Article

Chaotic dynamics in a discrete-time predator-prey food chain

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Abstract

In this paper, we consider a classical discrete-time food chain model describing predators-prey interaction. The Holling type I functional response is used as the uptake for both predators. The existence and local stability of fixed points of the discrete dynamical system are analyzed algebraically. Using growth rate of prey as the bifurcation parameter, it is shown that the system undergoes a flip and Hopf bifurcations around planer or interior fixed point. It has been found that the dynamical behavior of the model is very sensitive to the parameter values and the initial conditions. Numerical simulations not only illustrate the key points of analytical findings but also exhibit complex dynamical behaviors of the model, such as the phase portraits, cascade of period-doubling bifurcation and determine the effects of operating parameters of the model on its dynamics. The Lyapunov exponents are numerically computed to characterize the asymptotic stability of the system dynamic response and estimate the amount of chaos in the system.

Keywords discrete-time food chain; stability; Flip and Hopf bifurcations; Lyapunov exponents.

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1 Introduction

The dynamics of predator-prey interaction is the starting point for many variations (food chain, food web etc.) that yield more realistic biological and mathematical problems in population ecology. Predation is a direct interaction which occurs when individuals from one population derive their nourishment by capturing and ingesting individuals from another population. There are many articles devoted to the study of predator-prey interaction both from the experimental and the modeling point of view. It is well known the Lotka-Voltera predator-prey model is one of the fundamental population models, a predator-prey interaction has been described firstly by two pioneers Lotka (1924) and Voltera (1926) in two independent works. After them, more realistic prey-predator model were introduced by Holling suggesting three types of functional responses for different species to model the phenomena of predation (Holling, 1965).

Qualitative analyses of prey-predator models describe by set of differential equations were studied by

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