Article

Method of extreme points: Characteristics of feasible sets for forecast of population dynamics

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Abstract

In current publication within the framework of method of extreme points various characteristics of feasible sets are applied for forecast of population dynamics. Following characteristics were used: maximum, minimum and average values for all trajectories with parameters belonging to feasible set, trajectory with best characteristics for Kolmogorov - Smirnov criterion, and trajectory with lowest value of sum of squared deviations between theoretical and empirical values. Analyses were provided for larch bud moth population (*Zeiraphera diniana* Gn.) time series (GPDD 1407; sample size is 38) and for Moran – Ricker model. Time series was divided onto two parts: for first part (first 21 values or more) feasible sets were determined and for tails of time series pointed out characteristics were applied. Forecasting properties of used characteristics are under discussion.

Keywords larch bud moth population dynamics; time series; statistical analysis; Moran-Ricker model; fitting, forecast; method of extreme points.

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1 Introduction

Search of suitable mathematical model and estimation of model parameters using empirical datasets are among of main elements of population dynamics analysis (McCallum, 2000; Isaev et al., 1984, 2001; Turchin, 2003; Wood, 2001; Nedorezov et al., 2008; Tonnang et al., 2009 a, b, 2010, 2012; Gao et al., 2012). Without finding of suitable model (or without constructing of new suitable model) it is impossible to prepare scientifically based forecasts of population size changing, optimal methods of its management etc. But up to current moment there are no criterions, which can help in finding suitable model before comparison of theoretical and empirical results (Isaev et al., 1984, 2001; Nedorezov and Utyupin, 2011; Nedorezov and Sadykova, 2010). In such a situation various methods of preliminary statistical analysis, which can help in creation of plausible hypothesis about the character of population fluctuations, can play important, key role in choosing of mathematical models (Nedorezov, 2012, 2013a, 2014). Moreover, in various situations comparison of theoretical and empirical/experimental datasets doesn’t allow choosing of best model: several models with similar properties can demonstrate very close final results (see, for example, Tonnang et al., 2009 a, b, 2010, 2012).

Time series on population dynamics of larch bud moth (*Zeiraphera diniana* Gn.) is very popular object of scientific investigations (Auer, 1977; Baltensweiler and Fischlin, 1988). In our previous publications (Nedorezov, 2011; Sadykova and Nedorezov, 2013; Nedorezov and Sadykova, 2015) it was proved that it is possible to obtain good fitting of empirical time series of larch bud moth population using well-known Moran – Ricker model (Moran, 1950; Ricker, 1954):

$$x_{k+1} = Ax_k e^{-\alpha x_k}. \quad (1)$$

In model (1) x_k is population density at moment k; parameter A is a maximum birth rate which is observed when population density is close to zero; α is a coefficient of self-regulation; both model parameters are non-negative, $A, \alpha = const \geq 0$. This model (1) has very rich set of dynamic regimes including cycles of various lengths and chaos, and it led to very wide application of this model to description of dynamics of various populations (McCallum, 2000; Nedorezov and Nedorezova, 1994; Turchin, 2003 and many others).

In current publication we use model (1) in the following manner. For part of time series (21 first elements of initial sample or more, $m = 21,...,30$) we determine points of feasible set (for 5% significance level). After that we use points from feasible sets for determination of various forecasting characteristics (for tails of time series): maximum and minimum values of population density, and average values. Trajectories with some extreme properties (with minimum value of characteristics of Kolmogorov – Smirnov test and with minimum value of sum of squared deviations) were also used for forecast.

It is assumed additionally that subsets of negative and positive deviations between theoretical and empirical values must contain 40% or more of all points of set of deviations. It isn’t obligatory condition and percentages can be changed. On the other hand, it is much better to compare samples with close sample sizes than in situation when difference of sample sizes is rather big. Respectively, use of this additional condition leads to decreasing of measures of feasible sets and to changing of behavior of used for forecast characteristics.

For some particular cases one more criterion was used: it was assumed that in 60% cases sign of increment/decrement of model trajectory must be equal to sign of increment/decrement of time series. It is important to note that this assumption looks strange for situation when we observe stochastic fluctuations near stable level. Respectively, before use of this criterion we have to be sure that observed population fluctuations don’t correspond to this dynamic regime: it was demonstrated in our previous publications (Nedorezov, 2011, 2013b, 2014; Sadykova and Nedorezov, 2013; Nedorezov and Sadykova, 2015). It was also proved that larch bud moth dynamics doesn’t correspond to strong 8- or 9-year cycle.

2 Datasets

Regular observations of larch bud moth fluctuations had been started in Swiss Alps (in Upper Engadine Valley) in 1949 (Auer, 1977; Baltensweiler and Fischlin, 1988). Used in current publication time series can be free downloaded in Internet (NERC Centre for Population Biology, Imperial College (1999) The Global Population Dynamics Database, N 1407). Population densities are presented in units “number of larvae per kilogram of branches”. Data were collected in Upper Engadine Valley on 1800 m above the sea level. Total sample size is 38 values (from 1949 to 1986).

3 Statistical Tests

Let $\{x_k^*\}$, $k = 0,1,..., N$, be empirical time series of population density changing in time; $N+1$ is sample size. Using this sample $\{x_k^*\}$ we have to estimate model parameters A, α, and initial population size x_0.

Use of least square method (Bard, 1974; Borovkov, 1984) is based on assumption that best estimations of model parameters can be found with minimizing of sum of squared deviations between theoretical (model) and empirical datasets. If time series is approximated by model (1) trajectory loss-function can be presented in the following form (note, that presented form is one of particular cases, and it is possible to present a lot of other forms which is used in practice):

$$Q(\vec{\alpha}, x_0) = \sum_{k=0}^{N}\left(x_k(\vec{\alpha}, x_0) - x_k^*\right)^2 . \quad (2)$$

In (2) $\{x_k(\vec{\alpha}, x_0)\}$ is model (1) trajectory obtained for fixed values of vector $\vec{\alpha} = \{A, \alpha\}$ and fixed initial population density x_0. Let's also assume that for certain point $(\vec{\alpha}^{**}, x_0^{**})$ there is a global minimum in (2):

$$Q(\vec{\alpha}^{**}, x_0^{**}) = \min_{\vec{\alpha}, x_0}\left(\sum_{k=0}^{N}\left(x_k(\vec{\alpha}, x_0) - x_k^*\right)^2\right). \quad (3)$$

Following a traditional approach of solution of considering problem of estimation of nonlinear model parameter estimation (Bard, 1974; Borovkov, 1984; Draper and Smith, 1981) after determination of parameter estimations $(\vec{\alpha}^{**}, x_0^{**})$ (3) analysis of set of deviations $\{e_k\}$ between theoretical (model) and empirical time series must be provided:

$$e_k = x_k(\vec{\alpha}^{**}, x_0^{**}) - x_k^* . \quad (4)$$

Model (1) is recognized to be suitable for fitting of considering time series if following conditions are truthful: deviations $\{e_k\}$ (4) are values of independent stochastic variables with Normal distribution with zero average. Following these assumptions Kolmogorov – Smirnov, Lilliefors, Shapiro – Wilk or other statistical tests are used for checking of Normality of deviations (Bolshev and Smirnov, 1983; Lilliefors, 1967; Shapiro et al., 1968). For checking of independence of stochastic variables Durbin – Watson and/or Swed – Eisenhart tests are used (Draper and Smith, 1981; Hollander and Wolfe, 1973; Likes and Laga, 1985).

If in a sequence of residuals (4) serial correlation is observed (for selected significance level) it gives a background for conclusion that considering model isn't suitable for fitting and needs in further modification (or we have to use other model). It means also that some of important factors or processes were not taken into account within the framework of used model. Similar conclusion about model and its applicability to fitting can be made in situation when hypothesis about Normality of deviations must be rejected (it is also for selected significance level). In other words, final conclusion about suitability of model for approximation of considering time series is based on analysis of properties of unique point $(\vec{\alpha}^{**}, x_0^{**})$ in a space of model parameters.

In our opinion, this is one of basic problems of least squared method: a'priori it is impossible to exclude from consideration a situation when nearest to $(\vec{\alpha}^{**}, x_0^{**})$ points have required properties. Below we'll consider situations when deviations between theoretical and empirical datasets are satisfied to following conditions. First of all, distribution of deviations (4) must be symmetric with respect to origin. Branches of density function must be monotonic curves – it must increase in negative part of straight line, and it must decrease in right part. In other words, density function must be symmetric with respect to ordinate line. Hypotheses about existence of serial correlation in sequences of residuals must be rejected (for selected significance level).

For checking of these properties of residuals we used Kolmogorov – Smirnov test, Lehmann – Rosenblatt test, and Mann – Whitney test (for checking of symmetry of distribution; Bolshev and Smirov, 1983; Hollander and Wolfe, 1973; Likes and Laga, 1985). For checking of monotonic behavior of branches of density function Spearman rank correlation coefficient was used. For testing of absence of serial correlation Swed – Eisenhart test (Draper, Smith, 1981) and test "jump up –jump down" (Likes and Laga, 1985) were used. Note that below non-parametric statistical tests were used only.

Point of space of model parameters (x_0, A, α) belongs to feasible set if and only if for selected significance level all statistical tests show requirement results: hypotheses about symmetry of density function with respect to ordinate line cannot be rejected, hypothesis about equivalence of Spearman rank correlation coefficient ρ to zero *must be rejected* (with alternative hypothesis $\rho > 0$), hypotheses about existence of serial correlation must be rejected. It is obvious that geometry of feasible set depends on statistical criterions we use for checking of properties of deviations and on selected significance level. Note, that for various tests we can use different significance levels.

As it was pointed out in Introduction, for some particular cases one more non-statistical (deterministic) criterion was used: it was assumed that in 60% cases sign of increment or decrement of model trajectory must be equal to respective sign of increment or decrement of time series. This assumption looks very strange for situation when we have stochastic fluctuations of population density near stable level. Respectively, before use of this deterministic criterion we have to be sure that observed population fluctuations don't correspond to this dynamic regime: it was demonstrated in our previous publications (Nedorezov, 2011, 2013b, 2014; Sadykova and Nedorezov, 2013; Nedorezov and Sadykova, 2015). Moreover, it was also proved that larch bud moth dynamics doesn't correspond to strong 8- or 9-year cycle. On the other hand, we have no reasons to say that we have good model if for every increment of time series model demonstrates decreasing of density and vice versa. We cannot talk about good model if number of cases with inverse changing of time series and model is rather big.

Below we used this criterion in deterministic form (with fixed 60%). But it can be used in standard statistical form: under an assumption that cases of equality and inequality of signs of increment/decrement of time series and model trajectory are independent events, we can calculate frequency of cases when we have equivalence for signs (in such a situation we can talk about Bernoulli trials). After that we can check Null hypothesis about equivalence of probability to 0.5 with alternative hypothesis "probability is bigger than 0.5". For *good model* Null hypothesis must be rejected for selected significance level.

4 Results

Stochastic points (x_0, A, α) were determined in set $\Delta = [0,1000] \times [0,1000] \times [0,100]$ with uniform distribution. First two limits of set Δ were determined from condition that observed values in two times less than pointed out amounts. Maximum of population density is 450, $\forall k \quad x_k^* \le 450$, maximum value of birth rate is 333.33, $A \le 333.33$. It was additionally assumed that number of positive and negative deviations (4) cannot be less than 40% of considering sample. For every stochastic point (x_0, A, α) properties of deviations between model trajectory and time series were determined (with statistical tests pointed out above).

4.1 Variant: 21 points

First sample contained 21 points of initial sample (values from 1949 and up to 1969). Respectively, in this case forecast was prepared for time interval 1970-1986. If it was checked that point belongs to feasible set (all statistical tests were satisfied), for these parameters trajectory of model (1) was determined, and obtained values were used for maximum, minimum and average forecasting values. All these values were determined for all 150000 stochastic trajectories (for 150000 elements of feasible sets).

Results of calculations for considering variant are presented in Fig. 1. Minimum values cannot be presented in this figure: all obtained values are rather small and less than 10^{-81}. As we can see in this figure, behaviors of presented curves are qualitatively different.

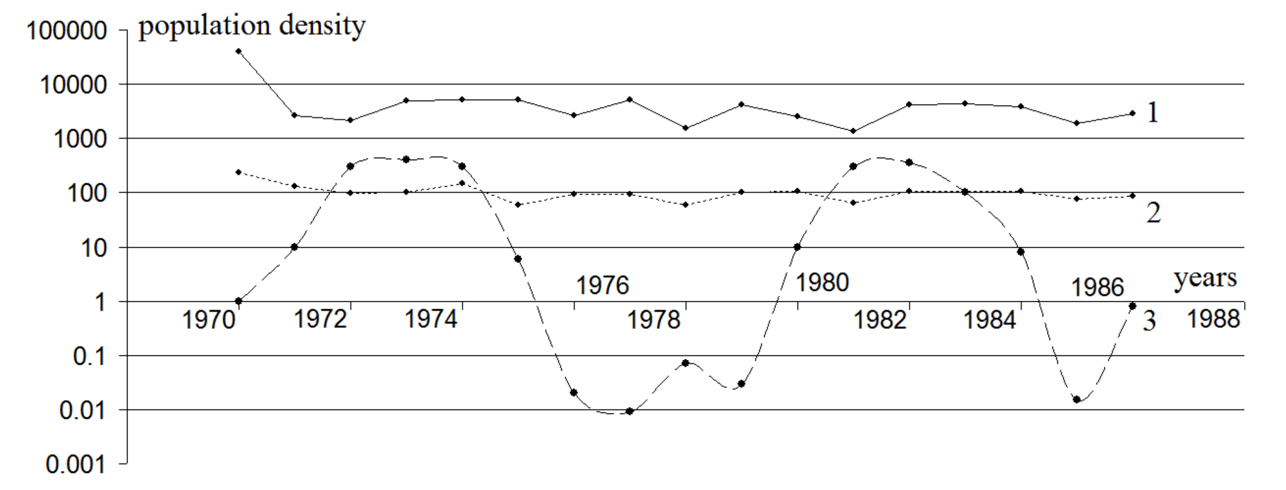

Fig. 1 Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 1 is maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is empirical time series. Results are presented in logarithmic scale.

Correspondence between increments/decrements of maximums (curve 1, Fig. 1) and increments/decrements of empirical time series is observed in 0.375 cases. For minimums this characteristics is equal to 0.4375. The best result was obtained for average line (curve 2, Fig. 1): frequency of cases when signs of increments/decrements of this curve are equal to increments/decrements of time series is equal to 0.5. It means that presented characteristics (Fig. 1) cannot be used for prediction of increasing or decreasing of population density (on qualitative level). Correlation coefficients for pointed out variables (maximum, minimum and average curves) are following: -0.15108, -0.23665, -0.00033. It shows also that considering characteristics are not suitable for forecast.

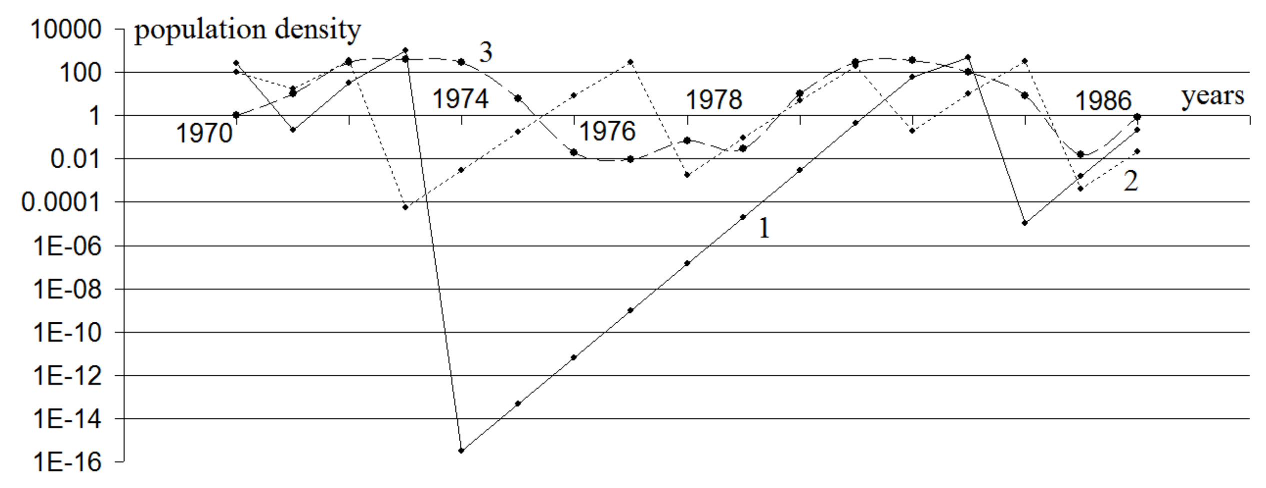

Fig. 2 Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 1 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 2 is model (1) trajectory corresponding to minimum value of functional (2)-(3) on feasible set. Curve 3 is empirical time series. Results are presented in logarithmic scale.

In Fig. 2 there are the trajectories of model (1) with parameters from feasible set: curve 1 corresponds to minimum value of Kolmogorov – Smirnov test (0.457738; hypothesis about symmetry cannot be rejected with 98.4% significance level; in other words, this hypothesis *must be accepted*), curve 2 corresponds to minimum value of functional (2)-(3) within the limits of 5%-feasible set. Correspondence between increments/decrements of curve 1 (Fig. 2) and increments/decrements of empirical time series is observed in 56.25% (9 of 16 cases), for curve 2 it is equal to 31.25%. Correlation coefficients for pointed out variables are following: 0.4485, 0.1031. Thus, we can conclude that best results were obtained for curve which corresponds to minimum value of Kolmogorov – Smirnov test. On the other hand, even for this curve we cannot say that this curve has good forecasting properties.

It is naturally to assume that forecast can be organized better if for prediction we'll use trajectories (within the limits of feasible set) which follow changes of time series. Below we consider the situations when in 60% (or more) cases increasing/decreasing of model trajectory is the same like it is observed in time series.

Results of calculations for these variants are presented in Fig. 3. Like in previous case, minimum values cannot be presented in this figure: all obtained values are rather small and less than 10^{-15} (it is better than in previous case but it has no sense for forecast). Correspondence between increments/decrements of maximums (curve 1, Fig. 3) and increments/decrements of empirical time series is observed in 43.75% of all cases; for minimums we have the same value. The best result was obtained for average line (curve 2, Fig. 3): frequency of cases when signs of increments/decrements of this curve are equal to increments/decrements of time series is equal to 0.5. It means that presented characteristics (Fig. 3) cannot be used for prediction of increasing or decreasing of population density (on qualitative level). Correlation coefficients for pointed out variables (maximum, minimum and average curves) are following: -0.07886, -0.1774, -0.03919. It shows also that considering characteristics are not suitable for forecast.

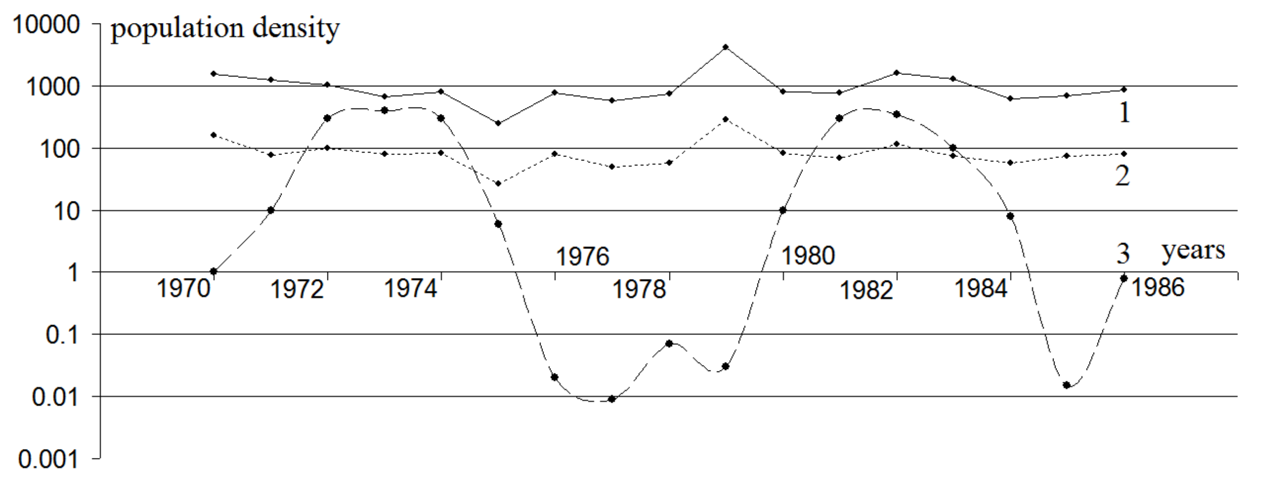

Fig. 3 Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 1 is maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is empirical time series.

In Fig. 4 there are the trajectories of model (1) with parameters from feasible set: curve 1 corresponds to minimum value of Kolmogorov – Smirnov test (0.561769; hypothesis about symmetry cannot be rejected with 90.13% significance level; value of Lehmann – Rosenblatt test is equal to 0.08557; Null hypothesis cannot be rejected with 68% significance level).

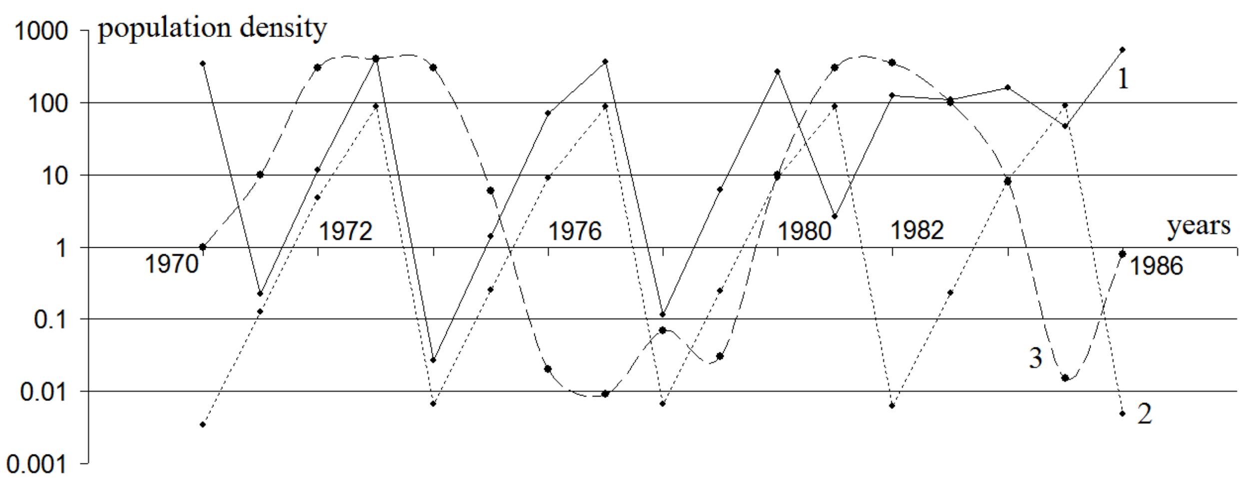

Fig. 4 Forecast of changing of larch bud moth dynamics for time interval 1970-1986. Curve 1 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 2 is model (1) trajectory corresponding to minimum value of functional (2)-(3) on feasible set. Curve 3 is empirical time series.

Curve 2 corresponds to minimum value of functional (2)-(3) within the limits of 5%-feasible set (value of Kolmogorov – Smirnov test is 1.007905; hypothesis about symmetry cannot be rejected with 25.94% significance level; value of Lehmann – Rosenblatt test is equal to 0.345238; Null hypothesis cannot be rejected with 9.833% significance level).

Correspondence between increments/decrements of curve 1 (Fig. 4) and increments/decrements of empirical time series is observed in 50% (8 of 16 cases), for curve 2 it is equal to 37.5% (6 of 16 cases). Correlation coefficients for pointed out variables are following: -0.06586, 0.24472. Thus, we can conclude that all considered curves haven't good forecasting properties. For first 21 elements of initial sample we had a situation when in 60% cases model trajectory followed to changing of time series; for forecasting tail of time series this property wasn't saved.

4.2 Variant: 22 points

Let's consider a situation when *starting* sample contains 22 points of initial sample (values from 1949 and up to 1970). In this case forecast was prepared for time interval 1971-1986. If it was checked that point belongs to feasible set, for these parameters trajectory of model (1) was determined, and obtained values were used for maximum, minimum and average forecasting values. Like in previous case all these values were determined for all 150000 stochastic trajectories (for 150000 elements of feasible sets).

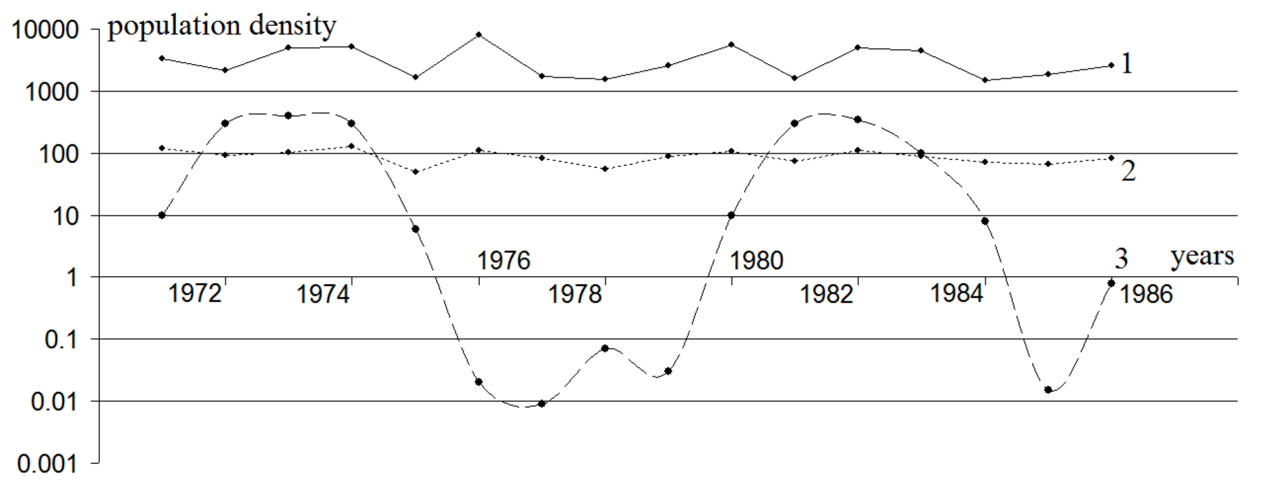

Fig. 5 Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 1 is a set of maximum forecasting values of population density. Curve 2 contains average values of population densities. Curve 3 is empirical time series.

Results of calculations for considering variant are presented in Fig. 5. Minimum values cannot be presented in this figure: all obtained values are rather small and less than 10^{-76}. As we can see in this figure, behaviors of presented curves are qualitatively different.

Correspondence between increments/decrements of curve 1 (Fig. 5) and increments/decrements of empirical time series is observed in 53.33% of all cases (8 of 15); for curve of minimums we have the same result; for curve 2 frequency of cases when signs of increments/decrements of this curve are equal to increments/decrements of time series is equal to 0.6. Correlation coefficients for pointed out variables (maximum, minimum and average curves) are following: 0.215725, -0.18991, and 0.390333. It means that presented characteristics (Fig. 5) haven't good forecasting properties.

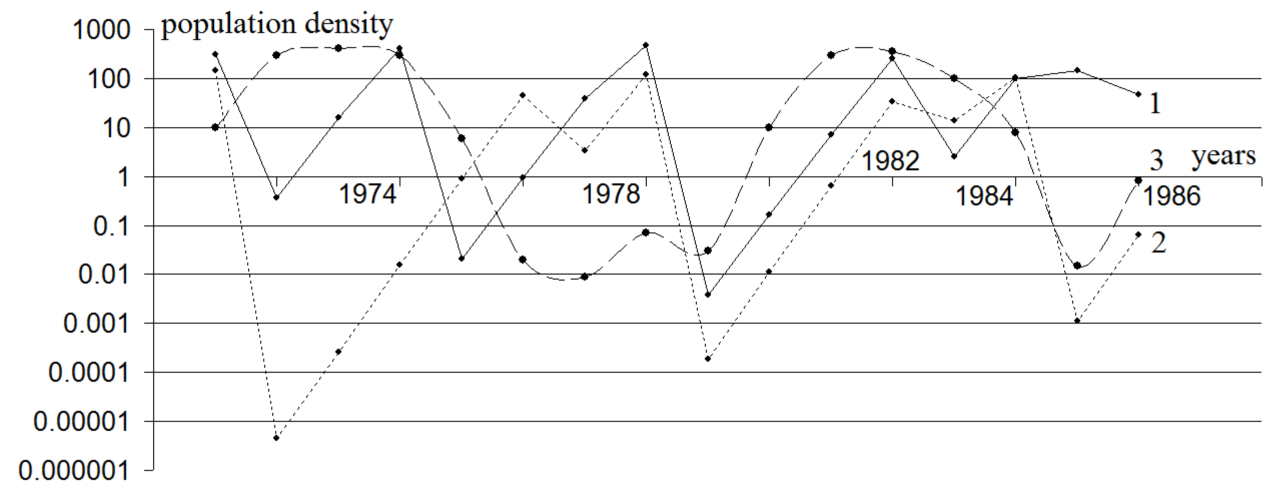

Fig. 6 Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 1 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 2 is model (1) trajectory corresponding to minimum value of functional (2)-(3) on feasible set. Curve 3 is empirical time series.

In Fig. 6 there are the trajectories of model (1) with parameters from feasible set: curve 1 corresponds to minimum value of Kolmogorov – Smirnov test (0.426401; hypothesis about symmetry cannot be rejected with 99.26% significance level; in other words, this hypothesis *must be accepted*; value of Lehmann – Rosenblatt test is equal to 0.055785; Null hypothesis cannot be rejected with 81.14% significance level). Curve 2 corresponds to minimum value of functional (2)-(3) within the limits of 5%-feasible set (value of Kolmogorov – Smirnov test is equal to 0.985521; hypothesis about symmetry cannot be rejected with 28.09% significance level; value of Lehmann – Rosenblatt test is equal to 0.245726; Null hypothesis cannot be rejected with 18.84% significance level).

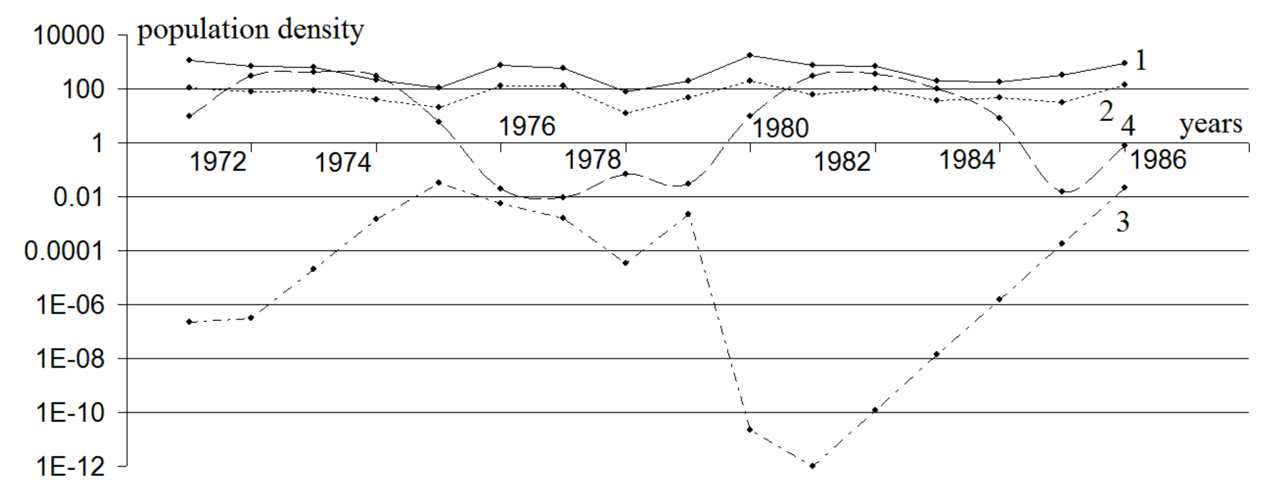

Fig. 7 Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 1 is a set of maximum forecasting values of population density. Curve 2 contains minimum forecasting values of population density. Curve 3 contains average values of population densities. Curve 4 is empirical time series.

Correspondence between increments/decrements of curve 1 (Fig. 6) and increments/decrements of empirical time series is observed in 53.33% (8 of 15 cases), for curve 2 it is equal to 66.67% (10 of 15 cases).

Correlation coefficients for pointed out variables are following: 0.059764, -0.31821. For these curves we cannot also say that curves have good forecasting properties.

Below we consider the situation when in 60% (or more) cases increasing/decreasing of model trajectory is the same like it is observed in time series.

Results of calculations for considering variant are presented in Fig. 7. As we can see in this figure, curves 1 and 2 may have a certain sense for long-term forecast: sometimes values of maximums of set of trajectories are very close to values of time series (1972-1974 and 1981-1983) and minimums are also close to values of time series (1976-1977, 1979). In a result of additional condition for samples of deviations (4) that sizes of samples of negative and positive deviations cannot be less than 40% of initial sample size, we got rather small feasible set and maximum value (1974) for all trajectories with parameters from feasible set is less than respective value from time series. Anyway, during several years values of time series are close to values of curve 1 (Fig. 7).

Correspondence between increments/decrements of curve 1 (Fig. 7) and increments/decrements of empirical time series is observed in 46.67% of all cases (7 of 15); for curve 2 we have 40% only; for curve 3 frequency of cases when signs of increments/decrements of this curve are equal to increments/decrements of time series is equal to 0.6. Correlation coefficients for pointed out variables (maximum, minimum and average curves) are following: 0.034029, -0.305, -0.09706. It means that presented characteristics (Fig. 7) can *sometimes* be used for prediction of changing of population density.

In Fig. 8 there are the trajectories of model (1) with parameters from 5%-feasible set: curve 1 corresponds to minimum value of Kolmogorov – Smirnov test (0.639602; hypothesis about symmetry cannot be rejected with 80.73% significance level; value of Lehmann – Rosenblatt test is equal to 0.130165; Null hypothesis cannot be rejected with 45% significance level). Curve 2 corresponds to minimum value of functional (2)-(3) within the limits of 5%-feasible set (value of Kolmogorov – Smirnov test is equal to 1.024941; hypothesis about symmetry cannot be rejected with 23.92% significance level; value of Lehmann – Rosenblatt test is equal to 0.258936; Null hypothesis cannot be rejected with 17.6% significance level). Note, that pointed out results were obtained for starting part of initial sample which was used for finding elements of feasible set.

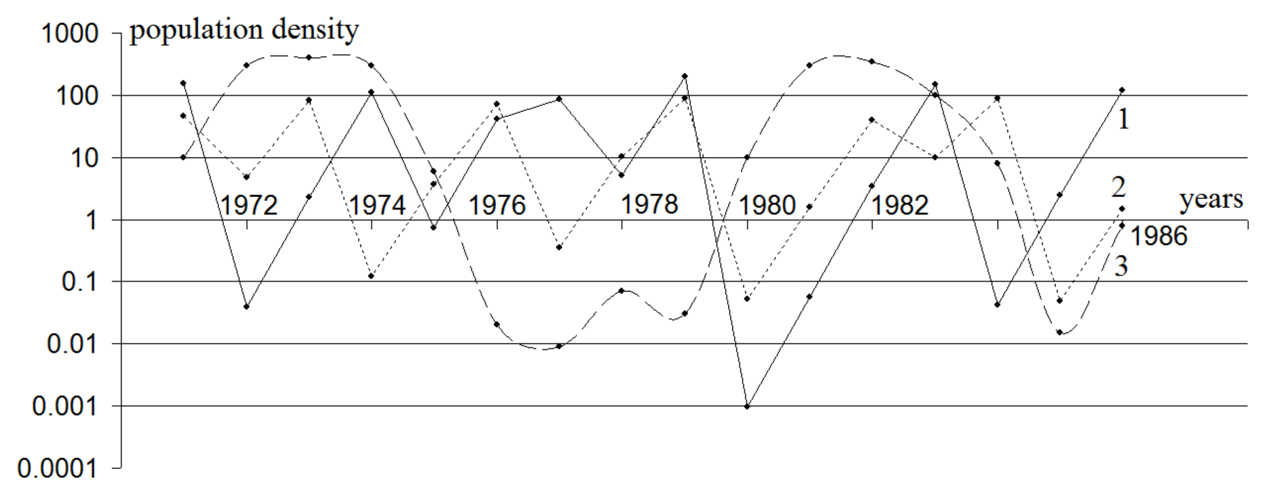

Fig. 8 Forecast of changing of larch bud moth dynamics for time interval 1971-1986. Curve 1 is model (1) trajectory corresponding to minimum value of characteristics of Kolmogorov – Smirnov test. Curve 2 is model (1) trajectory corresponding to minimum value of functional (2)-(3) on feasible set. Curve 3 is empirical time series.

Let's consider results which were found for *tail of sample*. Correspondence between increments/decrements of curve 1 (Fig. 8) and increments/decrements of empirical time series is observed in 40% (6 of 15 cases), for curve 2 it is equal to 60% (9 of 15). Correlation coefficients for pointed out variables are following: -0.27527, 0.00221. Thus, like in previous variants we haven't a background for conclusion that considering curves can play important role in forecasting of population density changing.

5 Conclusion

Constructing of feasible sets in space of model parameters allows obtaining and using for forecast various characteristics of these sets. In particular, it allows using trajectories with extreme properties (of sets of deviations between theoretical and empirical datasets). In ideal situation we have one trajectory only which has extreme properties for deviations. But such situations are very rare. In most cases we can find trajectories which have, for example, symmetric distribution for deviations with a certain guarantee (in a case when hypothesis about symmetry cannot be rejected with 99.999% or bigger significance level). It is also possible to use least squared method within the limits of feasible sets etc. But up to current moment it isn't obvious what kind of characteristics of feasible sets we have to use for constructing best forecasts.

Results obtaining for feasible sets with 5% significance level (and with some additional conditions: it was assumed that samples of deviations between theoretical and empirical values must contain 40% values of total sample; for special cases it was assumed that model trajectory must follow empirical dataset in 60% cases of changing of population density) are presented in current publication. Note that all requirements (correspondence to statistical criterions plus additional conditions) were applied to first parts of initial sample: these parts contain 21 and 22 values of changing of larch bud moth density in Swiss Alps. Forecasting properties of Moran – Ricker model were tested using *tails of samples* (total number of points in sample is 38).

First of all, for all trajectories with parameters from feasible sets maximum and minimum values were determined. In first case (when number of points is equal to 21) these curves have no sense: obtained values were much bigger and much smaller than empirical values. But in second case (number of points is equal to 22 and feasible set was constructed with additional conditions) for some years these curves were rather close to empirical values.

For constructing of forecasts the following curves were also used: averages for all trajectories with parameters from feasible sets; trajectories of Moran – Ricker model which corresponds to situation when hypothesis about symmetry of deviations cannot be rejected with biggest significance level (Kolmogorov – Smirnov test); trajectories of Moran – Ricker model which corresponds to situation when sum of squared deviations has its minimum value (within the limits of feasible sets; direct analog of Least Squared Methods with restriction on feasible sets). In all these cases pointed out curves had no relation to behavior of tail of sample: correlation coefficients were very small, and in some cases these coefficients were negative.

Obtained results can be explained in two possible ways. First of all, Moran – Ricker model has the following property: there are two points (beginning and end) on the phase of population decreasing. At the same time empirical time series may contain several points on this phase. It means that a'priori we cannot give sufficient approximation of empirical sample with the help of considering model. For better approximation we have to use models (may be based on Moran – Ricker model) which take into account influence of time lags in reactions of regulative mechanisms on population density changing. The second, may be used curves are not the best for forecast, and we must use for it other curves with extreme characteristics.

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