	61	435.00	-1.23×10^{6}	-3.56×10^{8}	2.02×10^{10}	II/V
14	66	299.00	4.87×10^{5}	4.38×10^{7}	-3.24×10^{9}	IV/V
15	71	53.00	1.41×10^{7}	1.06×10^{9}	-7.83×10^{10}	IV

(a) (b)

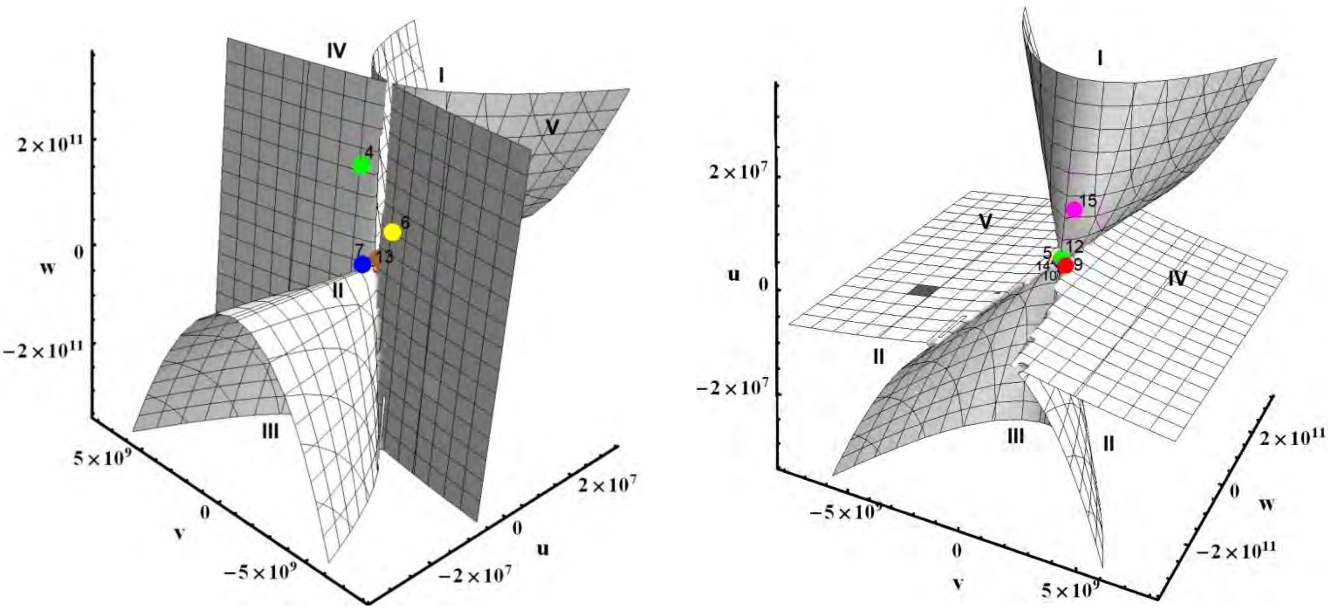

Fig. 4 Three dimensional control space with plotted data points for the year 1986. (a) Three points (No. 4, 6 and 7) can clearly be seen in the region II, also, No. 13 can slightly be seen; (b) points located in the region IV and V, but only visible points are labeled; point colors are used to identify each point separately and therefore no any meaning related with colors used for points.

In the year 1985 (Table 2 and Fig 3), there is one point located in region II (day 46) and changed to region IV or V at the day 51. According to above explanation, a catastrophic change could be expected to occur at this point, and similarly in the data, the aphid population density is changed considerably at day 46 to day 51 (Table 2). The same behavior can also be seen in the year 1986 (Table 3 and Fig. 4) and year 2007 (Table 4 and Fig. 5), but the changes are slightly matched with the actual situation. Although the model prediction indicates rapid changes at day 16 to 21 and 31 to 36 (Table 2) in 1986, only slight changes of aphid population can be seen in actual data in the same year. Similarly in year 2007, prediction indicates three changing points (day 6 to 11, day 21 to 26 and day 46 to 51) with a decline at day 46 in the aphid population (Table 3 and Fig. 4). It is clear that rapid changing points are correctly predicted by the model while the decline point is slightly shifted when compared to actual data.

Table 4 *u*, *v* and *w* values for the year 2007 calculated by APHIDSim with relevant catastrophic regions

No	Day	Aphid Population /100 stems	u	v	w	Region in the control space
1	1	3.21	-4.93×10^{6}	-3.63×10^{8}	2.68×10^{10}	II
2	6	41.38	-1.23×10^{7}	-1.51×10^{9}	9.82×10^{10}	II
3	11	111.21	3.03×10^{7}	1.17×10^{10}	-6.52×10^{11}	IV
4	16	120.80	5.80×10^{6}	6.55×10^{8}	-4.37×10^{10}	IV
5	21	187.70	-3.07×10^{7}	-2.63×10^{9}	1.88×10^{11}	II
6	26	648.40	4.39×10^{7}	3.64×10^{9}	-2.62×10^{11}	IV
7	31	1381.74	5.83×10^{6}	4.70×10^{8}	-3.42×10^{10}	IV
8	36	1759.09	9.57×10^{6}	7.45×10^{8}	-5.47×10^{10}	IV
9	41	686.80	6.02×10^{6}	4.93×10^{8}	-3.57×10^{10}	IV
10	46	80.80	-5.66×10^{6}	-4.62×10^{8}	3.32×10^{10}	II
11	51	12.11	6.97×10^{7}	6.05×10^{9}	-4.29×10^{11}	IV

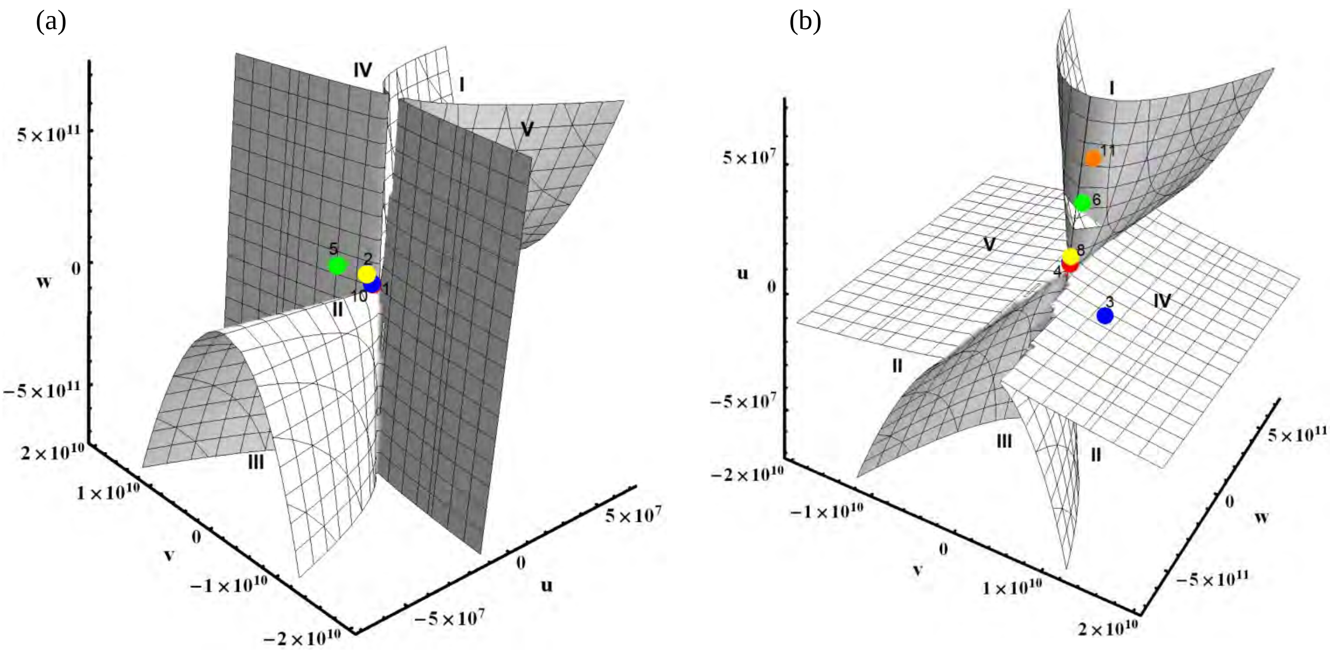

Fig. 5 Three dimensional control space with plotted data points for the year 2007. (a) Four points (No. 1, 2, 5 and 10) can clearly be seen in the region II; (b) points located in the region IV and V, but only visible points are labeled; point colors are used to identify each point separately and therefore no any meaning related with colors used for points.

When analyzing these results, it appears that catastrophic behavior of aphid population growth can be modeled using swallowtail catastrophe theory interacting with crop, weather and predator effects. Although there are some changing points which are not perfectly predicted by the simulated results, especially the points when aphid population decline, other major changes are predicted accurately. However, the parameter estimation is critically important since the change of the equilibrium points mainly depend on the negativity or positivity of the simulated control variables (u, v and w). Therefore it is highly suggested that positivity of the estimated parameters should be guaranteed.

4 Conclusion

APHIDSim software package which is developed as a higher dimensional catastrophe model application to analyze wheat aphid population dynamics has further been verified successfully with actual field data, proving that the model is reliable for field experiments. Model results show that catastrophic changes (sudden jumps) are exist in growth of aphid populations, but all the data sets used for this experiment are not shown the same behavior. Still the data used in this experiment were five day interval field observations. Therefore, short time interval (may be daily) field observations are suggested to ensure a precise prediction of aphid population growth pattern. However, as concluded in Piyaratne et al. (2013) further improvement of the program can be done including different control factors to increase the accuracy and prediction capabilities. Likewise, we believe that interesting results simulated by this model would be a new approach for short-term prediction of wheat aphid population dynamics. Since lack of practical application of catastrophe theory models on aphid population dynamics, these results would also be a new approach for scientists to study more about catastrophe theory and encourage more application of catastrophe theory models on aphid population dynamics.

Acknowledgment

This research was supported by National Natural Science Foundation China (grants 39970112 and 30470268).

References

Arnold VI. 1984. Catastrophe Theory (2nd ed). Translated from the Russian by Wassermann GS. Springer, Berlin, Germany

Bonabeau E, Theraulaz G, Deneubourg JL, et al. 1997. Self-organization in social insects. Trends in Ecology and Evolution, 12(5): 188-193

Bonabeau E. 1998. Social Insect Colonies as Complex Adaptive Systems, Ecosystems, 1(5): 437-443

Casti J. 1982. Catastrophes, control and the inevitability of spruce budworm outbreaks. Ecological Modelling, 14: 293-300

Deakin MAB. 1998. Catastrophe modelling in the biological sciences. Acta Biotheoretica, 38: 3-22

Glimore R. 1981. Catastrophe Theory for Scientists and Engineers. Wiley, New York, USA

Holt J, Cheke RA. 1996. Models of desert locust phase changes. Ecological Modelling, 91: 131-137

Hufbauer RA. 2002. Aphid population dynamics: does resistance to parasitism influence population size? Ecological Entomology, 27: 25-32

Jarošík V, Honěk A, Dixon AFG. 2003. Natural enemy ravine revisited: the importance of sample size for determining population growth. Ecological Entomology, 28: 85-91

Jones DD. 1977. Catastrophe theory applied to ecological systems. Simulation, 29: 1-15

Karley AJ, Parker WE, Pitchford JW, Douglas AE. 2004. The mid-season crash in aphid populations: why and how does it occur? Ecological Entomology, 29: 383-388

Kayacan E, Ulutas B, Kaynak O. 2010. Grey system theory-based models in time series prediction. Expert

Systems with Applications, 37: 1784-1789

Kummel M, Brown D, Bruder A. 2013. How the aphids got their spots: predation drives self-organization of aphid colonies in a patchy habitat. Oikos, 122(6): 896–906

Li J, Zhao H, Du C, Meng Q, Piyaratne MKDK. 2012. Studies on catastrophe model and stability for arthropod community succession in kiwifruit orchards. Journal of Northwest A & F University (Nat Sci Ed), 40: 135-140

Li Z, Zhao H, Liu GZ, Liu JJ. 2012. The population dynamics of insect pests of butterfly catastrophe model and analysis. Journal of Northwest A & F University, 40: 103-108

Lockwood D, Lockwood J. 1989. Application of catastrophe theory to population dynamics of rangeland grasshoppers. In: Estimation and Analysis of Insect Populations (McDonald L, Manly BJ, Lockwood J, Logan J, eds). 268-277, Springer, New York, USA

Lockwood JA, Lockwood DR. 1991. Rangeland grasshopper (Orthoptera: Acrididae) population dynamics: Insights from catastrophe theory. Environmental Entomology, 20: 970-980

Lockwood JA, Lockwood DR. 1993. Catastrophe theory: a unified paradigm for rangeland ecosystem dynamics. Journal of Range Management, 46: 282-288

Ludwig D, Jones DD, Holling CS. 1978. Qualitative analysis of insect outbreak systems: the spruce budworm and forest. Journal of Animal Ecology, 47: 315-332

Ma ZS, Bechinski EJ. 2009. An approach to the nonlinear dynamics of Russian wheat aphid population growth with the cusp catastrophe model. Entomological Research, 39: 175-181

Ouimet C, Legendre P. 1988. Practical aspects of modelling ecological phenomena using the cusp catastrophe. Ecological Modelling, 42: 265-287

Petrovskii S, Petrovskaya N, Bearup D. 2014. Multiscale approach to pest insect monitoring: Random walks, pattern formation, synchronization, and networks. Physics of Life Reviews (In Press)

Piyaratne MKDK, Zhao HY, Meng QX. 2013. APHIDSim: A population dynamics model for wheat aphids based on swallowtail catastrophe theory. Ecological Modelling, 253: 9-16

Price PW, Denno RF, Eubanks MD, Finke DL, Kaplan I. 2011. Insect Ecology: Behavior, Populations and Communities. Cambridge University Press, UK

Recknagel F. 1985. Analysis of structural stability of aquatic ecosystems as an aid for ecosystem control. Ecological Modelling, 27: 221-234

Schowalter TD. 2011. Insect Ecology: An Ecosystem Approach. Elsevier/Academic, San Diego, CA, USA

Sifeng L, Yi L. 2006. Grey Information, Theory and Practical Applications. Springer, London, UK

Tien TL. 2009. A new grey prediction model FGM (1, 1). Mathematical and Computer Modelling, 49: 1416-1426

Trivedi HV, Singh JK. 2005. Application of grey system theory in the development of a runoff prediction model. Biosystems Engineering, 92: 521-526

Wei X, Zhao H, Liu G, Wu Y. 2009. Analysis of pest population dynamics model using swallowtail catastrophe theory. Acta Ecologica Sinica, 29: 5478-5484

Wei X. 2009. Swallowtail catastrophe analysis and system analysis on pest population dynamic model. Dissertation, Northwest A & F University, China

Wu L, Wang Y. 2011. Estimation the parameters of Lotka–Volterra model based on grey direct modelling method and its application. Expert Systems with Applications, 38: 6412-6416

Zeeman EC. 1976. Catastrophe theory. Science America, 234: 65-83

Zhang WJ. 2013. Selforganizology: A science that deals with self-organization. Network Biology, 3(1): 1-14

Zhao H. 1991. Study on the cusp catastrophe model, sudden change region and controlling target during strategy of wheat aphid control. Systems Engineering, 9: 30-35

Zhao H, Wang S, Yue M. 1993. Cusp mutation model of cotton seedling aphids and its analysis. Chinese Journal of Ecology, 12: 62-66

Zhao H, Wang S. 1993a. Catastrophe forecast of pest in agriculture and analysis on its exactness. Journal of Catastrophology, 8: 10-14

Zhao H, Wang S. 1993b. Catastrophe theory and application of some main pest in agriculture. Acta Agriculturae Boreali-occidentalis Sinica, 2: 48-56

Zhao X, Zhao H, Liu G. 2005. Catastrophe analysis of a predator model with prey. Journal of Northwest A & F University (Nat Sci Ed), 33: 65-68