Article

A discrete homotopy perturbation method for non-linear Schrödinger equation

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Abstract

A general analysis is made by homotopy perturbation method while taking the advantages of the initial guess, appearance of the embedding parameter, different choices of the linear operator to the approximated solution to the non-linear Schrodinger equation. We are not dependent upon the Adomian polynomials and find the linear forms of the components without these calculations. The discretised forms of the nonlinear Schrodinger equation allow us whether to apply any numerical technique on the discritisation forms or proceed for perturbation solution of the problem. The discretised forms obtained by constructed homotopy provide the linear parts of the components of the solution series and hence a new discretised form is obtained. The general discretised form for the NLSE allows us to choose any initial guess and the solution in the closed form.

Keywords discrete homotopy perturbation method; nonlinear models; discretisation.

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1 Introduction

Erwin Schrödinger was born on 12 august 1887 in Vienna Austria- Hungry. In the field of quantum mechanics, Schrödinger developed a basic result and was awarded by the noble prize as Austrian physicist, by which the basis of wave mechanics is formed. Schrödinger suggested the new interpretation of physical meanings of wave function. Moreover in different fields of physics, he was the author of many remarkable works; such as color theory, electrodynamics theory, statistical mechanics, Physics of dielectrics, cosmology and general relativity. He also made a number of attempts to describe and establish the unified theory. In his known book "What is Life?" Schrödinger lectured the problem of Genetics while looking at the phenomena of life on the basis of the physical point of view. Schrodinger also made experiments in the fields of atmospheric electricity, atmospheric radioactivity, electrical engineering, and studied theory of vibrations. He performed his last physical experiment on coherent light and mainly focused subsequently on theoretical aspects in 1919. The Schrödinger quantum mechanical wave function is described as

$$i\text{h}\frac{\partial\Psi}{\partial t}=\hat{H}\,\Psi.$$

Here the quantities are;

- Hamiltonian operator $\hat{H}$
- Imaginary unit i
- Wave function Ψ
- Plank's constant h

The Schrödinger equation being a partial differential equation describes the quantum state of a physical system and the changes in that system with respect to time. It was first time formulated by Schrödinger in 1925(Schrödinger, 1926) . In the sense of classical mechanics, the governing equation predicts the behavior of a system mathematically at any time after the initial state of the system is set and this corresponds to the Newton's law $(F=ma)$. In the language of quantum mechanics, the Schrödinger equation is analogous to Newton's law for quantum mechanical system (which usually involves molecules, atoms, sub-atomic particles, whether moving freely, bounded or localized). It is not as simple algebraic operation but in general form it is a linear partial differential equation that describes the evaluation of time of the wave function system (Griffiths, 2004).

The basic postulate of quantum mechanics is the idea of wave function. The Schrödinger equation may also be presented as separate postulate, by the statement of some authors it can also be derived from symmetry principle. Commonly, for describing the wave particle duality, the Schrödinger equation (SE) comes to define its mathematical probability. The most general form of Schrödinger equation is either consisting in special relativity or classical mechanics. According to Schrödinger himself, the original invention was non relativistic. In quantum mechanics, the Schrödinger equation is not only a way to make its prediction but other applications can also be considered, such as matrix mechanic by Werner Heisenberg and path integral formulation by Richard Feynmans.

We have developed the discrete models of predator prey interactions (Shakil, 2014), where the quasi-chemical approach has been use for modeling the predator prey interactions. These discrete models have nonlinear systems of partial differential equations for different types of mechanisms. The idea of discrete homotopy perturbation method can be surely extended to these models as it is applicable here. The other models of ecology and mathematical biology can also be dealt with this approach.

2 The Problem

In this work, we want to present a discrete homotopy perturbation method (DHPM) applied to the non-linear Schrödinger equation (NLS) with cubic non-linearity. In 1998, J.H. He of Shanghai University introduced the homotopy perturbation method to solve the nonlinear ordinary and partial differential equations. This method gives an efficient analytical approximate solution with high accuracy in the presence of prescribed conditions. The nonlinear Schrödinger equation (NLS) is the nonlinear partial differential equation which contributes to the various fields of Mathematical Physics. This reveals the spatio-temporal evolution of a complex field $\Psi=\Psi(x,t)\in £$ and its general form is given as,

$$i\frac{\partial \Psi}{\partial t}+\frac{\partial^2 \Psi}{\partial x^2}+q|\Psi|^2\,\Psi=0, \qquad x\in ¡\ ,\ t>0, \tag{1}$$

$$\Psi(x,0)=f(x), \tag{2}$$

where the parameter $q\in ¡$,and which corresponds to the focusing $(q>0)$ or defocusing $(q<0)$ effect of the nonlinearity. The above form of the equation (1) describes many problems in mathematical Physics. The variety of the applications of this field varies from optics (Kivshar and Agrawal, 2003), the electric field propagation in optical fibers (Hasegawa,1995), the collapse of Langmuir waves and self-focusing in plasma physics (Zakharov, 1972) for the modeling of deep and freak water waves (so-called rogue waves) in the oceans (Onorato et al., 2001). Now we know that the equation (1) is completely S- integrable (in the sense of Calogero, 1991) which is the inverse scattering method (ISM) (Zakharov and Shabat, 1972) and the solution of the single soliton is given by,

$$\Psi(x,t)=\left(\frac{2a}{q}\right)^{\frac{1}{2}} e^{\left[i\left(\frac{c}{2}x-\theta t\right)\right]}\sec h\left[a^{\frac{1}{2}}(x-ct)\right], \tag{3}$$

where $\theta=\frac{c^2}{4-a}$. For a fixed t, the function Ψ in equation (3) exponentially decays as $|x|\to\infty$. It travels in the envelope speed c and its amplitude is obtained by parameter $a\in R$. Then an N – soliton solution for $q\neq 0$ is given by

$$\Psi(x,t)=\left(\frac{2a}{q}\right)^{\frac{1}{2}}\sum_{p=1}^{N} e^{\left[i\left(\frac{c_p}{2}x_p-\theta_p t\right)\right]}\sec h\left[a^{\frac{1}{2}}\left(x_p-\theta_p t\right)\right], \tag{4}$$

Where $\theta=\frac{c^2}{4-a}$, is the position of p - soliton and its velocity is c_p. At last the particular simple solutions of the Schrödinger equation (1) are then become plane wave solutions.

3 The Homotopy Perturbation Method

The homotopy perturbation method (HPM) will be briefly sketched in this section for the application to the nonlinear Schrödinger equation (NLS). To do this we consider (1) as,

$$L_t\Psi=i\partial^2{}_x\Psi+iqF(\Psi); \qquad x\in R,\ \ t>0, \tag{5}$$

where the notation are $L_t=\partial_t$ and the nonlinear cubic term is $F(\Psi)=|\Psi|^2\,\Psi$. Then the inverse operator of L_t is designated as a definite integral,

$$L^{-1}{}_t\Psi(t)=\int_0^t\Psi(t)dt; \qquad t>0, \tag{6}$$

$$L_t\Psi-i\partial^2{}_x\Psi-iqF(\Psi)=0, \qquad x\in ¡\ ,\ t>0, \tag{7}$$

The homotopy perturbation method assumes a solution in the form of series, $\Psi=\sum_{l=0}^{\infty}\Psi_l$, where each component of $\Psi(x,t)$ can be determined recursively. The term $F(\Psi)$ which is nonlinear is decomposed into infinite series of sub-linear terms while making the nonlinear problem as the series of sub-linear problems. Now, we construct homotopy for the equation (7) which is assumes the following form as,

$$H(\psi,p)=(1-p)\left[L\psi-L\Psi_0\right]+p\left[N(\psi)-f(r)\right]=0 \qquad (8)$$

where $p\in[0,1]$ is termed as an embedding parameter, Ψ_0 is the initial approximation and $f(r)$ is as an analytic function. Then

$$H(\psi,p)=(1-p)L[\psi-\Psi_0]+p\left[L\psi-i\partial^2{}_x\psi-iq\psi^2\overline{\psi}\right]=0. \qquad (9)$$

Suppose that ψ can be expressed in series of power p given as,

$$\psi=\psi_0+p\psi_1+p^2\psi_2+...$$

Thus from equation (9), we then have,

$$H(\psi,p)=L\left(\psi_0+p\psi_1+p^2\psi_2+...\right)-L\psi_0+pL\psi_0-ip\left(\psi_{0xx}+p\psi_{1xx}+p^2\psi_{2xx}+p^3\psi_{3xx}+...\right)$$
$$-ipq\left(\left(\psi_0+p\psi_1+p^2\psi_2+...\right)^2\left(\overline{\psi}_0+p\overline{\psi}_1+p^2\overline{\psi}_2+p^3\overline{\psi}_3+...\right)\right)=0.$$

Equating the coefficient of p^0,p^1,p^2,p^3

$$p^0;\qquad L\psi_0-L\Psi_0=0;\qquad \Psi(x,0)=f(x), \qquad (10)$$

$$p^1;\qquad L\psi_1+L\Psi_0-i\left(\psi_{0xx}+q\psi_0{}^2\overline{\psi}_0\right)=0;\qquad \psi_1(x,0)=0, \qquad (11)$$

$$p^2;\qquad L\psi_2-i\left[\left(\psi_{1xx}+q\left(2\psi_0\overline{\psi}_0\psi_1+\psi_0{}^2\overline{\psi}_1\right)\right)\right]=0,\qquad \psi_2(x,0)=0, \qquad (12)$$

$$p^3;\qquad L\psi_3-i\left[\psi_{2xx}+q\left(\psi_1{}^2\overline{\psi}_0+2\psi_0\overline{\psi}_0\psi_2+2\psi_0\psi_1\overline{\psi}_1+2\psi_0{}^2\overline{\psi}_2\right)\right]=0;\quad \psi_3(x,0)=0,$$

and so on. The above equations are recursively solved using the initial approximation, and a series of solutions will be obtained step by step iterations.

4 The Standard Discrete Nonlinear Schrödinger Equation (NLS)

When we apply the discretization to (1) and $|\Psi|^2\Psi$ is replaced by diagonal discretization $|\Psi_j|^2\Psi_j$, then we obtain Discrete Nonlinear Schrödinger equation (DNL) as;

$$i\partial_t\psi_j+D^2{}_h\psi_j+q|\psi_j|^2\psi_j=0,\quad j\in ¢,\quad t>0, \qquad (13)$$

$$\psi_j(0)=f_j,\qquad j\in ¢, \qquad (14)$$

where $\Psi_j=\Psi_j(t)$, $h=\Delta x$, and $D^2{}_x\Psi_j=\dfrac{\left(\Psi_{j+1}-2\Psi_j+\Psi_{j-1}\right)}{h^2}$ represents the quotient of the second order difference. The parameters q and $\varepsilon=h^{-2}$ are called (discrete) dispersion and a harmonicity respectively. The equation (13) defines uncoupled set of an harmonic oscillators when $\varepsilon = 0$. But the standard discrete nonlinear Schrödinger (DNLS) equation is not an accurately integrable discrete nonlinear Schrödinger (DNLS). The particular discrete plane wave solution for the discrete nonlinear Schrödinger (DNLS) equation (13) is given by

$$\Psi_j(t) = e^{i(jkh-\omega t)}, \quad j \in ¢, \ t > 0. \qquad (15)$$

Put (15) into the discrete nonlinear Schrödinger (DNLS) equation (13) then we get,

$$\omega e^{i(jkh-\omega t)} + \frac{2e^{i(jkh-\omega t)}}{h^2}\left[\cos(kh)-1\right] + q\cos^2(jkh-\omega t)e^{i(jkh-\omega t)} + q\sin^2(jkh-\omega t)e^{i(jkh-\omega t)} = 0,$$

Or we find that,
$$\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right) - \omega = q. \qquad (16)$$

Also,
$$D^2{}_x\Psi_j = \frac{\left(\Psi_{j+1} - 2\Psi_j + \Psi_{j-1}\right)}{h^2}, \qquad (17)$$

5 The Ablowitz- Ladik Equation

From equation (1), when we discretize the cubic nonlinear term $\left|\Psi_j\right|^2\Psi_j$, then it is replaced by the off-diagonal discretization as;

$$\Psi_j\left|\Psi_j\right|^2 = \frac{\left(\Psi_{j+1} + \Psi_{j-1}\right)}{2}\left|\Psi_j\right|^2,$$

and the Ablowitz-Ladik equation (Ablowitz, 1976) is obtained when the time variable is continuous, that is

$$i\partial_t\Psi_j + D^2{}_x\Psi_j + q\left[\frac{\Psi_{j+1} + \Psi_{j-1}}{2}\right]\left|\Psi_j\right|^2 = 0, \quad j \in ¢, \ t > 0, \ (18)$$

With
$$\Psi_j(0) = f_j, \quad j \in ¢, \ (19)$$

And $\Psi_j = \left[\frac{\Psi_{j-1} + \Psi_{j+1}}{2}\right]$, (20)

Put $\Psi_j(t) = e^{i(jkh-\omega t)}$ in equation (18), and then we have,

$$i\partial_t\Psi_j + D^2{}_x\Psi_j + q\left|\Psi_j\right|^2\left[\frac{\Psi_{j+1} + \Psi_{j-1}}{2}\right] = \frac{2e^{i(jkh-\omega t)}}{h^2}\left[\cos(kh)-1\right] + \left[\omega + q\cos(kh)\right]e^{i(jkh-\omega t)},$$

$$\frac{4}{h^2}\sin\left(\frac{kh}{2}\right) - \omega = q\cos(kh)$$

6 The Semi-Discrete Homotopy Perturbation Method (HPM)

The discretized form of the homotopy perturbation method is simply the solution to the discrete nonlinear Schrödinger equation (DNLS) (13) or the AL-equation (18). Since we have constructed a homotopy for the equation (7), then from the equations (10), (11), (12), we have;

$$L\psi_0 - L\Psi_0 = 0; \qquad \Psi(x,0) = f(x), \tag{21}$$

$$L\psi_1 + L\Psi_0 - i\left(\psi_{0xx} + q\psi_0{}^2\bar{\psi}_0\right) = 0; \qquad \psi_1(x,0) = 0, \tag{22}$$

$$L\psi_2 - i\left[\left(\psi_{1xx} + q\left(2\psi_0\bar{\psi}_0\psi_1 + \psi_0{}^2\bar{\psi}_1\right)\right)\right] = 0, \qquad \psi_2(x,0) = 0, \tag{23}$$

$$L\psi_3 - i\left[\psi_{2xx} + q\left(\psi_1{}^2\bar{\psi}_0 + 2\psi_0\bar{\psi}_0\psi_2 + 2\psi_0\psi_1\bar{\psi}_1 + 2\psi_0{}^2\bar{\psi}_2\right)\right] = 0; \quad \psi_3(x,0) = 0, \tag{24}$$

From equation (21) and using the given condition we have the so called zero-th order equation which contains the initial approximation to start the analysis for the proposed method. Then we have,

$$\psi_0 = \Psi_0.$$

where Ψ_0 is the initial approximation. The equation (22) yields,

$$\psi_1 = \Psi_0 + iL^{-1}\left(\psi_{0xx} + q\psi_0{}^2\bar{\psi}_0\right); \qquad \psi_1(x,0) = 0, \tag{25}$$

Similarly, the equations (23)- (24) yield the following set of equations;

$$\psi_2 = iL_t^{-1}\left[\left(\psi_{1xx} + q\left(2\psi_0\bar{\psi}_0\psi_1 + \psi_0{}^2\bar{\psi}_1\right)\right)\right]; \qquad \psi_2(x,0) = 0, \tag{26}$$

$$\psi_3 = iL_t^{-1}\left[\psi_{2xx} + q\left(\psi_1{}^2\bar{\psi}_0 + 2\psi_0\bar{\psi}_0\psi_2 + 2\psi_0\psi_1\bar{\psi}_1 + 2\psi_0{}^2\bar{\psi}_2\right)\right]; \qquad \psi_3(x,0) = 0, \tag{27}$$

Since we know that $\Psi_j\left|\Psi_j\right|^2 = \dfrac{\left(\Psi_{j+1} + \Psi_{j-1}\right)}{2}\left|\Psi_j\right|^2$, then the equations (25)-(27) take the following forms accordingly as;

$$\psi_1 = \Psi_0 + iL_t^{-1}\left(\psi_{0xx} + q\left[\frac{\Psi_{j+1,l} + \Psi_{j-1,l}}{2}\bar{\Psi}_{j,0}\right]_0\right); \qquad \psi_1(x,0) = 0, \tag{28}$$

$$\psi_2 = iL_t^{-1}\left[\psi_{1xx} + q\left(\left[\Psi_{j,0}\frac{\Psi_{j+1,1} + \Psi_{j-1,1}}{2} + \frac{\Psi_{j+1,0} + \Psi_{j-1,0}}{2}\Psi_{j,1}\right]\bar{\Psi}_{j,0} + \Psi_{j,0}\frac{\Psi_{j+1.0} + \Psi_{j-1,0}}{2}\bar{\Psi}_{j,1}\right)\right];$$
$$\psi_2(x,0) = 0, \tag{29}$$

$$\psi_3 = iL_t^{-1}\left[\psi_{2xx} + q\left[\left[\Psi_{j,0}\frac{\Psi_{j+1,2}+\Psi_{j-1,2}}{2} + \frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,2}\right]\bar{\Psi}_{j,0} + \Psi_{j,1}\frac{\Psi_{j+1,1+}\Psi_{j-1,1}}{2}\bar{\Psi}_{j,0}\right.\right.$$
$$\left.\left.+\left[\Psi_{j,0}\frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2} + \frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,1}\right]\bar{\Psi}_{j,1} + \Psi_{j,0}\frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\bar{\Psi}_{j,2}\right]\right];$$
$$\psi_3(x,0) = 0,$$

(30)

$$\psi_4 = iL_t^{-1}\left[\psi_{3xx} + q\left[\left[\Psi_{j,0}\frac{\Psi_{j+1,3}+\Psi_{j-1,3}}{2} + \frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,3}\right]\bar{\Psi}_{j,0}\right.\right.$$
$$+\left[\Psi_{j,1}\frac{\Psi_{j+1,2}+\Psi_{j-1,2}}{2} + \frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2}\Psi_{j,2}\right]\bar{\Psi}_{j,0} + \left[\Psi_{j,0}\frac{\Psi_{j+1,2}+\Psi_{j-1,2}}{2} + \frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,2}\right]\bar{\Psi}_{j,1}$$
$$\left.\left.+\left[\Psi_{j,0}\frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2} + \frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,1}\right]\bar{\Psi}_{j,2} + \left[\Psi_{j,1}\frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2}\bar{\Psi}_{j,1} + \Psi_{j,0}\frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\bar{\Psi}_{j,3},\ldots\right]\right];$$
$$\psi_4(x,0) = 0,$$

(31)

and so on.

Now we are going to discuss an ordinary example to the plane wave solution (15) to the discrete nonlinear Schrödinger equation (13) with the help of our analysis of homotopy perturbation method and using the relation, $\omega = \left(\frac{4}{h^2}\right)\sin^2\left(\frac{kh}{2}\right) - q$ from the semi-discrete dispersion given in (16).

On the other hand, we can calculate the x-independent solution $\Psi_j(t) = fe^{\left(iq|f|^2 t\right)}$. The DNLS equation (13) and the AL equation (18) agree in this special case and it is properly easy to see that HPM produces exactly the wanted solution. Now we are going to consider the plane wave solution (15) and AL equation (18).Now we start with the initial condition given below as,

$$\psi_{j,0}(t) = f_j = e^{ijkh} \qquad (32)$$

From Equation (25) and (28), we can write in connection with the equation (32)

$$\psi_{j,1} = iL^{-1}{}_t D^2{}_x \psi_{j,0}(t) + iqL^{-1}{}_t\left(\psi_0{}^2\bar{\psi}_0\right),$$
$$= -iL^{-1}{}_t\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right)e^{ijkh} + iqL^{-1}{}_t\left[\psi_{j,0}\frac{\psi_{j+1,0}+\psi_{j-1,0}}{2}\bar{\psi}_{j,0}\right],$$
$$= -ite^{ijkh}\left(\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right) - q\cos(kh)\right) = -it\omega e^{ijkh}.$$

From Equation (26) and (29), we can write in connection with the equation (32)

$$\psi_{j,2}(t) = iL^{-1}{}_t\left(\left(-\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right)\left(-i\omega te^{ijkh}\right)\right)\right) + iqL^{-1}{}_t\left(\psi_{j,0}\frac{\psi_{j+1}+\psi_{j-1}}{2}\bar{\psi}_{j,1}\right)$$
$$+iqL^{-1}{}_t\left(\psi_{j,0}\frac{\psi_{j+1,1}+\psi_{j-1,1}}{2}+\frac{\psi_{j+1,0}+\psi_{j-1,0}}{2}\psi_{j,1}\right)\bar{\psi}_{j,0},$$
$$= -\frac{1}{2}\omega t^2\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right)e^{ijkh} + iq\frac{L^{-1}{}_t}{2}\left(-2i\omega te^{ijkh}\cos(kh)\right) = -\frac{1}{2}\omega^2t^2e^{ijkh},$$

$$\Psi_{j,3}(t) = iL^{-1}{}_tD^2{}_x\Psi_{j,2}(t) + iqL^{-1}{}_t\left(\psi_1{}^2\bar{\psi}_0 + 2\psi_0\bar{\psi}_0\psi_2 + 2\psi_0\psi_1\bar{\psi}_1 + 2\psi_0{}^2\bar{\psi}_2\right)$$
$$= iL^{-1}{}_t\left(\left(-\frac{1}{2}\omega^2t^2e^{ijkh}\right)\left(-\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right)\right)\right) + iqL^{-1}{}_t\left(\Psi_{j,1}\frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2}\bar{\Psi}_{j,0}\right)$$
$$+iqL^{-1}{}_t\left(\Psi_{j,0}\frac{\Psi_{j+1,2}+\Psi_{j-1,2}}{2}+\frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,2}\right)\bar{\Psi}_{j,0}$$
$$+iqL^{-1}{}_t\left(\Psi_{j,0}\frac{\Psi_{j+1,1}+\Psi_{j-1,1}}{2}+\frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\Psi_{j,1}\right)\bar{\Psi}_{j,0}$$
$$+iqL^{-1}{}_t\left(\Psi_{j,0}\frac{\Psi_{j+1,0}+\Psi_{j-1,0}}{2}\bar{\Psi}_{j,2}\right)$$
$$= \frac{i}{2}\omega^2\frac{t^3}{3}\frac{4}{h^2}\sin^2\left(\frac{kh}{2}\right)e^{ijkh} + iq\frac{L^{-1}{}_t}{2}\left(-i\omega te^{ijkh}\left(-i\omega te^{i(j+1)kh} - i\omega te^{i(j-1)kh}\right)e^{ie^{i\theta}jkh}\right)$$
$$+iq\frac{L^{-1}{}_t}{2}\left(\left(-\frac{1}{2}\omega^2t^2e^{i(j+1)kh} - \frac{1}{2}\omega^2t^2e^{i(j-1)kh}\right)\right) + iq\frac{L^{-1}{}_t}{2}\left(\left(e^{i(j+1)kh}+e^{i(j-1)kh}\right)\left(-\frac{1}{2}\omega^2t^2\right)\right)$$
$$+iq\frac{L^{-1}{}_t}{2}\left(\left(-i\omega te^{i(j+1)kh} - i\omega te^{i(j-1)kh}\right)(i\omega t)\right) + iq\frac{L^{-1}{}_t}{2}\left(i\omega t\left(e^{i(j+1)kh}+e^{i(j-1)kh}\right)(-i\omega t)\right)$$
$$+iq\frac{L^{-1}{}_t}{2}\left(\left(e^{i(j+1)kh}+e^{i(j-1)kh}\right)\left(-\frac{1}{2}\omega^2t^2\right)\right)$$
$$= \frac{i}{6}\omega^3t^3e^{ijkh}$$

$$\Psi_j(t) = \sum_{l=0}^{\infty}\Psi_{j,l}(t) = e^{ijkh}\left\{1 - i\omega t - \frac{1}{2}\omega^2t^2 + \frac{i}{6}\omega^3t^3 +, ...\right\}$$
$$= e^{ijkh}e^{-i\omega t} = e^{i(jkh-\omega t)}.$$

7 Conclusion

As a general analysis, we have seen that the homotopy perturbation method has the advantage over the other methods that we are free to choose the initial guess of different types, not bounded to apply this technique in spite of the existence of the large parameter because of the appearance of the embedding parameter, different choices of the linear operator. The solution is approximated by the series in the form of the parameter which is supposed to be small and takes the values between 0 and 1. The approximate solution gives, in some cases, closed form or the exact solution of the problems when the parameter is equal to 1.

Here, we have applied the homotopy perturbation method to the non-linear Schrodinger equation. In

Adomian decomposition method, we have to tackle with the so called Adomian polynomials and their hectic calculations. But in our analysis, we are not dependent upon these terms and find the linear forms of the components without these calculations.

We deal with the discretised forms of the nonlinear Schrodinger equation. Then at this stage, we are free whether to apply any numerical technique on the discritisation forms or proceed for perturbation solution of the problem. The discretised forms are obtained directly from the constructed homotopy, which on the first step provides the linear parts of the components of the solution series and then the partial derivatives are discretised using the standard technique. And hence a new discretised form is obtained with the help of this technique and the other standard method to discretise the derivatives.

We have seen that we consider the general form for the NLSE and then chose a general form of the initial guess and find the solution in the closed form.

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