Article

Modeling the dynamics of carbon dioxide removal in the atmosphere

Shyam Sundar[1], **Ram Naresh**[2], **Ashish Kumar Mishra**[1], **Agraj Tripathi**[3]
[1]Department of Mathematics, P. S. Institute of Technology, Kanpur-208020, India
[2]Department of Mathematics, H. B. Technological Institute, Kanpur-208002, India
[3]Department of Mathematics, Bhabha Institute of Technology, Kanpur D.-209204, India
E-mail: ssmishra15@gmail.com

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Abstract

The temperature of Earth's surface is increasing over the past few years due to emission of global warming gases such as CO_2, CH_4 and NO_x from industries, power plants, etc., leading to several adverse effects on human and his environment. Therefore, the question of their removal/reduction from the atmosphere is very important. In this paper, a nonlinear mathematical model to study the removal/reduction of carbon dioxide by using suitable absorbent (such as aqueous ammonia solution, amines, sodium hydroxide, etc.) near the source of emission and externally introducing liquid species in the atmosphere is presented. Dynamical properties of the model which include local and global stabilities for the equilibrium are analyzed carefully. Model analysis is performed by considering three physical situations i.e. when both absorbent and the liquid species are used, only absorbent is used and only liquid species is used. It is shown that the concentration of carbon dioxide decreases as the rate of introduction of absorbent in the absorber increases. It decreases further as the rate of introduction of liquid species. Thus, the concentration of carbon dioxide would be reduced by a large amount if adequate amount of absorbent is used near the source of emission. The remaining amount can be reduced further by infusing liquid drops in the atmosphere. Numerical simulations are also carried out to support the analytical results.

Keywords carbon dioxide; absorbent; liquid species; stability; simulations.

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1 Introduction

It is well known that global warming is caused due to emission of greenhouse gases (GHG) such as carbon dioxide (CO_2), methane (CH_4), oxide of nitrogen (NO_x), hydro fluorocarbons (HFCs), sulphur hexafluoride (SF_6), etc., leading to increase in average temperature of Earth's atmosphere (Ferrarini, 2012; Wu and Zhang, 2012; Rai and Rai, 2013a, b). Warming of the climate system is unequivocal, and scientists are more than 90% certain that most of it is caused by elevated level of greenhouse gases, mainly due to carbon dioxide, produced by human activities such as deforestation and the burning of fossil fuels (Millstein and Harley, 2009; Schneising et al., 2013). An increase in global temperature will cause sea levels to rise and will change the amount and pattern of precipitation, and a probable expansion of subtropical deserts (Gregory and Oerlemans, 1998; Overpeck and Weiss, 2009). Climate responds to several types of external forcing, such as radiative forcing due to changes in atmospheric composition (mainly greenhouse gas concentrations), changes in solar luminosity, volcanic eruptions, and variations in Earth's orbit around the Sun (Botzen et al., 2008; Wu et al., 2008; Höhne et al., 2011). The problem of lower agricultural production, melting of glaciers and lesser summer stream flows has been found as a result of global warming threat. Due to the increase in Earth’s average temperature various new diseases are occurring frequently, since the bacteria can survive better in such type of situations and multiply faster in favorable conditions. Further, many undesirable consequences such as economical loss, loss of biodiversity, drought, flood, etc, may arise due to global warming (Douglas and Selden, 1995; Robinson et al., 2007; Gao et al., 2012; Wu et al., 2012).

Since global warming due to carbon dioxide potentially affects human and the environment, the mitigation of carbon dioxide from the atmosphere is absolutely necessary. In this direction, several investigations have been made to reduce the concentration of carbon dioxide into the atmosphere significantly by introducing liquid droplet (Dixon and Russell, 1950; Lackner, 2009; Chen et al., 2011, 2012a, b, 2013; Han et al., 2013). Chen et al. (2012a) made a theoretical analysis to study the dynamics of carbon dioxide uptake by a liquid droplet and have observed the abatement in carbon dioxide concentration absorbed by liquid droplets. Han et al. (2013) have established an experimental set-up to calculate the liquid phase mass transfer coefficient of carbon dioxide absorption by single water droplet. They have shown that the mass transfer coefficient decreases as droplet formation time increases.

Some investigations have also been made to study the removal of carbon dioxide by using suitable absorbents such as sodium hydroxide, amines and aqueous ammonia solution (Bai and Yeh, 1997; Resnik et al., 2004; Zeman and Lackner, 2004; Rao et al., 2006; Park et al., 2008; Stolaroff et al., 2008; Mahmoudkhani and Keith, 2009; Kamarudin et al., 2010; Zeng et al., 2011; Choi et al., 2012; Darde et al., 2012; Niu et al., 2012; Kale et al., 2013; Voice et al., 2013; Yoo et al., 2013; Zhang and Guo, 2013). In this regard, Bai and Yeh (1997) have shown experimentally that under appropriate conditions, the efficiency of carbon dioxide removal by ammonia scrubbing could be more than 95 %. The process of absorption of carbon dioxide by sodium hydroxide forming calcium carbonate has been explored (Mahmoudkhani and Keith, 2009). Kale et al. (2013) have modeled a phenomenon to study the mechanism of carbon dioxide absorption by aqueous mono-ethanolamine (MEA). It has also been observed that the reduction in carbon dioxide concentration by aqueous ammonia solution is more predominant (Yeh and Bai, 1999; Pellegrini et al., 2010). In this regard, Yeh and Bai (1999) have made an experiment to compare the performance of two absorbents, aqueous ammonia solution and mono-ethanolamine (MEA), for scrubbing carbon dioxide in terms of high CO_2 removal efficiency and absorbing capacity. They have ascertained that the CO_2 removal efficiency and absorbing capacity of aqueous ammonia solution are more salutary than those of MEA. The CO_2 removal efficiency and absorbing capacity of aqueous ammonia solution was found up to 99% and 1.2 Kg CO_2 per Kg NH_3 respectively while for MEA these were found 94% and 0.40 Kg CO_2 per Kg MEA.

It is worth mentioning here the following chemical reactions showing the interaction of carbon dioxide with water and aqueous ammonia (Knoche, 1980; Jiang et al., 2012),

$$\mathrm{CO_2 + H_2O \rightarrow H_2CO_3}$$

$$\mathrm{2NH_3 + CO_2 \rightarrow NH_2COONH_4}$$

$$\mathrm{NH_2COONH_4 + H_2O \rightarrow NH_4HCO_3(s) + NH_3}$$

Several modeling studies have been conducted to understand the phenomenon of removal of gases and particulate matters from the atmosphere (Slinn, 1977; Shukla et al., 2008; Sundar et al., 2011; Shukla et al., 2014). Shukla et al. (2008) proposed and analyzed a nonlinear mathematical model to study the removal of gases and particulate matters by raindrops. In their analysis they found considerable decrease in the concentration of gases and particulate matters. Shukla et al. (2014) have proposed a nonlinear mathematical model to study the removal of carbon dioxide from the atmosphere to reduce global warming considering two cases; (i) removal of carbon dioxide from the near Earth atmosphere by greenbelt plantation and introducing liquid species and (ii) removal of carbon dioxide from the upper Earth atmosphere by using external liquid species particulate matters. In their study, it has been shown that the concentration of carbon dioxide can be reduced in the atmosphere.

If the concentration of carbon dioxide into the atmosphere keeps on increasing hereafter, probably the menace of global warming becomes more frightening and therefore the diminution in carbon dioxide concentration is crucial. Therefore, in the present paper, we have developed a mathematical model which explores the removal of carbon dioxide using suitable absorbent and externally introduced liquid species in the atmosphere. The paper is organized as follows: In section 2, the model framework is put in place and it enables us to derive the model equations and provides the region of feasible solutions. The equilibrium and stability (local and global) of the steady states are provided in section 3 together with variation of dependent variables with different parameters, followed by numerical simulation in section 4. Finally we round up the paper by conclusion in section 5.

2 Mathematical Model

Consider an atmosphere affected by global warming gases containing CO_2. Carbon dioxide is passed through a chamber containing absorbent near the source of emission to reduce its concentration. Further, externally introduced liquid species, in the atmosphere, interacts with carbon dioxide to form particulate matter, being removed by gravitation. To model the phenomenon, the following assumptions are made,

1. The rate of emission of carbon dioxide is constant.
2. There exists a threshold concentration of carbon dioxide below which harmful effects are insignificant.
3. The rate of introduction of liquid species in the atmosphere is in direct proportion to the difference of cumulative and threshold concentrations of carbon dioxide. Here, cumulative concentration stands for global average temperature of carbon dioxide in the atmosphere.
4. The rate of inflow of absorbent in absorption chamber is in direct proportion to the cumulative concentration of carbon dioxide.
5. The decrease in cumulative concentration of carbon dioxide is directly proportional to the cumulative concentration of CO_2 itself and the concentration of externally introduced liquid species. The decrease in concentration of CO_2 is also proportional to the product of cumulative concentration of carbon dioxide and the concentration of suitable absorbent.
6. During the interaction of carbon dioxide and externally introduced liquid species, particulate matter is formed which is removed from the atmosphere by gravity, lowering the concentration of carbon dioxide.
7. The natural depletion rates of carbon dioxide and externally introduced liquid species are assumed to be directly proportional to their respective concentrations.

To develop a mathematical model, let C be the cumulative concentration of carbon dioxide, A be the concentration of externally introduced liquid species, C_p be the density of particulate matter formed due to interaction of carbon dioxide with liquid species and C_m be the concentration of a suitable absorbent. Further, let Q be the rate of discharge of carbon dioxide from different sources such as chimneys of power plants, industries, etc. with its natural depletion rate $\delta_0 C$. The depletion of carbon dioxide due to interaction with liquid species is assumed to be in direct proportion to the cumulative concentration of carbon dioxide and the concentration of liquid species (i.e. $\lambda_1 CA$) and by the same amount liquid species is also consumed, λ_1 being the interaction rate coefficient. The rate of introduction of liquid species needed to reduce the carbon dioxide concentration is directly proportional to the difference of its cumulative concentration C and the threshold concentration C_0 (i.e. $\lambda(C-C_0)$) where λ is its growth rate coefficient. It may be noted that if $C < C_0$ i.e. if the cumulative concentration of CO_2 is below the threshold, the introduction of external liquid species is not required. The constant λ_0 is the natural depletion rate coefficient of liquid species, for example by evaporation. Let θ be the rate by which particulate matter is formed in the atmosphere as a result of interaction of carbon dioxide with liquid species, and θ_0 is its natural depletion rate coefficient. Let μ be the rate of inflow of absorbent in absorption chamber with its natural depletion rate $\mu_0 C_m$. The depletion of carbon dioxide due to interaction with absorbent is assumed to be in direct proportion to the cumulative concentration of carbon dioxide and that of absorbent and by the same amount absorbent is also used. The constant μ_1 is the interaction rate coefficient of carbon dioxide with absorbent. All the above constants are assumed to be positive.

In view of the above assumptions and considerations, the dynamics of the model is governed by the following system of nonlinear ordinary differential equations,

$$\frac{dC}{dt} = Q - \delta_0 C - \lambda_1 CA - \mu_1 CC_m \tag{1}$$

$$\frac{dA}{dt} = \lambda (C - C_0) - \lambda_0 A - \lambda_1 CA \tag{2}$$

$$\frac{dC_p}{dt} = \theta\lambda_1 CA - \theta_0 C_p \tag{3}$$

$$\frac{dC_m}{dt} = \mu C - \mu_0 C_m - \mu_1 CC_m \tag{4}$$

with $C(0) = C_0 \geq 0, A(0) \geq 0, C_p(0) \geq 0, C_m(0) \geq 0$.

Remark For the physical importance of the model, it is required that $C > C_0$, and then the introduction of liquid species in the atmosphere would continue till $C \leq C_0$. If $C \leq C_0$, then there is no need to neutralize carbon dioxide by introducing external liquid species. Further, if λ_1 and δ_0 are so large that $\frac{dC}{dt}$ becomes negative, this implies that the global warming gas is removed completely from the atmosphere. It is also noted here that $\frac{dC_p}{dt}$ may become negative for very large value of θ_0 implying that the particulate matter, so formed due the interaction of carbon dioxide with liquid species, is removed completely.

Lemma The region of attraction for all solutions of model (1) – (4) initiating in the positive octant is given by the set,

$$\Omega = \left\{ (C, A, C_P, C_m) \in R_+^4 : 0 \le C \le \frac{Q}{\delta_0}, 0 \le A \le \tilde{A},\ 0 \le C_P \le \tilde{C}_p,\ 0 \le C_m \le \tilde{C}_m \right\} \tag{5}$$

where $\tilde{A} = \frac{\lambda}{\lambda_0}\left(\frac{Q}{\delta_0} - C_0\right)$, $\tilde{C}_p = \theta\lambda_1 \frac{Q}{\delta_0}\frac{\lambda}{\lambda_0}\left(\frac{Q}{\delta_0} - C_0\right)$ and $\tilde{C}_m = \frac{\mu}{\mu_0}\frac{Q}{\delta_0}$

3 Equilibrium and Stability Analysis

In this section, we analyze the model under the following three cases:

Case I When both the absorbent and the externally introduced liquid species are used

In this case, it is assumed that the if absorbent is not able to bring down the concentration of carbon dioxide below its threshold, then we have to use external liquid species also into the atmosphere to reduce the remaining amount of carbon dioxide. Thus, in this case, we have $C > C_0$, $\mu > 0$ i.e. the cumulative concentration of $\mathrm{CO_2}$ in the atmosphere is larger than its threshold value. We use absorbent near the source and liquid species in the atmosphere.

Case II When only absorbent is used

This case corresponds to the reduction of carbon dioxide only by using absorbent assuming that absorbent is enough to bring down the concentration of carbon dioxide below its threshold. Thus, in this case, we have $C < C_0$, $\mu > 0$ i.e. the cumulative concentration of $\mathrm{CO_2}$ in the atmosphere is less than its threshold value and no external liquid species is required.

Case III When only externally introduced liquid species is used

In this case, $C > C_0$, $\mu < 0$ i.e. the cumulative concentration of $\mathrm{CO_2}$ in the atmosphere is larger than its threshold value. We explore the role of externally introduced liquid species only and not the absorbent to reduce the concentration of carbon dioxide.

3.1 Case I When both the absorbent and the externally introduced liquid species are used

The model system (1) – (4) has a unique non-trivial equilibrium, $E^*(C^*, A^*, C_P{}^*, C_m{}^*)$ where $C^*, A^*, C_P{}^*$ and $C_m{}^*$ are positive solutions of the following system of algebraic equations,

$$Q - \delta_0 C - \lambda_1 CA - \mu_1 CC_m = 0 \tag{6}$$

$$A = \frac{\lambda(C - C_0)}{\lambda_0 + \lambda_1 C} = f(C) \tag{7}$$

$$C_P = \frac{\theta\lambda_1 CA}{\theta_0} \tag{8}$$

$$C_m = \frac{\mu C}{\mu_0 + \mu_1 C} = g(C) \tag{9}$$

Using equation (6), we assume

$$F(C) = Q - \delta_0 C - \lambda_1 C f(C) - \mu_1 C g(C) = 0 \tag{10}$$

from which we get,

$$F(0) = Q > 0$$

$$F\left(\frac{Q}{\delta_0}\right) = -\lambda_1\left(\frac{Q}{\delta_0}\right) f\left(\frac{Q}{\delta_0}\right) - \mu_1\left(\frac{Q}{\delta_0}\right) g\left(\frac{Q}{\delta_0}\right) < 0$$

This implies that there exists a root of $F(C) = 0$ (say C^*) in $0 \le C \le \frac{Q}{\delta_0}$.

For the uniqueness of the root, we have

$$F'(C) = -\delta_0 - \lambda_1 C f'(C) - \lambda_1 f(C) - \mu_1 C g'(C) - \mu_1 g(C) < 0$$

where $f'(C) = \frac{\lambda\lambda_0 + \lambda\lambda_1 C_0}{(\lambda_0 + \lambda_1 C)^2} > 0$ and $g'(C) = \frac{\mu\mu_0}{(\mu_0 + \mu_1 C)^2} > 0$

This guarantees the uniqueness of the root in $0 \le C \le \frac{Q}{\delta_0}$. By knowing the value of C^*, we can find the values of $A^*, C_p{}^*$ and $C_m{}^*$ from the equations (7), (8) and (9) respectively.

In the following, we check the variations of dependent variables with respect to relevant parameters, Using equations (7) and (9) in (6), we get

$$\lambda_1\mu_1\delta_0 C^3 + \{\mu\mu_1 + (\lambda_1\mu_0 + \lambda_0\mu_1)\delta_0 + \lambda\lambda_1 - Q\lambda_1\mu_1\}C^2$$

$$-\{Q(\lambda_1\mu_0 + \lambda_0\mu_1) + \lambda\lambda_1 C_0 - \lambda_0\mu_0\delta_0\}C - Q\lambda_0\mu_0 = 0 \tag{11}$$

3.1.1 Variation of C with Q

Differentiating (11) with respect to 'Q', we get,

$$\frac{dC}{dQ} = \frac{C\{\lambda_1\mu_1 C^2 + (\lambda_1\mu_0 + \lambda_0\mu_1)C + \lambda_0\mu_0\}}{\lambda_1\mu_1\delta_0 C^3 + Q\lambda_0\mu_0 + \{Q(\lambda_1\mu_0 + \lambda_0\mu_1) + \lambda\lambda_1 C_0\}C + \lambda_0\mu_0\delta_0\left(\frac{Q}{\delta_0} - C\right)} > 0, \text{ since } \frac{Q}{\delta_0} > C$$

This implies that the cumulative concentration C of carbon dioxide increases with increase in its rate of discharge Q.

3.1.2 Variation of A with Q

Differentiating (7) with respect to ‘ C ’, we get

$$\frac{dA}{dC}=\frac{\lambda\lambda_0+\lambda\lambda_1 C_0}{(\lambda_0+\lambda_1 C)^2}>0$$

Since $\frac{dC}{dQ}>0$, therefore, from $\frac{dA}{dQ}=\frac{dA}{dC}\frac{dC}{dQ}$ we have $\frac{dA}{dQ}>0$

This implies increase in the requirement of externally introduced liquid species with the increase in the rate of emission of carbon dioxide.

3.1.3 Variation of C_m with Q

Differentiating (9) with respect to ‘ C ’, we get

$$\frac{dC_m}{dC}=\frac{\mu\mu_0}{(\mu_0+\mu_1 C)^2}>0$$

Since $\frac{dC}{dQ}>0$, therefore, from $\frac{dC_m}{dQ}=\frac{dC_m}{dC}\frac{dC}{dQ}$ we have $\frac{dC_m}{dQ}>0$

This implies that if the rate of emission of carbon dioxide increases, the requirement of absorbent is also increases.

3.1.4 Variation of C with λ

Differentiating (11) with respect to ‘ λ ’, we get

$$\frac{dC}{d\lambda}=-\frac{\lambda_1 C^2(C-C_0)}{\lambda_1\mu_1\delta_0 C^3+Q\lambda_0\mu_0+\{Q(\lambda_1\mu_0+\lambda_0\mu_1)+\lambda\lambda_1 C_0\}C+\lambda_0\mu_0\delta_0\left(\frac{Q}{\delta_0}-C\right)}<0 \text{ since } \frac{Q}{\delta_0}>C$$

This implies that the cumulative concentration C of carbon dioxide decreases as growth rate of externally introduced liquid species λ increases.

3.1.5 Variation of C with μ

Differentiating (11) with respect to ‘ μ ’, we get,

$$\frac{dC}{d\mu}=-\frac{\mu_1 C^3}{\lambda_1\mu_1\delta_0 C^3+Q\lambda_0\mu_0+\{Q(\lambda_1\mu_0+\lambda_0\mu_1)+\lambda\lambda_1 C_0\}C+\lambda_0\mu_0\delta_0\left(\frac{Q}{\delta_0}-C\right)}<0, \text{ since } \frac{Q}{\delta_0}>C$$

This implies that the concentration of carbon dioxide C decreases as the rate of introduction of absorbent (i.e. μ) in the absorption chamber increases.

Similarly, we can show that $\frac{dC}{d\lambda_1}<0$, $\frac{dC}{d\mu_1}<0$, $\frac{dA}{d\lambda}>0$, $\frac{dC_P}{d\lambda}>0$ and $\frac{dC_P}{d\theta_0}<0$

From the above analysis, it is observed that more the concentration of carbon dioxide in the atmosphere, more amount of liquid species is required for its removal. As the rate of externally introduced liquid species increases, the concentration of carbon dioxide decreases.

3.1.6 Stability analysis

In the following two theorems we state the results regarding the stability of equilibrium.

Theorem 3.1.1 The non-trivial equilibrium E^* is locally asymptotically stable, without any condition.

Proof The Jacobian matrix of the system (1) – (4) about E^* is

$$J(E^*) = \begin{bmatrix} -\dfrac{Q}{C^*} & -\lambda_1 C^* & 0 & -\mu_1 C^* \\ \lambda - \lambda_1 A^* & -(\lambda_0 + \lambda_1 C^*) & 0 & 0 \\ \theta\lambda_1 A^* & \theta\lambda_1 C^* & -\theta_0 & 0 \\ \mu - \mu_1 {C_m}^* & 0 & 0 & -(\mu_0 + \mu_1 C^*) \end{bmatrix}$$

It is clear that one eigenvalue of the above Jacobian is negative and the others are given by the following characteristic equation,

$$F(x) = x^3 + a_1 x^2 + a_2 x + a_3 = 0 \tag{12}$$

where $a_1 = (\mu_0 + \mu_1 C^*) + \dfrac{Q}{C^*} + (\lambda_0 + \lambda_1 C^*) > 0$

$$a_2 = (\mu_0 + \mu_1 C^*)\frac{Q}{C^*} + \frac{Q}{C^*}(\lambda_0 + \lambda_1 C^*) + (\lambda_0 + \lambda_1 C^*)(\mu_0 + \mu_1 C^*) > 0$$

$$a_3 = (\mu_0 + \mu_1 C^*)\frac{Q}{C^*}(\lambda_0 + \lambda_1 C^*) + (\mu - \mu_1 {C_m}^*)(\lambda_0 + \lambda_1 C^*)\mu_1 C^* + (\mu_0 + \mu_1 C^*)(\lambda - \lambda_1 A^*)\lambda_1 C^* > 0$$

Since $\lambda - \lambda_1 A^* > 0$, $\mu - \mu_1 {C_m}^* > 0$ we find that $a_1 > 0,\ a_2 > 0,\ a_3 > 0$ and $a_1 a_2 > a_3$. Thus, equilibrium E^* is locally asymptotically stable by Routh Hurwitz criteria (Barauer and Castillo-Chavez, 2001) without any condition.

Theorem 3.1.2 The equilibrium E^* is nonlinearly asymptotically stable in the positive octant Ω, without any condition.

Proof To prove this theorem, we consider the following positive definite function

$$V = \frac{1}{2}m_1(C - C^*)^2 + \frac{1}{2}m_2(A - A^*)^2 + \frac{1}{2}m_3(C_P - {C_P}^*)^2 + \frac{1}{2}m_4(C_m - {C_m}^*)^2 \tag{13}$$

where $m_i\ (i = 1,\ 2,\ 3, 4)$ are positive constants, to be chosen appropriately.

Differentiating *V* with respect to *t*, we get

$$\frac{dV}{dt} = m_1(C - C^*)\dot{C} + m_2(A - A^*)\dot{A} + m_3(C_P - {C_P}^*)\dot{C}_p + m_4(C_m - {C_m}^*)\dot{C}_m$$

Putting the values of derivatives and simplifying, we get,

$$\frac{dV}{dt} = -m_1(\delta_0 + \lambda_1 A + \mu_1 C_m)(C - C^*)^2 - m_2(\lambda_0 + \lambda_1 C)(A - A^*)^2$$

$$-m_3\theta_0(C_P - {C_P}^*)^2 - m_4(\mu_0 + \mu_1 C)(C_m - {C_m}^*)^2$$

$$+[-m_1\lambda_1 C^* + m_2(\lambda - \lambda_1 A^*)](C - C^*)(A - A^*)$$

$$+m_3\theta\lambda_1 A^*(C-C^*)(C_P-C_P{}^*)+m_3\theta\lambda_1 C(A-A^*)(C_P-C_P{}^*)$$

$$+[-m_1\mu_1 C^*+m_4(\mu-\mu_1 C_m{}^*)](C-C^*)(C-C_m{}^*)$$

After some algebraic manipulations and by choosing $m_1=1$, $m_2=\frac{\lambda_1 C^*}{\lambda-\lambda_1 A^*}$, $m_4=\frac{\mu_1 C^*}{\mu-\mu_1 C_m{}^*}$, and

$$m_3<\frac{\theta_0}{(\lambda_1\theta)^2}\min\left\{\frac{2\delta_0}{3A^{*2}},\ \frac{\lambda_0\lambda_1 C^*}{\left(\frac{Q}{\delta_0}\right)^2(\lambda-\lambda_1 A^*)}\right\} \text{where}\ \lambda-\lambda_1 A^*>0\ \text{ and }\ \mu-\mu_1 C_m{}^*>0,$$

E^* is nonlinearly stable without any condition.

The above theorems imply that the concentration of carbon dioxide can be controlled by introducing liquid species in the atmosphere and absorbent near the source of emission.

3.2 Case II When only absorbent is used

In this case, we use only absorbent near source of emission assuming that the concentration of carbon dioxide in emerging flue gases would be less than its threshold and then there is no need to use external liquid species. In this case, the model (1) – (4) is reduced to the following form

$$\frac{dC}{dt}=Q-\delta_0 C-\mu_1 CC_m \tag{14}$$

$$\frac{dC_m}{dt}=\mu C-\mu_0 C_m-\mu_1 CC_m \tag{15}$$

with $C(0)=C_0\geq 0, C_m(0)\geq 0$

The model has a unique equilibrium $\overline{E}(\overline{C},\overline{C}_m)$. The positive solution of $\overline{E}(\overline{C},\overline{C}_m)$ is given by the following set of algebraic equations,

$$Q-\delta_0 C-\mu_1 CC_m=0 \tag{16}$$

$$C_m=\frac{\mu C}{\mu_0+\mu_1 C} \tag{17}$$

Using (16) and (17), we get

$$\mu_1(\delta_0+\mu)C^2-(Q\mu_1-\mu_0\delta_0)C-Q\mu_0=0 \tag{18}$$

from which we get positive value of C as

$$C=\frac{(Q\mu_1-\mu_0\delta_0)+\sqrt{(Q\mu_1-\mu_0\delta_0)^2+\mu_0\mu_1(\delta_0+\mu)}}{2\mu_1(\delta_0+\mu)} \quad (\text{say } \overline{C}) \tag{19}$$

By knowing the value of $\overline{C}$, we can find the value of $\overline{C}_m$ from equation (17).

We also note from equation (14) that $\lim\limits_{\mu_1 \to 0} C(t) = \frac{Q}{\delta_0}$

This implies that if due to unfavourable conditions, absorbent is unable to absorb CO_2 then it remains at its maximum concentration.

As before, it can also be checked that $\frac{dC}{d\mu} < 0$ and $\frac{dC}{d\mu_1} < 0$. It means that the cumulative concentration of CO_2 decreases as the inflow rate and interaction of absorbent (with CO_2) increases.

Lemma The region of attraction for all solution of (14) – (15) initiating in the positive octant is given by the set,

$$\Pi = \left\{ (C, C_m) \in R_+^2 : 0 \le C \le \frac{Q}{\delta_0},\ 0 \le C_m \le \frac{\mu}{\mu_0} \frac{Q}{\delta_0} \right\} \tag{20}$$

Now we state following theorems regarding the stability of equilibrium $\overline{E}$.

Theorem 3.2.1 The equilibrium $\overline{E}(\overline{C}, \overline{C}_m)$ is locally asymptotically stable, without any condition.

Proof The eigenvalues corresponding to $\overline{E}(\overline{C}, \overline{C}_m)$ of Jacobian matrix, in this case, are given by the following quadratic equation

$$G(y) = y^2 + b_1 y + b_2 = 0 \tag{21}$$

where $b_1 = \frac{Q}{\overline{C}} + \mu_0 + \mu_1 \overline{C} > 0$

$$b_2 = \frac{Q}{\overline{C}}(\mu_0 + \mu_1 \overline{C}) + (\mu - \mu_1 \overline{C}_m)\mu_1 \overline{C} > 0$$

Since $b_1 > 0$, $b_2 > 0$, therefore both the roots of equation (21) are either negative or have negative real part.

Thus, $\overline{E}(\overline{C}, \overline{C}_m)$ is locally asymptotically stable.

Theorem 3.2.2 The equilibrium $\overline{E}(\overline{C}, \overline{C}_m)$ is nonlinearly asymptotically stable, without any condition within the region of attraction Π.

Proof Consider the following positive definite function about $\overline{E}(\overline{C}, \overline{C}_m)$

$$U = \frac{1}{2} l_1 (C - \overline{C})^2 + \frac{1}{2} l_2 (C_m - \overline{C}_m)^2 \tag{22}$$

where l_1, l_2 are positive constants, to be chosen appropriately.

Differentiating U with respect to *t*, we get

$$\frac{dU}{dt} = l_1 (C - \overline{C})\dot{C} + l_2 (C_m - \overline{C}_m)\dot{C}_m$$

Putting the values of derivatives and simplifying, we get

$$\frac{dU}{dt} = -l_1(\delta_0 + \mu_1 C_m)(C - \overline{C})^2 - l_2(\mu_0 + \mu_1 C)(C_m - \overline{C}_m)^2$$

$$+[-l_1\mu_1\overline{C} + l_2(\mu - \mu_1\overline{C}_m)](C - \overline{C})(C_m - \overline{C}_m)$$

Choosing $l_2 = \dfrac{\mu_1\overline{C}}{\mu - \mu_1\overline{C}_m} l_1$, where $\mu - \mu_1\overline{C}_m > 0$, we note that $\dfrac{dU}{dt}$ will be negative definite showing that E^* is nonlinearly stable, without any condition.

The above theorems imply that the cumulative concentration of $\mathrm{CO_2}$ remains at its maximum equilibrium level if there is no interaction of $\mathrm{CO_2}$ with aqueous ammonia solution. Moreover, the concentration of $\mathrm{CO_2}$ decreases as the rate of introduction of absorbent increases.

3.3 Case III When only externally introduced liquid species is used

This case pertains to the situation when the cumulative concentration of carbon dioxide gas in the atmosphere is larger than its threshold value and the absorbent is not used to reduce the concentration of carbon dioxide. In this case, our aim is to study the removal/reduction of carbon dioxide from the atmosphere only by introducing suitable liquid species which interact with carbon dioxide to form particulate matter, being removed from the atmosphere by gravity.

In this case, the model (1) – (4) is reduced to the following form

$$\frac{dC}{dt} = Q - \delta_0 C - \lambda_1 CA \tag{23}$$

$$\frac{dA}{dt} = \lambda(C - C_0) - \lambda_0 A - \lambda_1 CA \tag{24}$$

$$\frac{dC_P}{dt} = \theta\lambda_1 CA - \theta_0 C_P \tag{25}$$

with $C(0) = C_0 \geq 0, A(0) \geq 0, C_p(0) \geq 0$

The model (23) – (25) again has a unique non-trivial equilibrium $\hat{E}(\hat{C}, \hat{A}, \hat{C}_P)$ where $\hat{C}, \hat{A}$ and $\hat{C}_P$ are positive solutions of the following system of algebraic equations,

$$Q - \delta_0 C - \lambda_1 CA = 0 \tag{26}$$

$$A = \frac{\lambda(C - C_0)}{\lambda_0 + \lambda_1 C} = f(C) \tag{27}$$

$$C_P = \frac{\theta\lambda_1 CA}{\theta_0} \tag{28}$$

Using equation (26), we assume

$$F(C) = Q - \delta_0 C - \lambda_1 C f(C) = 0 \tag{29}$$

from which we note that,

$$F(0) = Q > 0 \quad \text{and} \quad F\left(\frac{Q}{\delta_0}\right) = -\alpha\left(\frac{Q}{\delta_0}\right) f\left(\frac{Q}{\delta_0}\right) < 0$$

This implies that there exists a root of $F(C) = 0$ (say $\hat{C}$) in $0 \le C \le \dfrac{Q}{\delta_0}$.

For the uniqueness of the root, we have

$$F'(C) = -\delta_0 - \lambda_1 C f'(C) - \lambda_1 f(C) < 0$$

where $f'(C) = \dfrac{\lambda\lambda_0 + \lambda\lambda_1 C_0}{(\lambda_0 + \lambda_1 C)^2} > 0$

This guarantees the uniqueness of the root in $0 \le C \le \dfrac{Q}{\delta_0}$.

Lemma The region of attraction for all solutions of system (23) – (25) initiating in the positive octant is given by the set,

$$\Gamma = \left\{ (C, A, C_P) \in R_+^3 : 0 \le C \le \frac{Q}{\delta_0}, 0 \le A \le \frac{\lambda}{\lambda_0}\left(\frac{Q}{\delta_0} - C_0\right), 0 \le C_P \le \frac{\theta\lambda_1 Q}{\delta_0}\frac{\lambda}{\lambda_0}\left(\frac{Q}{\delta_0} - C_0\right) \right\} \tag{30}$$

The following theorems are proposed for the stability of equilibrium $\hat{E}$.

Theorem 3.3.1 The non-trivial equilibrium $\hat{E}$ is locally asymptotically stable, without any condition.

Proof The Jacobian matrix of the system (23) – (25) about $\hat{E}$ is

$$J(\hat{E}) = \begin{bmatrix} -(\delta_0 + \lambda_1 \hat{A}) & -\lambda_1 \hat{C} & 0 \\ (\lambda - \lambda_1 \hat{A}) & -(\lambda_0 + \lambda_1 \hat{C}) & 0 \\ \theta\lambda_1 \hat{A} & \theta\lambda_1 \hat{C} & -\theta_0 \end{bmatrix}$$

It is again noted that one eigenvalue of above Jacobian is negative and others are given by the following characteristic equation,

$$F(z) = z^2 + c_1 z + c_2 = 0 \tag{31}$$

where $c_1 = \delta_0 + \lambda_1 \hat{A} + \lambda_0 + \lambda_1 \hat{C} > 0$

$$c_2 = (\delta_0 + \lambda_1 \hat{A})(\lambda_0 + \lambda_1 \hat{C}) + \lambda_1 \hat{C}(\lambda - \lambda_1 \hat{A}) > 0$$

Since $\lambda - \lambda_1 \hat{A} > 0$, we find that $c_1 > 0, c_2 > 0$. Thus, equilibrium $\hat{E}$ is locally asymptotically stable by Routh Hurwitz criteria, without any condition.

Theorem 3.3.2 The equilibrium $\hat{E}$ is nonlinearly asymptotically stable in the positive octant Γ, without any condition.

Proof To prove the theorem, we consider the following positive definite function

$$W = \frac{1}{2}k_1(C-\hat{C})^2 + \frac{1}{2}k_2(A-\hat{A})^2 + \frac{1}{2}k_3(C_P-\hat{C}_P)^2 \qquad (32)$$

where k_i $(i = 1,\ 2,\ 3)$ are positive constants, to be chosen appropriately.

Differentiating W with respect to *t*, we get

$$\frac{dW}{dt} = k_1(C-\hat{C})\dot{C} + k_2(A-\hat{A})\dot{A} + k_3(C_P-\hat{C}_P)\dot{C}_p$$

Putting the values of derivatives and simplifying, we get,

$$\frac{dW}{dt} = -k_1(\delta_0+\lambda_1 A)(C-\hat{C})^2 - k_2(\lambda_0+\lambda_1 C)(A-\hat{A})^2 - k_3\theta_0(C_P-\hat{C}_P)^2$$

$$+[-k_1\lambda_1\hat{C} + k_2(\lambda-\lambda_1\hat{A})](C-\hat{C})(A-\hat{A})$$

$$+k_3\theta\lambda_1 A^*(C-\hat{C})(C_P-\hat{C}_P) + k_3\theta\lambda_1 C(A-\hat{A})(C_P-\hat{C}_P)$$

After some algebraic manipulations and by choosing $k_1 = 1$ and $k_2 = \dfrac{\lambda_1\hat{C}}{\lambda-\lambda_1\hat{A}}$, $\lambda-\lambda_1\hat{A} > 0$ and

$$k_3 < \frac{\theta_0}{(\lambda_1\theta)^2}\min\left\{\frac{\delta_0}{\hat{A}^2},\ \frac{\lambda_0\lambda_1\hat{C}}{\left(\dfrac{Q}{\delta_0}\right)^2(\lambda-\lambda_1\hat{A})}\right\}$$

we see that $\hat{E}$ is nonlinearly asymptotically stable, without any condition.

The above theorems imply that within the region of attraction the cumulative concentration of carbon dioxide can be reduced by introducing liquid species in the atmosphere.

4 Numerical Simulation

In this section, we perform a numerical simulation of model (1) – (4) with respect to E^*, for the different values of parameters for the validation of analytical results and to study the dynamical behavior of the model system. For that the system (1) – (4) is integrated numerically with the help of MAPLE 7 by considering the following set of parameter values,

$$Q = 1, \delta_0 = 0.1, \lambda_1 = 0.5, \lambda = 0.4, \lambda_0 = 0.2, \theta = 0.8,$$

$$\theta_0 = 0.7, C_0 = 0.60, \mu = 1, \mu_0 = 0.02, \mu_1 = 0.6$$

The equilibrium E^* is obtained as,

$$C^* = 0.85105666,\ A^* = 0.22540130,\ C_p{}^* = 0.10961673, C_m{}^* = 1.60384872$$

Eigenvalues of the Jacobian matrix corresponding to E^* are -0.70,

$-0.818760+0.115886\,i,\quad -0.818760-0.115886\,i,\quad -0.513650$. It is noted here that, two eigenvalues are negative and the other two have negative real part, therefore E^* is locally asymptotically stable.

To see the nonlinear stability behavior, for the above set of parameters, computer generated graphs in $C-A$ and $C-C_p$ plane have been shown in figures 1 and 2 respectively with different initial starts as below. From these figures, we note that all trajectories starting at any point always reach to its equilibrium.

1. $C(0)=0.7,\ A(0)=0.2,\ C_p(0)=0.10, C_m=1.0$
2. $C(0)=1.2,\ A(0)=0.2,\ C_p(0)=0.15, C_m=0.5$
3. $C(0)=1.3,\ A(0)=0.35,\ C_p(0)=0.20, C_m=1.0$
4. $C(0)=1.2,\ A(0)=0.4,\ C_p(0)=0.25, C_m=1.0$

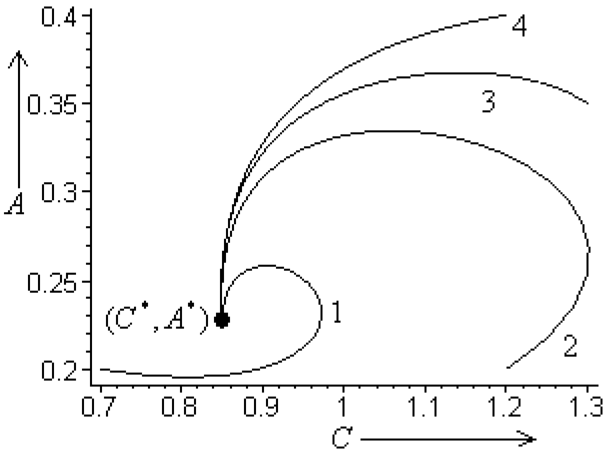

Fig. 1 Phase plane diagram for nonlinear stability in $C-A$ plane.

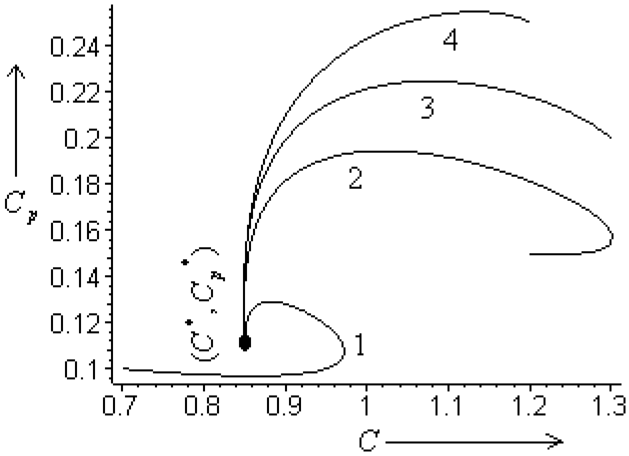

Fig. 2 Phase plane diagram for nonlinear stability in $C-C_p$ plane.

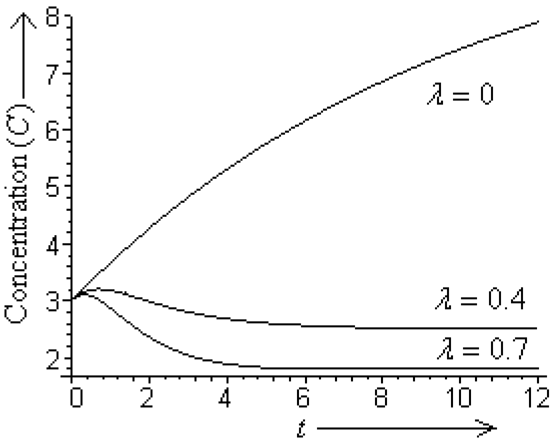

Fig. 3 Variation of CO_2 concentration C with time 't' for different values of λ at $\mu = 0$ (other parameters remains same).

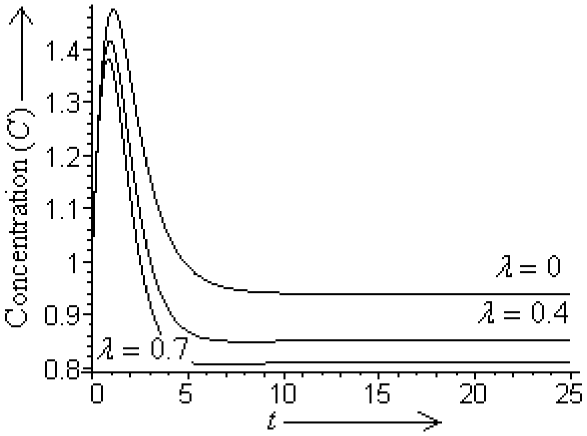

Fig. 4 Variation of CO_2 concentration C with time 't' for different values of λ at $\mu = 1$ (other parameters remains same).

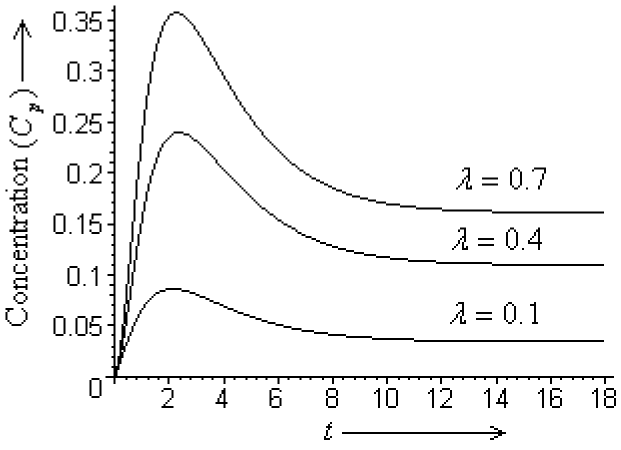

Fig. 5 Variation of density of particulate matters C_p with time 't' for different values of λ at $\mu = 1$ (other parameters remains same).

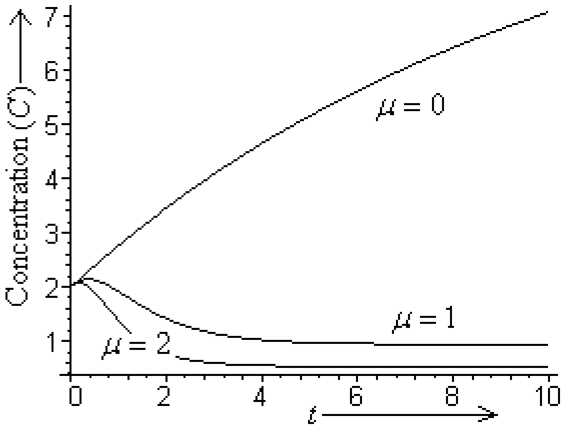

Fig. 6 Variation of CO_2 concentration *C* with time '*t*' for different values of μ at $\lambda = 0$ (other parameters remains same).

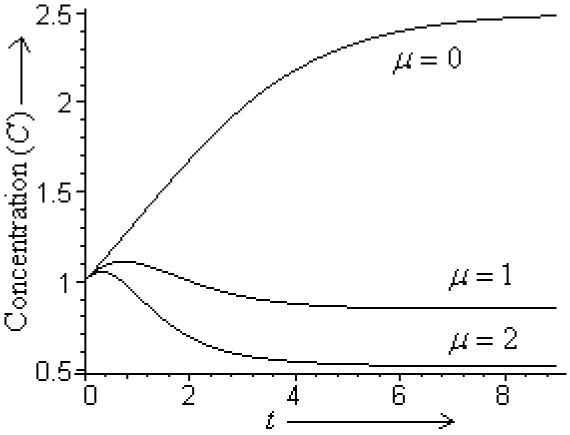

Fig. 7 Variation of CO_2 concentration *C* with time '*t*' for different values of μ at $\lambda = 0.4$ (other parameters remains same).

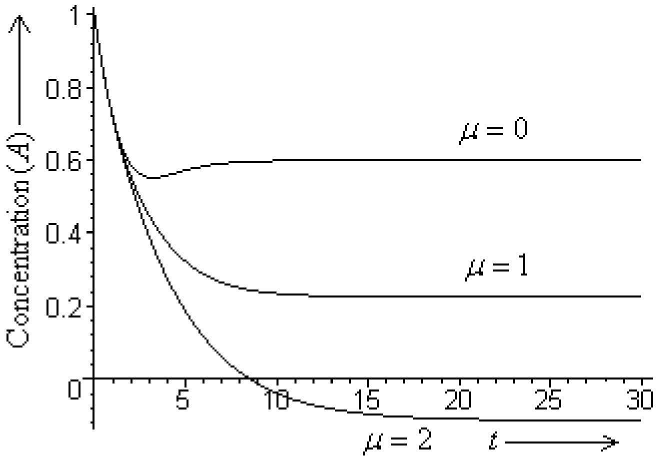

Fig. 8 Variation of concentration of external species A with time '*t*' for different values of μ (other parameters remains same).

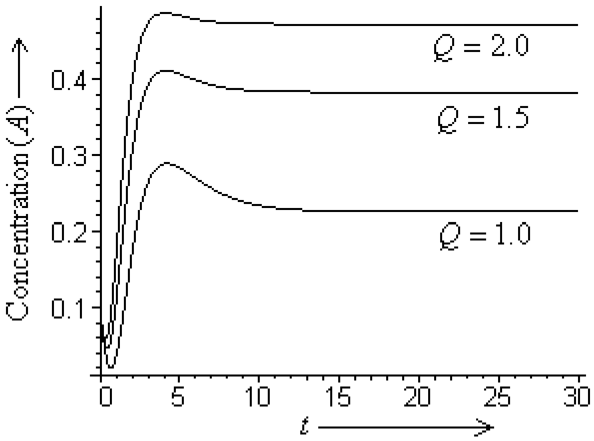

Fig. 9 Variation of concentration of external species A with time '*t*' for different values of Q (other parameters remains same).

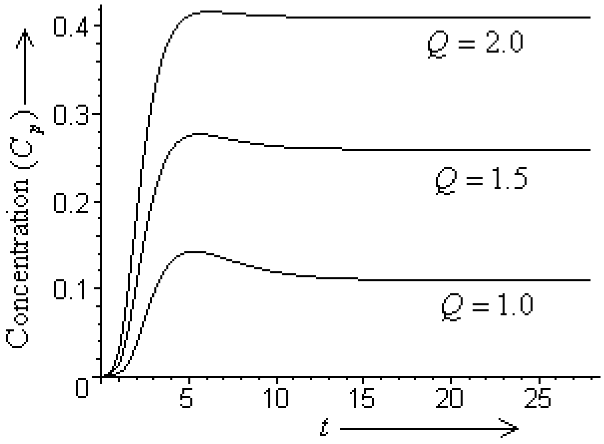

Fig. 10 Variation of density of particulate matters C_p with time '*t*' for different values of Q (other parameters remains same).

In Figs. 3 and 4, the variation of cumulative concentration *C* of carbon dioxide gas with time t is shown for different values of rate of introduction of externally introduced liquid species i.e. $\lambda = 0, 0.4, 0.7$ keeping $\mu = 0$ (without absorbent) and $\mu = 1$(with absorbent) respectively. From these figures, we note that the concentration of carbon dioxide decreases as λ increases. When λ = 0 and μ = 0 (Fig. 3) i.e. neither the absorbent near the source nor externally introduced liquid species in the atmosphere are used, the concentration of CO_2 increases continuously and remains at its maximum equilibrium level. On comparing these two figures, it is also noted that the reduction in CO_2 concentration is higher if both the removal mechanisms are used to reduce CO_2. Moreover, the amount of the liquid species required to reduce/remove CO_2 is less when the concentration of CO_2 is already being reduced at source by using absorbent at the source. In Fig. 5, the variation of concentration of particulate matter, formed due to interaction of carbon dioxide with liquid species, with time t is shown for different values of rate of introduction of externally introduced liquid species i.e. $\lambda = 0.1, 0.4, 0.7$. From this figure, we note that the density of particulate matter increases as λ increases. This means that large amount of particulate matter is formed after interaction of CO_2 with liquid species and is being removed by gravity which ultimately results in decline of CO_2 concentration. The variation of cumulative concentration *C* of carbon dioxide gas with time t for different values of inflow rate of absorbent in the absorption chamber i.e. $\mu = 0, 1, 2$ keeping $\lambda = 0$ (with no removal by liquid species) and $\lambda = 0.4$ (with removal by liquid species) is shown in Figs. 6 and 7 respectively. From these figures, it is observed that the concentration of CO_2 decreases with increase in the inflow rate of absorbent near the source. This decrease of CO_2 concentration is further enhanced if the liquid species is used in the atmosphere (Fig. 7). When $\lambda = 0$ and $\mu = 0$ (absence of both the removal mechanisms), the cumulative concentration of CO_2 continues to increase and settles at its maximum equilibrium level (Fig. 6). In Fig. 8, the variation of concentration A of externally introduced liquid species with time t is shown for different values of inflow rate of absorbent in the absorption chamber to absorb CO_2 i.e. $\mu = 0,1,2$. We see that, if μ increases, the concentration of liquid species in the atmosphere decreases, which, in turn, means no more liquid species in the atmosphere is required as the concentration of CO_2 is already reduced at the source by absorbent. We further note that at $\mu = 2$, when the equilibrium concentration of CO_2 reaches below the threshold concentration i.e. $C < C_0$, the curve crosses the $'t'$ axis and we get negative equilibrium of A. It suggests that when the equilibrium concentration of CO_2 is less than its threshold concentration, the introduction of liquid species in the atmosphere would not be required. The variation of cumulative concentration C of CO_2 and particulate matter with time 't' for different values of emission rate of CO_2 i.e. $Q = 1, 1.5, 2$, is shown in Figs. 9 and 10 respectively. From these figures, it is seen that the cumulative concentration of carbon dioxide as well as particulate matter increase with time as the emission rate of carbon dioxide increases.

5 Conclusion

A nonlinear mathematical model has been proposed and analyzed for the removal of carbon dioxide from the atmosphere by using absorbent near the source and externally introduced liquid species in the atmosphere. The model is analyzed and its dynamical properties in terms of local and global stability are established under three situations namely, when both the absorbent near the source and externally introduced liquid species in the atmosphere are used, when only absorbent is used, and when only liquid species is used. It is found that the concentration of CO_2 decreases as the value of removal parameters increases. Moreover, the concentration of carbon dioxide would be reduced by a large amount if absorbent is used near the source of emission and the remaining amount can be reduced further by infusing liquid drops in the atmosphere. If the inflow rate of absorbent is higher enough then the concentration of CO_2 would be reduced below to its threshold concentration and in that case no removal by liquid species would be required. It is also observed that by infusing more liquid species in the atmosphere, CO_2 concentration is reduced significantly due to its conversion into particulate matter which is removed by gravity. Analysis also suggests that it is preferable to use absorbent near the source itself to reduce the concentration of carbon dioxide so that its equilibrium level would be maintained below its threshold concentration. Thus, to minimize the menace of global warming due to carbon dioxide, the above removal mechanisms could be very useful.

References

Bai H, Yeh AC. 1997. Removal of CO_2 greenhouse gas by ammonia scrubbing. Industrial & Engineering Chemistry Research, 36 (6): 2490-2493

Barauer F, Castillo-Chavez C. 2001. Mathematical Models in Population Biology and Epidemiology. Springer-Verlag, New York, USA

Botzen WJW, Gowdy JM, Van Den Bergh JCJM. 2008. Cumulative CO_2 emissions: shifting international responsibilities for climate debt. Climate Policy, 8(6): 569-576

Chen WH, Hou YL, Hung CI. 2011. A theoretical analysis of the capture of greenhouse gases by single water droplet at atmospheric and elevated pressures. Applied Energy, 88: 5120-5130

Chen WH, Hou YL, Hung CI. 2012a. A study of influence of acoustic excitation on carbon dioxide capture by a droplet. Energy, 37: 311-321

Chen WH, Hou YL, Hung CI. 2012b. Influence of droplet mutual interaction on carbon dioxide capture process in sprays. Applied Energy, 92: 185-193

Chen WH, Chen SM, Hung CI. 2013. Carbon dioxide capture by single droplet using Selexol, Rectisol and water as absorbents: A theoretical approach. Applied Energy, 111: 731-741

Choi BG, Kim GH, Yi KB, Kim JN, Hong WH. 2012. Influence of operating temperature on CO_2 - NH_3 reaction in an aqueous solution. Korean Journal of Chemical Engineering, 29(4): 478-482

Darde V, Maribo-Mogensen B, Willy JM, Willy JM van W, Stenby EH, Thomsen K. 2012. Process simulation of CO_2 capture with aqueous ammonia using the extended UNIQUAC model. International Journal of Greenhouse Gas Control, 10: 74-87

Dixon BE, Russell AAW. 1950. The Absorption of Carbon Dioxide by Liquid Drops. Journal of Society of Chemical Industry, 69(9): 284-288

Douglas HE, Selden TM. 1995. Stoking the fires? CO_2 emissions and economic growth. Journal of Pubic Economy, 57(1): 85-101

Ferrarini A. 2012. I think different: Models of climate warming impact on plant species are unrealistic. Environmental Skeptics and Critics, 1(2): 30-33

Gregory JM, Oerlemans J. 1998. Stimulated future see level rise due to glacier melt based on regionally and seasonally resolved temperature changes. Nature, 391: 474-476

Gao K, Xu J, Gao G, Li Y, Hutchins DA, Huang B, Wang L, Zheng Y, Jin P, Cai X, Häder D- P, Li W, Xu K, Liu N, Riebesell U. 2012. Rising CO_2 and increased light exposure synergistically reduce marine primary productivity. Nature Climate Change, 2: 519-523

Han J, Eimera DA, Melaaena MC. 2013. Liquid phase mass transfer coefficient of carbon dioxide absorption by water droplet. Energy Procedia, 37: 1728-1735

Höhne N, Blum H, Fuglestvedt J, Skeie RB, Kurosawa A, Hu G, Lowe J, Gohar L, Matthews B, Salles ACNde, Ellermann C. 2011. Contributions of individual countries' emissions to climate change and their uncertainty. Climatic Change, 106(3): 359-391

Jiang S, Zhong W, Peng R, Y, Zhang J. 2012. Experimental studies on the influence of HCO_3^- on absorption and desorption of CO_2 from ammonia solution. Advances in Materials Physics and Chemistry, 2: 240-243

Kale C, GorakA, Schoenmakers H. 2013. Modelling of the reactive absorption of CO_2 using mono-ethanolamine. International Journal of Greenhouse Gas Control, 17: 294-308

Kamarudin KSN, Bhatti I, Suahadah SN, Hamzah UN, MohdFaiz ZF. 2010. Removal of carbon dioxide using water-in-oil emulsion liquid membrane containing triethanolamine. Journal of Applied Sciences Research, 6(12): 2251-2256

Knoche W. 1980. Chemical reactions of CO_2 in water. biophysics and physiology of carbon dioxide. In: Proceedings in Life Sciences (Christian B, Gerolf G, Heinz B, eds). 3-11

Lackner KS. 2009. Capture of carbon dioxide from ambient air. European Physical Journal Special Topics, 176: 93-106

Mahmoudkhani M, Keith DW. 2009. Low-energy sodium hydroxide recovery for CO_2 capture from atmospheric air-thermodynamic analysis. International Journal of Greenhouse Gas Control, 3(4): 376-384

Millstein DE, Harley RA. 2009. Impact of climate change on photochemical air pollution in southern

California. Atmospheric Chemistry and Physics Discussion, 9(11): 3745-3754

Niu Z, GuoY, Zeng Q, Lin W. 2012. Experimental studies and rate-based process simulations of CO_2 absorption with aqueous ammonia solutions. Industrial & Engineering Chemistry Research, 51(14): 5309-5319

Overpeck JT, Weiss JL. 2009. Projections of future see level becoming direr. PNAS, 106(51): 21461-21462

Park HS, Jung YM, You JK, Hong W-Hi, Kim J-N. 2008. Analysis of the CO_2 and NH_3 reaction in an aqueous solution by 2D IR COS: Formation of bicarbonate and carbamate. Journal of Physical Chemistry A, 112(29): 6558-6562

Pellegrini G, Strube R, Manfrida G. 2010. Comparative study of chemical absorbents in postcombustion CO_2 capture. Energy, 35(2): 851-857

Rai PR, Rai PK. 2013a. Environmental and socio-economic impacts of global climate change: An overview on mitigation approaches. Environmental Skeptics and Critics, 2(4): 126-148

Rai PR, Rai PK. 2013b. Paradigms of global climate change and sustainable development: Issues and related policies. Environmental Skeptics and Critics, 2(2): 30-45

Rao AB, Rubin ES. 2006. Identifying cost effective CO_2 control levels for amine-based CO_2 capture systems. Industrial & Engineering Chemistry Research, 45: 2421-2429

Resnik KP, Yeh JT, Pennline HW. 2004. Aqua ammonia process for simultaneous removal of CO_2, SO_2 and NO_x. International Journal of Environmental Technology and Management, 4(1/2): 89-104

Robinson BA, Robinson NE, Willie S. 2007. Environmental effects of increased atmospheric carbon dioxide. Journal of American Physical Surg, 12: 79-90

Schneising O, Heymann J, Buchwitz M, Reuter M, Bovensmann H, Burrows JP. 2013. Anthropogenic carbon dioxide source areas observed from space: assessment of regional enhancements and trends. Atmospheric Chemistry and Physics, 13(5): 2445-2454

Shukla JB, Sundar S, Misra AK, Naresh R. 2008. Modelling the removal of gaseous pollutants and particulate matters from the atmosphere of a city by rain: Effect of cloud density. Environmental Modeling and Assessment, 13: 255-263

Shukla JB, Chauhan MS, Sundar S, Naresh R. 2014. Removal of carbon dioxide from the atmosphere to reduce global warming: a modelling study. International Journal of Global Warming, Accepted

Slinn WGN. 1977. Some approximations for the wet and dry removal of particles and gases from the atmosphere. Water, Air and Soil Pollution, 7: 513-543

Stolaroff JK, Keith DW, Lowry GV. 2008. Carbon dioxide capture from atmospheric air using sodium hydroxide spray. Environmental Science and Technology, 42: 2728-2735

Sundar S, Naresh R. 2011. Role of cloud droplets on the removal of gaseous pollutants from the atmosphere: A nonlinear model. International Journal of Applied Mathematics and Computation, 3(4): 274-282

Voice AK, Vevelstad SJ, Chen X, Nguyen T, Rochelle GT. 2013. Aqueous 3-(methylamino) propylamine for CO_2 capture. International Journal of Greenhouse Gas Control, 15: 70-77

Wu S, Mickley LJ, Leibensperger EM, Jacob DJ, Rind D, Streets DG. 2008. Effects of 2000–2050 global change on ozone air quality in the United States. Journal of Geophysical Research, 113(D6): 1-12

Wu SH, Pan T, He SF. 2012. Climate change risk research: A case study on flood disaster risk in China. Advances in Climate Change Research, 3(2): 92-98

Wu SH, Zhang WJ. 2012. Current status, crisis and conservation of coral reef ecosystems in China. Proceedings of the International Academy of Ecology and Environmental Sciences, 2(1): 1-11

Yeh AC, Bai H. 1999. Comparison of ammonia and monoethanolamine solvents to reduce CO_2 greenhouse gas emissions. Science of the Total Environment, 228(2-3): 121-133

Yoo M, Han SJ, Wee JH. 2013. Carbon dioxide capture capacity of sodium hydroxide aqueous solution. Journal of Environmental Management, 114: 512-519

Zhang M, Guo Y. 2013. Process simulations of large-scale CO_2 capture in coal-fired power plants using aqueous ammonia solution. International Journal of Greenhouse Gas Control, 16: 61-71

Zeman FS, Lackner KS. 2004. Capturing carbon dioxide directly from the atmosphere. World Resource Review, 16(2): 157-172

Zeng Q, Guo Y, Niu Z, Lin W. 2011. Mass transfer coefficients for CO_2 absorption into aqueous ammonia solution using a packed column. Industrial & Engineering Chemistry Research, 50(17): 10168–10175