Article

Local stability of an open-access anchovy fishery model

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Abstract
In this paper, we study the qualitative behavior of following open-access anchovy fishery model:

$$x_{n+1} = ax_n^b - d\alpha x_n^v y_n,\ y_{n+1} = y_n\left(\eta\left(p\alpha x_n^v - c\right)+1\right),$$

where $a, b, c, d, v, p, \alpha, \eta$ and the initial conditions x_0, y_0 are positive real numbers. More precisely, we investigate the necessary and sufficient condition for local asymptotic stability of the unique positive equilibrium point of this system. Some numerical examples are given to verify our theoretical results.

Keywords open-access anchovy fishery model; equilibrium points; local stability.

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1 Introduction

In this paper, we study the local asymptotic stability of an open-access anchovy fishery model which was proposed by Kulenovic' and Merino. This model describes the change in resource abundance and the level of investment or effort engaged in at the fishery. The model is given by

$$x_{n+1} = ax_n^b - d\alpha x_n^v y_n,\ y_{n+1} = y_n\left(\eta\left(p\alpha x_n^v - c\right)+1\right) \tag{1}$$

where x_n is the biomass of anchovy in year n and y_n is the effort in year n, d is the biological discount factor, c is the cost per unit effort, p is the price of the anchovy and η is the effort adjustment coefficient.

For basic theory of difference equations we refer Agarwal (2000), Kulenovic' and Ladas (2002), Camouzis and Ladas (2007), Elaydi (2005), Grove and Ladas (2004), Khan $et.al.$, (a, b, 2014) and for applications of difference equations we refer Keshet (1988), Sedaghat (2003), Kulenovic' and Merino (2002). In literature there are many papers on qualitative behavior of biological models Ahmad (1993), Tang and Zou (2006), Zhou and Zou (2003), Liu (2010), Din $et.al.$, (2013), Qureshi $et.al.$, (2014), Opsomer and Conrad (1994).

Our aim is to investigate the necessary and sufficient condition for local asymptotic stability of the unique positive equilibrium point of system (1).

2 Linearized Stability

Let us consider the two-dimensional discrete dynamical system of the form:

$$\left.\begin{aligned} x_{n+1} &= f(x_n, y_n) \\ y_{n+1} &= g(x_n, y_n), n = 0,1,\cdots, \end{aligned}\right\} \tag{2}$$

where $f: I \times J \to I$ and $g: I \times J \to J$ are continuously differentiable functions and I, J are some intervals of real numbers. Furthermore, a solution $\{(x_n, y_n)\}_{n=0}^{\infty}$ of system (2) is uniquely determined by the initial conditions $(x_0, y_0) \in I \times J$. An equilibrium point of system (2) is a point $(\bar{x}, \bar{y})$ that satisfies

$$\left.\begin{aligned} \bar{x} &= f(\bar{x}, \bar{y}) \\ \bar{y} &= g(\bar{x}, \bar{y}) \end{aligned}\right\}.$$

Definition 2.1. Let $(\bar{x}, \bar{y})$ be an equilibrium point of a map $F = \left(f(x, y), g(x, y)\right)$ where f and g are continuously differentiable functions at $(\bar{x}, \bar{y})$. The linearized system of (2) about the equilibrium point $(\bar{x}, \bar{y})$ is given by

$$X_{n+1} = F(X_n) = F_J X_n,$$

where $X_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$ and F_J is Jacobian matrix of system (2) about the equilibrium point $(\bar{x}, \bar{y})$.

Lemma 2.2. (Sedaghat, 2003): Consider the system $X_{n+1} = F(X_n)$, $n = 0,1,\cdots,$ where $\overline{X}$ is a fixed point of F. If all eigenvalues of the Jacobian matrix F_J about $\overline{X}$ lie inside the open unit disk $|\lambda| < 1$, then $\overline{X}$ is locally asymptotically stable. If any of the eigenvalue has a modulus greater than one, then $\overline{X}$ is unstable.

Lemma 2.3. (Grove and Ladas, 2004): Consider the second-degree polynomial equation

$$\lambda^2 + \mu\lambda + \nu = 0, \tag{3}$$

where μ and ν are real numbers.

(i) A necessary and sufficient condition for both roots of Equations (3) to lie inside the open disk $|\lambda| < 1$ is $|\mu| < 1 + \nu < 2.$ In this case the locally asymptotically stable equilibrium $(\bar{x}, \bar{y})$ is also called a sink.

(ii) A necessary and sufficient condition for both roots of Equations (3) to have absolute value greater than one is $|\nu| > 1, |\mu| < |1 + \nu|.$ In this case $(\bar{x}, \bar{y})$ is a repeller.

(iii) A necessary and sufficient condition for one root of Equations (3) to have absolute value greater than one and for the other to have absolute less than one is $\mu^2 - 4\nu > 0, |\mu| > |1 + \nu|.$ In this case unstable equilibrium point $(\bar{x}, \bar{y})$ is called a saddle point.

(iv) A necessary and sufficient condition for a root of Equations (3) to have absolute value equal to one is $|\mu| = |1 + \nu|.$ In this case the equilibrium $(\bar{x}, \bar{y})$ is called a non-hyperbolic point.

3 Main Results

Let $(\bar{x}, \bar{y})$ be equilibrium point of system (1) then

$$\bar{x} = a\bar{x}^b - d\alpha\bar{x}^v\bar{y}, \bar{y} = \bar{y}(\eta(p\alpha\bar{x}^v - c) + 1).$$

Hence, $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ and $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}},\frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ be equilibrium points of system (1). Note that if $a>\left(\frac{p\alpha}{c}\right)^{\frac{b-1}{v}}$ then system (1) has unique positive equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}},\frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ and if $a<\left(\frac{p\alpha}{c}\right)^{\frac{b-1}{v}}$ then system (1) has a unique equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$. The Jacobian matrix of linearized system of (1) about the equilibrium point $(\bar{x},\bar{y})$ is given by

$$F_J(\bar{x},\bar{y})=\begin{bmatrix} ab\bar{x}^{b-1}-dv\bar{x}^{v-1}\bar{y}\alpha & -d\bar{x}^{v}\alpha \\ pv\bar{x}^{v-1}\bar{y}\alpha\eta & 1+\left(p\bar{x}^{v}\alpha-c\right)\eta \end{bmatrix}.$$

Theorem 3.1. If

$$\left|b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right|+\left|b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)\right|<1, \tag{4}$$

then, equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ of system (1) is locally asymptotically stable.

Proof. The Jacobian matrix of linearized system of (1) about the equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ is given by

$$F_J(P)=\begin{bmatrix} b & -\alpha a^{\frac{v}{1-b}}d \\ 0 & 1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta \end{bmatrix}.$$

The characteristic polynomial of $F_J(P)$ about the equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ is given by

$$P(\lambda)=\lambda^2-\left(b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)\lambda+b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right). \quad (5)$$

Let

$$f(\lambda)=\lambda^2,\, g(\lambda)=\left(b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)\lambda-b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right).$$

Assume that condition (4) hold true and $|\lambda|=1$, one has

$$|g(\lambda)|\le\left|b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right|+\left|b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)\right|<1.$$

Then, by Rouche's theorem, $f(\lambda)$ and $f(\lambda)-g(\lambda)$ have the same number of zeroes in an open unit disk $|\lambda|<1$. Hence, both roots

$$\lambda_1=\frac{b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta+\sqrt{\left(b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)^2-4b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)}}{2},$$

and

$$\lambda_2=\frac{b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta-\sqrt{\left(b+1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)^2-4b\left(1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right)}}{2},$$

of (5) lie in an open disk $|\lambda|<1$, and it follows from Lemma 2.2 that equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ of system (1) is locally asymptotically stable.

Theorem 3.2. The equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ of system (1) is locally asymptotically stable if and only if $\alpha pa^{\frac{v}{1-b}}<c<\frac{2}{\eta}+\alpha pa^{\frac{v}{1-b}}$.

Proof. The zeroes of characteristic polynomial of $F_J(P)$ about the equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ are $\lambda_1=b$ and $\lambda_2=1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta$. As $|\lambda_1|=b<1$ and $|\lambda_2|=\left|1+\left(a^{\frac{v}{1-b}}p\alpha-c\right)\eta\right|<1$ if and only if $\alpha pa^{\frac{v}{1-b}}<c<\frac{2}{\eta}+\alpha pa^{\frac{v}{1-b}}$. Hence by Lemma 2.2, equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ of system (1) is locally asymptotically stable.

Theorem 3.3. If

$$\left|a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right|+\left|a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v+cv\eta\right|<1, \tag{6}$$

then, the unique positive equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ of system (1) is locally asymptotically stable.

Proof. The Jacobian matrix of linearized system of (1) about equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ is given by

$$F_J(P)=\begin{bmatrix} a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v & -\frac{dc}{p} \\ \frac{pv\eta\left(a\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-1\right)}{d} & 1 \end{bmatrix}.$$

The characteristic polynomial of $F_J(Q)$ about the equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ is given by

$$P(\lambda)=\lambda^2-\left(a(b-c)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right)\lambda+a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta\,. \tag{7}$$

Let

$$f(\lambda)=\lambda^2, g(\lambda)=\left(a(b-c)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right)\lambda-a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v+cv\eta\,.$$

Assume that condition (6) hold true and $|\lambda|=1$, one has

$$|g(\lambda)|\le\left|a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right|+\left|a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v+cv\eta\right|<1.$$

Then, by Rouche's theorem $f(\lambda)$ and $f(\lambda)-g(\lambda)$ have the same number of zeroes in an open unit disk $|\lambda|<1$. Hence, both roots

$$\lambda_1=\frac{a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1+\sqrt{\left(a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right)^2-4\left(a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v+cv\eta\right)}}{2},$$

and

$$\lambda_2=\frac{a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1-\sqrt{\left(a(b-v)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v+1\right)^2-4\left(a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v+cv\eta\right)}}{2},$$

of (7) lie in an open disk $|\lambda|<1$, and it follows from Lemma 2.2 that the unique positive equilibrium point

$$Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}},\frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$$

of system (1) is locally asymptotically stable.

The following theorem shows necessary and sufficient condition for local asymptotic stability of the unique positive equilibrium point of system (1).

Theorem 3.4. The unique positive equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}},\frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$ of system (1) is locally asymptotically stable if and only if

$$\left|a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v-1\right|<1+a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta<2. \quad (8)$$

Proof. Le $\mu=a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v-1, \nu=a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta$ .Then, it follows from Lemma 2.3 that the unique positive equilibrium point $Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)}{\alpha d}^{1-\frac{1}{v}}\right)$ of system (1) is locally asymptotically stable if and only if

$$\left|a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v-1\right|<1+a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta<2.$$

4 Examples

In order to verify our theoretical results we consider three interesting numerical examples. These examples represent different types of qualitative behavior of system (1). First two examples show that the unique positive equilibrium point of system (1) is locally asymptotically stable, *i.e.*, condition (8) of Theorem 3.4 is satisfied. Meanwhile, last example shows that the unique positive equilibrium point of system (1) is unstable, *i.e.*, condition (8) of Theorem 3.4 does not hold.

Example 1. Let $a=1.9, b=0.999, c=2.9, d=3.7, p=2.4, v=2.5, \alpha=0.05, \eta=0.2$ .Then, system (1) can be written as

$$x_{n+1}=1.9x_n^{0.999}-3.7\times0.05x_n^{2.5}y_n, y_{n+1}=y_n\big(0.2\big(2.4\times0.05x_n^{2.5}-2.9\big)+1\big), \quad (9)$$

with initial conditions $x_0=3.9, y_0=0.6$.

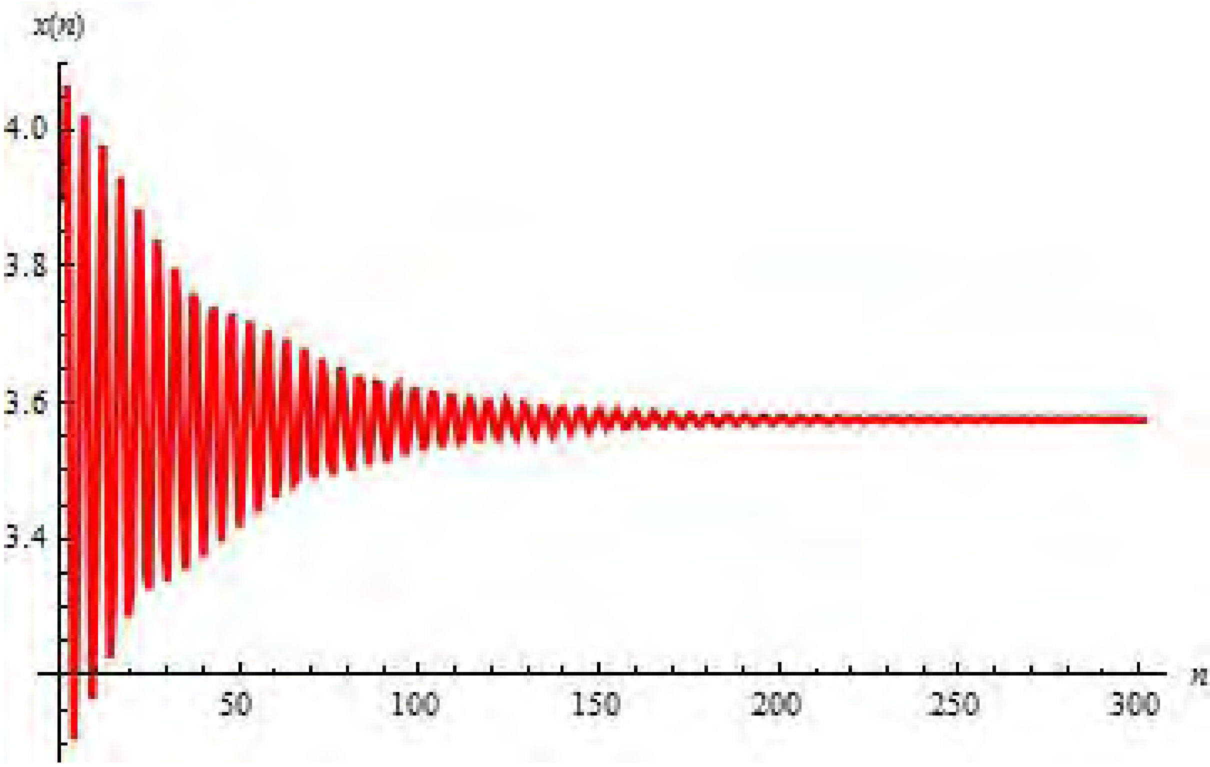
In this case

$$\left|a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v-1\right|=0.651731<1+a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta=1.95322<2, \textit{ i.e.,}$$

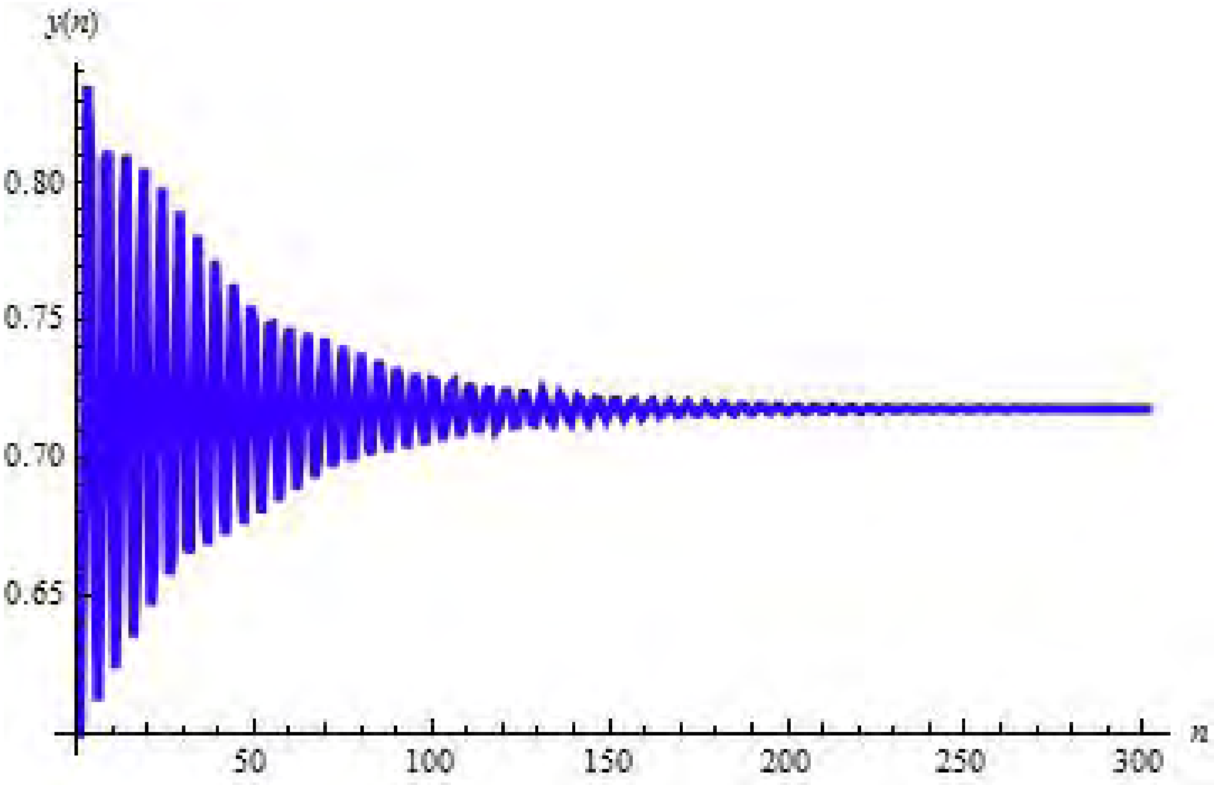
condition (8) of Theorem 3.4 is satisfied. The unique positive equilibrium of system (9) is given by

$$\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)=(3.57509,0.717748)$$

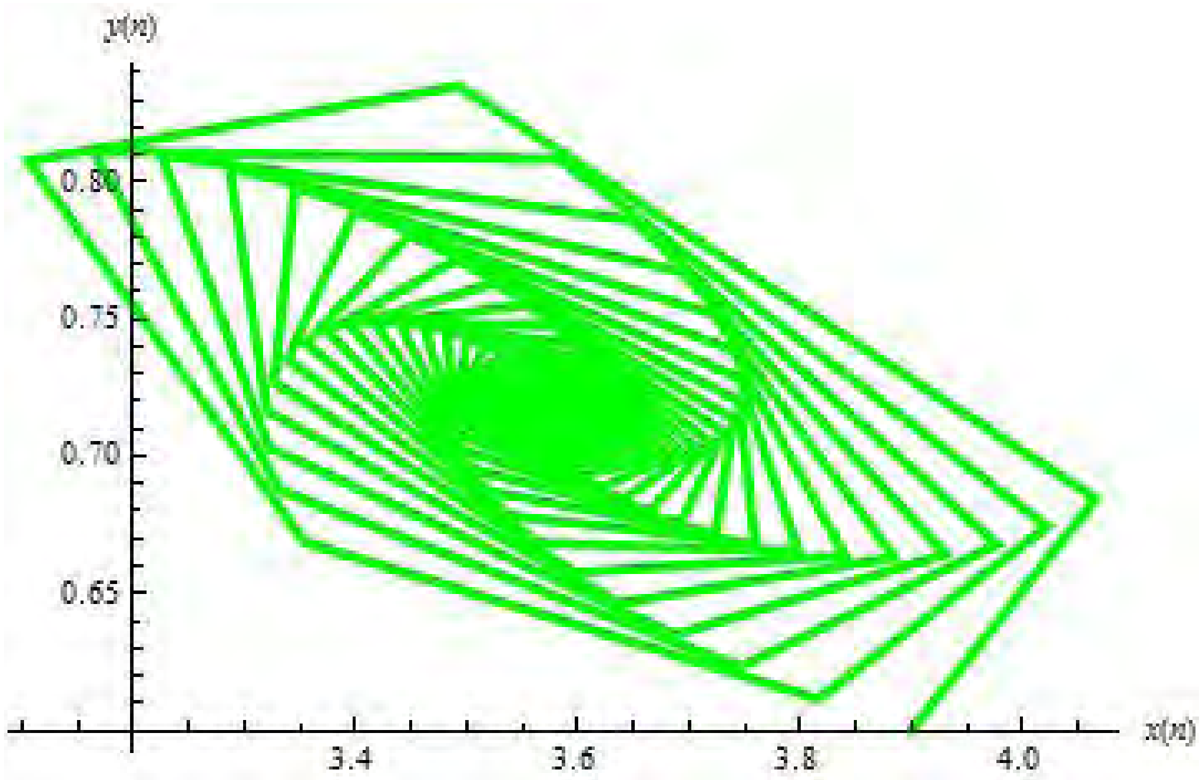
. Moreover, in Figure 1 the plot of x_n is shown in Fig. 1a, the plot of y_n is shown in Fig. 1b and an attractor of system (9) is shown in Fig. 1c.



(a) Plot of x_n for system (9)



(b) Plot of y_n for system (9)



(c) An attractor for system (9)

Fig. 1 Plots for system (9).

Example 2. Let $a = 2.2, b = 0.999, c = 3.1, d = 4.2, p = 4.5, v = 2.7, \alpha = 0.06, \eta = 0.2$. Then, system (1) can be written as

$$x_{n+1} = 2.2x_n^{0.999} - 4.2 \times 0.06x_n^{2.7} y_n, y_{n+1} = y_n\left(0.2\left(4.5 \times 0.06x_n^{2.7} - 3.1\right) + 1\right), \tag{10}$$

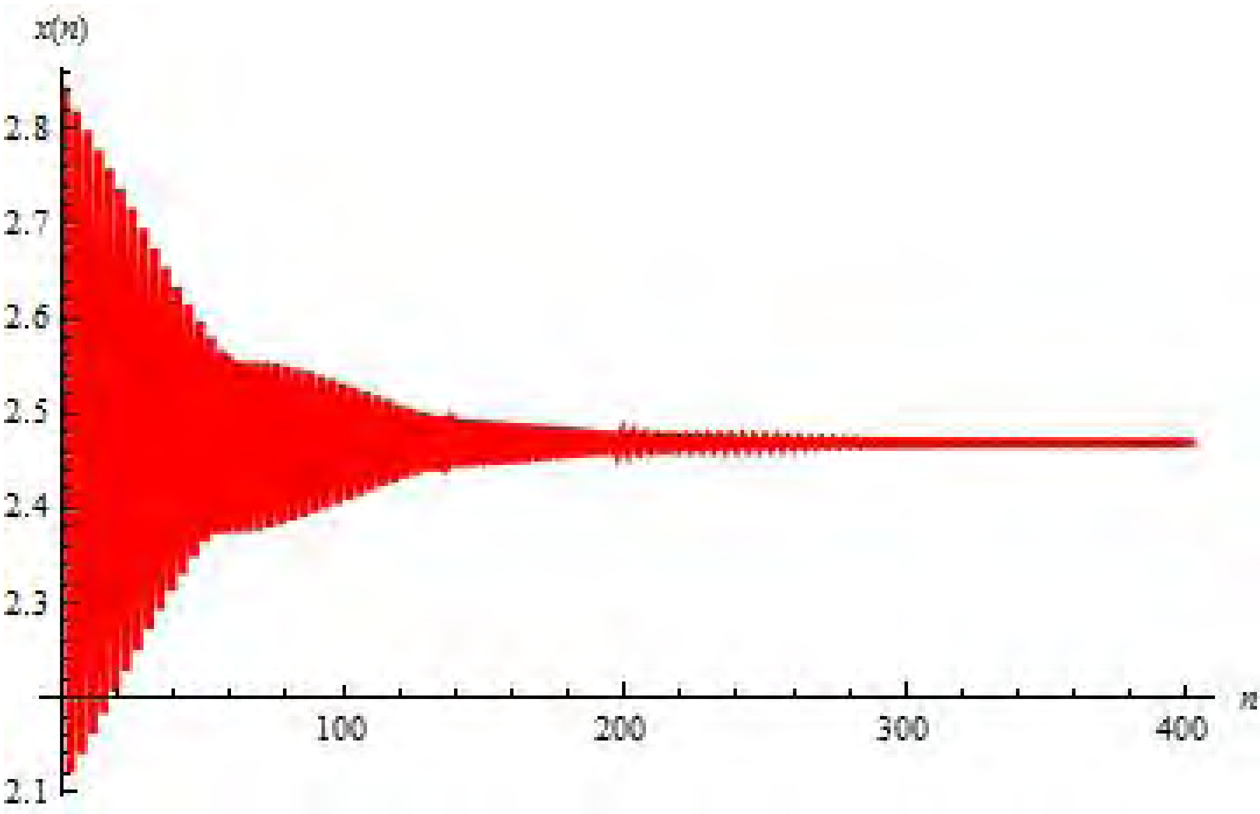
with initial conditions $x_0 = 2.84, y_0 = 0.9$.

In this case

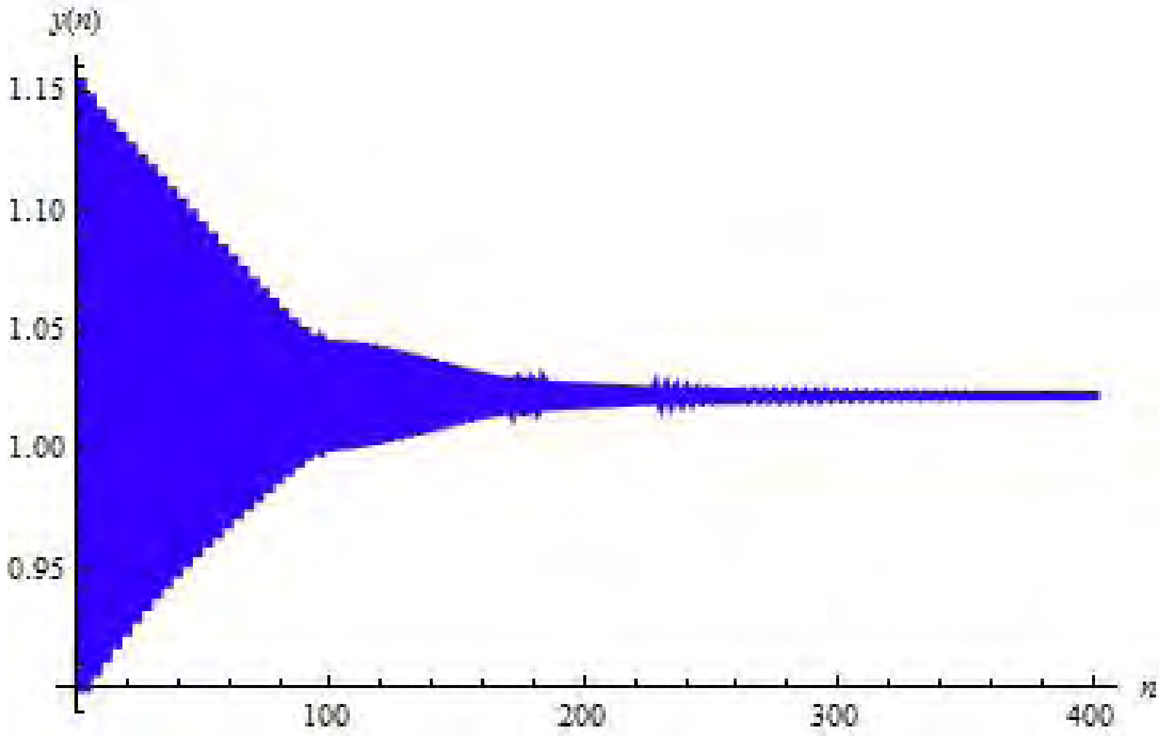
$$\left| a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}} - v - 1 \right| = 0.0388167 < 1 + a(b - v + cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}} + v - cv\eta = 1.96665 < 2, \textit{i.e.},$$

condition (8) of Theorem 3.4 is satisfied. The unique positive equilibrium of system (10) is given by

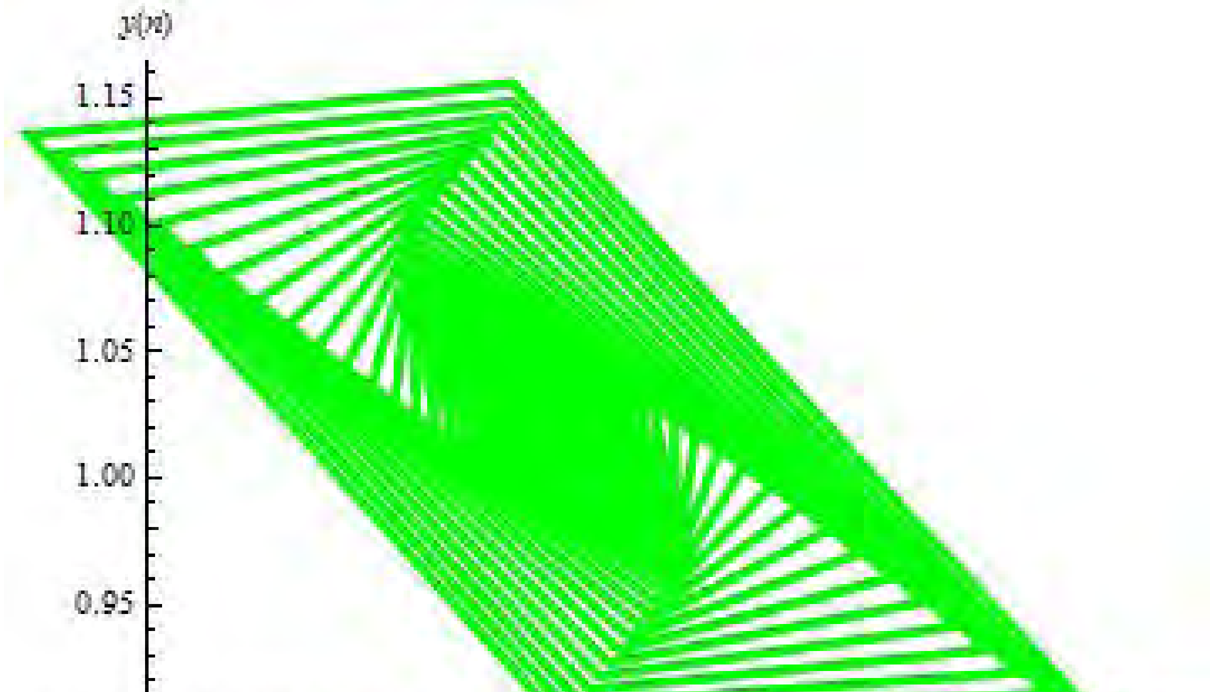
$$\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}}, \frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}} - \left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d} \right) = (2.4694, 1.02248).$$ Moreover, in Figure 2 the plot of x_n is shown in Fig. 2a, the plot of y_n is shown in Fig. 2b and an attractor of system (10) is shown in Fig. 2c.



(a) Plot of x_n for system (10)



(b) Plot of y_n for system (10)



(c) An attractor for system (10)

Fig. 2 Plots for system (10).

Example 3. Let $a = 2.6, b = 0.999, c = 3.0, d = 4.66, p = 3.1, v = 2.5, \alpha = 0.06, \eta = 0.2$ .Then, system (1) can be written as

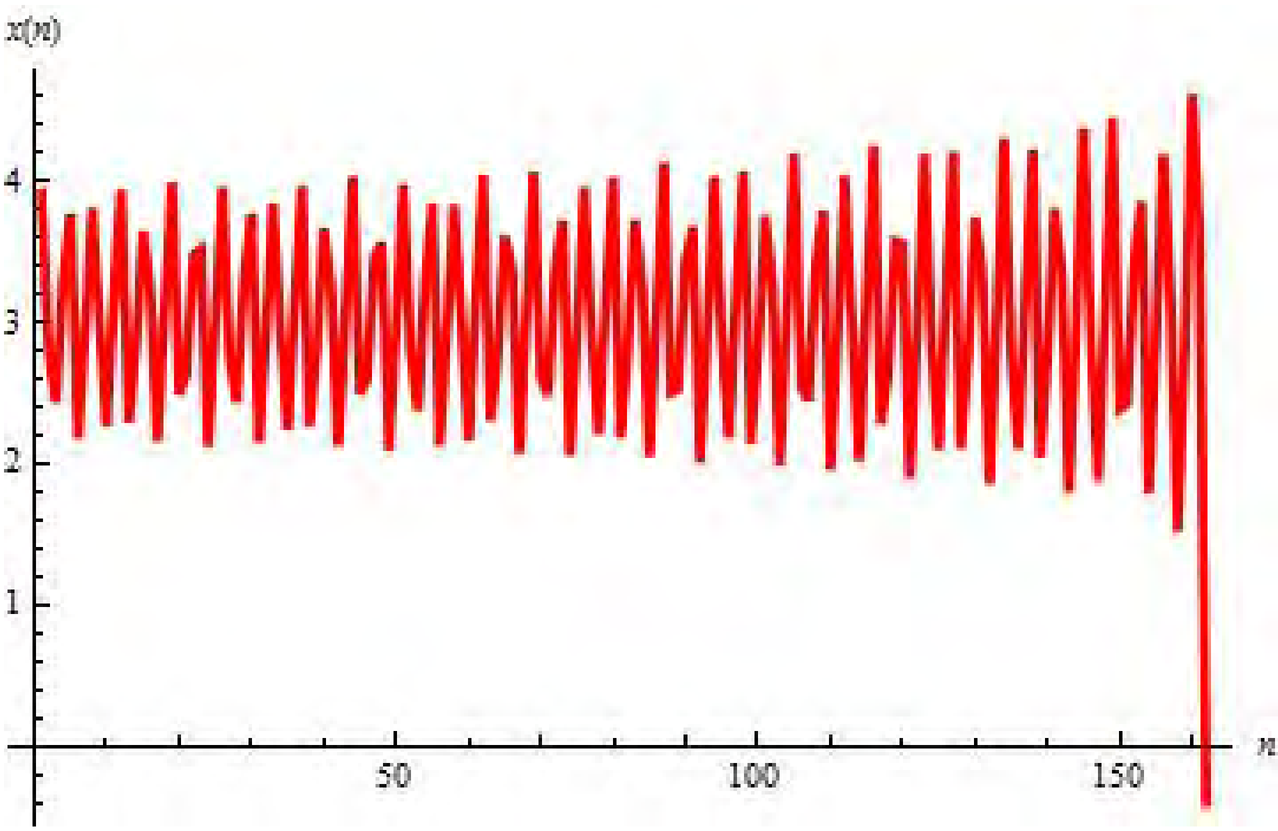
$$x_{n+1} = 2.6x_n^{0.999} - 4.66 \times 0.06 x_n^{2.5} y_n, y_{n+1} = y_n\left(0.2\left(3.1 \times 0.06 x_n^{2.5} - 3.0\right) + 1\right), \tag{11}$$

with initial conditions $x_0 = 2.999, y_0 = 0.89$.

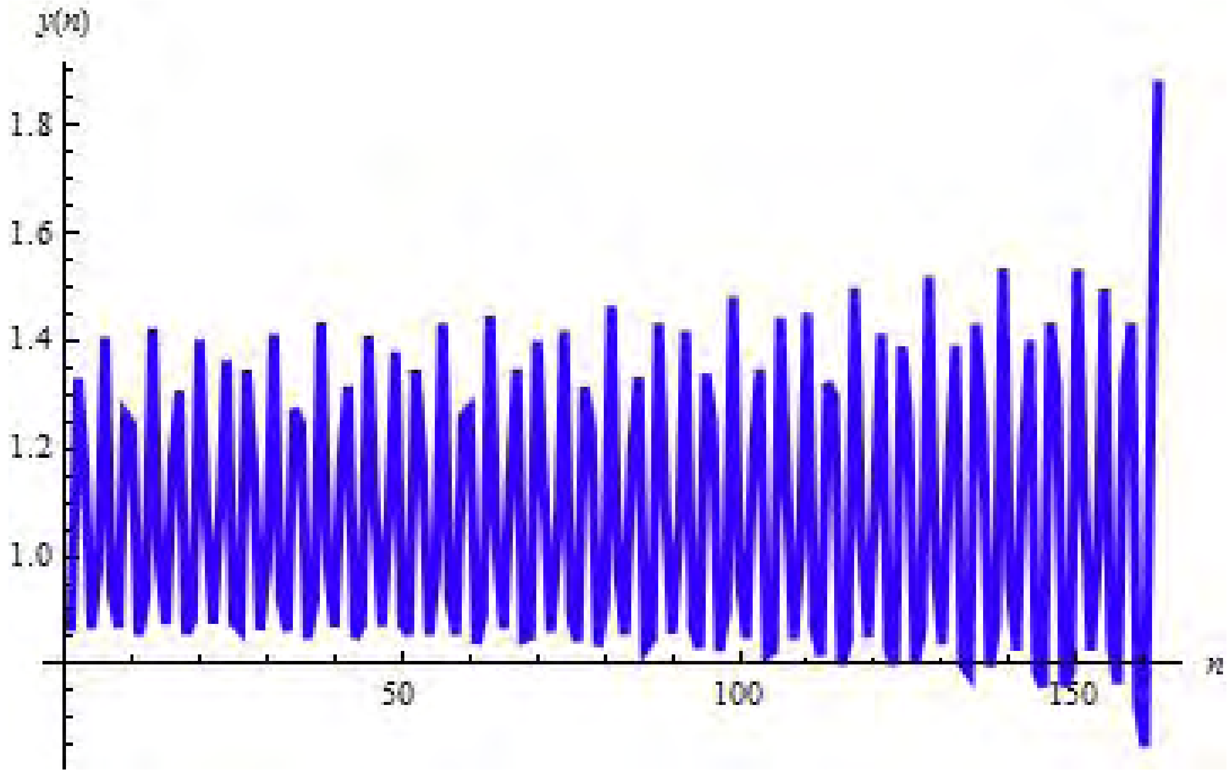
In this case

$$\left| a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}} - v - 1 \right| = 1.45238817 > 1 + a(b - v + cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}} + v - cv\eta = 0.91196665,$$

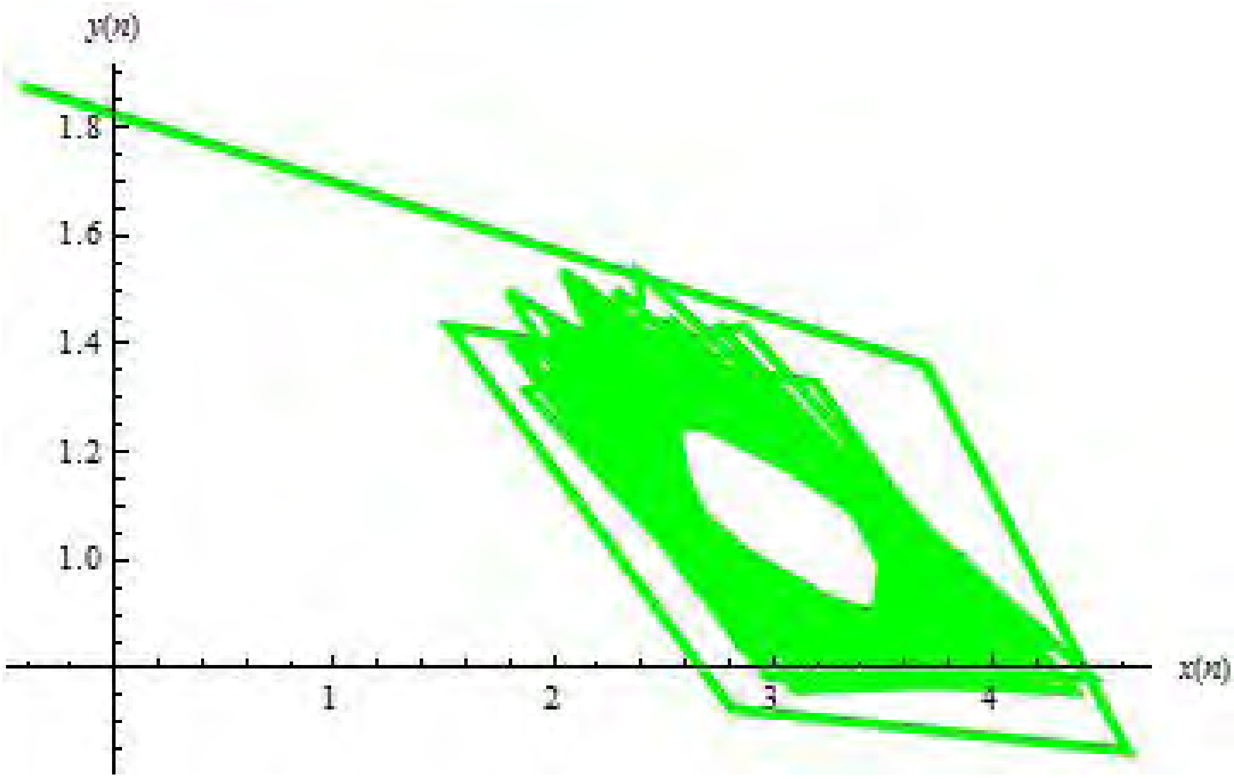
i.e., condition (8) of Theorem 3.4 is satisfied. The unique positive equilibrium of system (11) is unstable. Moreover, in Figure 3 the plot of x_n is shown in Fig. 3a, the plot of y_n is shown in Fig. 3b and Phase portrait of system (11) is shown in Fig. 3c.



(a) Plot of x_n for system (11)



(b) Plot of y_n for system (11)



(c) Phase portrait for system (11)

Fig. 3 Plots for system (11).

5 Conclusion

This work is related to the qualitative behavior of an open-access anchovy fishery model. We prove that system (1) have two equilibrium points, which are locally asymptotically stable. The method of linearization is used to prove the local asymptotic stability of unique positive equilibrium point. We prove that equilibrium point $P(\bar{x},\bar{y})=\left(a^{\frac{1}{1-b}},0\right)$ of system (1) is locally asymptotically stable if and only if $\alpha p a^{\frac{v}{1-b}}<c<\frac{2}{\eta}+\alpha p a^{\frac{v}{1-b}}$. Also, linear stability analysis shows that the unique positive equilibrium point

$$Q(\bar{x},\bar{y})=\left(\left(\frac{p\alpha}{c}\right)^{-\frac{1}{v}},\frac{a\left(\frac{p\alpha}{c}\right)^{1-\frac{b}{v}}-\left(\frac{p\alpha}{c}\right)^{1-\frac{1}{v}}}{\alpha d}\right)$$

of system (1) is locally asymptotically stable if and only if condition (8) of Theorem 3.4 is satisfied, *i.e.*, $\left|a(v-b)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}-v-1\right|<1+a(b-v+cv\eta)\left(\frac{p\alpha}{c}\right)^{\frac{1-b}{v}}+v-cv\eta<2$. Some numerical examples are provided to support our theoretical results. These examples are experimental verification of theoretical discussion. The global behavior of system (1) will be next our aim to study.

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