Article

A computational intensive method- Lubrication approximation theory for blade coating process

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Abstract

This paper presents the analysis of the process of blade coating through a computational intensive method for an incompressible Newtonian fluid along with Magnetohydrodynamics (MHD). The slip between the substrate and the fluid is also taken into account. The nature of the existing steady solutions has been investigated with the help of exact and numerical methods. Those obtained exact solutions include the solutions for the velocity profiles, volumetric flow rate and pressure gradient. The important engineering quantities like maximum pressure, pressure distribution and load are also computed. It is assumed that the relative velocity between the plate and the fluid is proportional to the shear rate at the plate. An external magnetic field is applied normal to the plates. It is observed and concluded that both slip parameter and the magnetic field parameter serve as the controlling parameters in the industrial blade coating process.

Keywords computational method; lubrication approximation theory; blade coating process; Newtonian fluid; MHD steady flows; slip conditions.

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1 Introduction

The process for blade-coating is characterized by the flow of a fluid between a plane and a blade in a narrow gap and for the improvement of a plane surface. In industry, this is the most common method. The importance of the blade coating process is obvious because it is involved in the process of manufacturing of newspapers, photographic film, magnetic storage media, fibers and catalogues. Further, the importance of the process of blade coating for economic point of view has aggravated an extensive research for better understanding of the involved underlying physical mechanisms, and hence for the improvement of its efficiency. Basically, the following actions are involved for a blade coater; a moving substrate supported by a backing roll which carries the fluid into the wedge which is formed by the substrate and the blade. The result is that, an adverse pressure gradient is produced due to the fact that the fluid is dragged in the wedge and this drag flow is opposed by the pressure gradient. Therefore, the blade removes excess coating since it is acting as the self-metering precise smoothing device. Typically, the metered coating layer ranges from 10 to 50 μm . A blade may be classified as stiff or flexible depending upon its elastic properties and how the blade is mounted. Only the edge of the blade remains in contact with the substrate in the case when the blade is in stiff mode. On the contrary, the blade flexes and it is in the bent mode then back side (underside) of the blade presses against the backing roll.

A Pioneer work regarding blade coating process can be seen in some references (Middleman, 1977; Ruschak, 1985; Kistler and Schweizer, 1997; Gaskell et al., 1996; Bourgin, 1998). In the geometry of the blade coating in case of viscous and viscoelastic fluids, the film thickness has been studied by Sullivan and Middleman (1986). Greener and Middleman (1974), Ross et al. (1999), and Tichy (1996) studied the viscoelastic fluid of blade coating with the limit of weak viscoelasticity, power law fluid and convected Maxwell model. The power law fluid was also studied by Hwang (1982) and Dien and Elrod (1983) for simple blade coater and the approximate solutions were proposed for the flow. The lubrication theory was adopted by Hsu et al. (1985) in which the obtained results were compared theoretically and experimentally. As far as the Magnetohydrodynamics is concerned, in our point of view, no attention was paid to the magnetohydrodynamic effects on the Newtonian model for blade coating. Magnetohydrodynamics (MHD) helps us to study the interactions of conducting fluids with electromagnetic phenomena. For the electrically conducting fluid flow, when the magnetic field is present, the reader is referred to the classical works by Hartmann (1937), Scercliff, (1965), Siddiqui et al. (2013), and Zahid et al. (2016).

Despite the above studies, attention has hardly been focused on the study of the effects of the magnetic field on Newtonian fluid flows with slip boundary conditions. One of the cornerstones on the basis of which the mechanics of the linearly viscous fluid is build is the non-slip boundary. It is well known that the slip velocity depends strongly on the shear stress, since mostly constitutive equations which are developed for slip, presume that it is only dependent on the shear stress. A slip boundary condition was supposed by Navier (1827) in which the slip velocity is dependent linearly upon the shear stress. The notion of the “coefficient of slip” was introduced by Helmholtz and Pitrowski (1882) for the slip which occurs adjacent to a wall. For the slip of a gas next to a solid surface, an expression was delivered by Maxwell (1890). Rao and Rajagopal (1999) studied the flow of the fluids in a channel where the slip is dependent upon both the normal stress and the shear stress. The numerical studies of the non-Newtonian fluid with wall slip have been discussed by Khaled and Vafai (2004), Debbaut (1993), Dong (1993), Flower (1982), Torres (1993), and Tarunin (1980).

According to the authors’ best of knowledge, no attention still has been paid to the MHD effects on Newtonian model of blade coating along with the slip. The major objective of this research is the presentation of the analytical solution of the blade coating of MHD along with the slip. This research is not only significant due to its technological significance but also it is more important because of its interesting mathematical features which are presented by the governing equations for the flow. The expressions for flow variables such as velocity, pressure distribution and pressure gradient are calculated. In the successive sections, the formulation of the problem with the governing equations is presented. The exact and numerical solutions for the flow variables have been given in the later section, as mentioned above. As a final point, the results, discussion and conclusions are presented in the last section.

2 The Governing Equations

In the presence of a magnetic field, the basic governing equations for the flow of an incompressible, isothermal and linearly viscous fluid are

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\rho \frac{D\mathbf{V}}{Dt} = \nabla.\mathbf{S} + \mathbf{J} \times \mathbf{B}, \tag{2}$$

In the above equation, **J** represents the current density, ρ the constant density, **V** is the velocity field, **B** is the total magnetic field with $\mathbf{B} = B_0 + \mathbf{b}$, (where **b** is termed as the induced magnetic field). The quantity $\frac{D}{Dt}$ is conventionally the material derivative and it is defined by,

$$\frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V}, \tag{3}$$

and for the Newtonian fluid, the Cauchy stress tensor is given by the following equation

$$\mathbf{S} = -p\mathbf{I} + \mu \mathbf{A}_1, \tag{4}$$

In the above equation, $\mathbf{A}_1$ is the termed as the first Rivilin Ericksen tensor, *p* represents the pressure, **I** is the unit tensor. $\mathbf{A}_1$ is defined by

$$\mathbf{A}_1 = (\nabla \mathbf{V}) + (\nabla \mathbf{V})^T. \tag{5}$$

The superscript *T*, here is to represent the transpose of a matrix.
The **MHD** theory leads us to take the advantage of the following equations;

$$\left.\begin{array}{l} \nabla.\mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_m \mathbf{J}, \quad \nabla \times \mathbf{B} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \end{array}\right\}, \tag{6}$$

here μ_m presents the magnetic permeability, **E** the electric field and σ the electric conductivity. In the flow, an electric field *E* is induced with the assumption that no excess charge density is present. This leads us to write $\nabla \cdot \boldsymbol{E} = 0$. Further, if the induced magnetic field is neglected then it implies that $\nabla \times \boldsymbol{E} = 0$ which means that the induced electric field is negligible. Thus, the linearized body force which is included in the Eq. (2) may take the following form

$$\mathbf{J} \times \mathbf{B} = -\sigma \mathbf{B}_0^2 \mathbf{V}, \tag{7}$$

In view of equation (7), the momentum equation then may be rewritten as

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V} \right) = -\nabla p + \mu \nabla.\mathbf{A}_1 - \sigma \mathbf{B}_0^2 \mathbf{V}. \tag{8}$$

3 Formulation of the Problem

For our model, we consider a steady state, isothermal and two dimensional blade coater in the system of Cartesian coordinates as shown in Fig.1. The geometry of the blade coater comprises a moving substrate at $y = 0$, and a stiff blade of approximate shape $y = h(x)$.

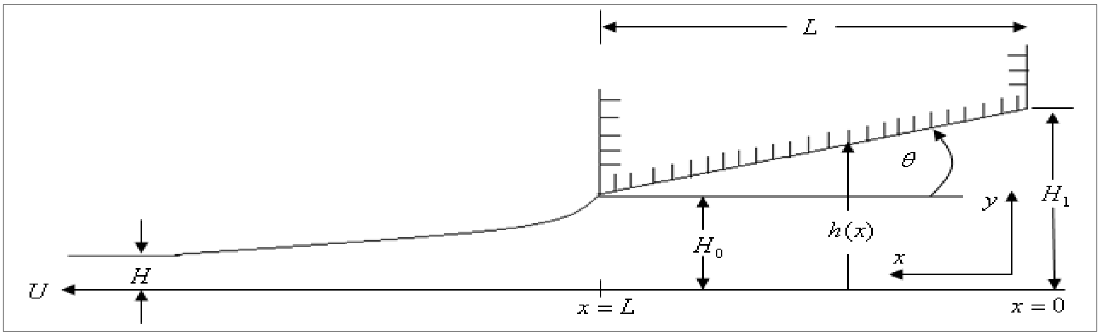

Fig. 1 Blade-Coater Geometry

The velocity U of the moving substrate is taken constant in the x direction. The length of the stiff blade is L while the edges are of height H_0 (at $x = L$) and H_1 (at $x = 0$). The blade is held fixed at an angle with $\tan\theta = \frac{H_1 - H_0}{L}$. An external magnetic field is applied normal to the plate. The lubrication approximation is supposed to be valid here in the flow field.

If the velocity components u and v are taken in x and y directions respectively then the boundary conditions along with slip are appropriately written in the following form

$$u = U \text{ at } y = 0,$$
$$u + \gamma\frac{du}{dy} = 0 \text{ at } y = h(x). \tag{9}$$

The velocity profile for two dimensional flow will have the following form,

$$\mathbf{V} = \left[u(x, y), v(x, y)\right]. \tag{10}$$

This velocity profile helps us to write Eqs. (1) and (8) as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{11}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B_0^2 u, \tag{12}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \sigma B_0^2 v. \tag{13}$$

Here, there are three equations with three unknowns. It is not easy to solve these equations analytically, therefore, one can apply lubrication approximation theory to overcome this problem.

We note from the geometry of the problem that there is a most important dynamical event in occurring in the nip of the blade. The minimum distance between the blade and the web at the nip, H_0, is so small that it can be negligible when compared with the length of substrate. It is reasonable then to suppose that the flow is nearly parallel and that the general movement of the fluid is mainly in *x*-direction. Further, the velocity of the fluid is small in the *y*-direction.

For the velocity and pressure, we obtain the characteristic scale for them. For which, we conduct an order of magnitude analysis for brevity. We then can identify the following scales for x, y and u, as $x \sim L_c,\ \ y \sim H_0,\ \ u \sim U.$ Taking into account the above relationship and from the continuity Eq.(11), we obtain $\frac{v_c}{U} \sim \frac{H_0}{L_c} = 1,$ which shows that the order of magnitude of a transversal velocity, v_c, is smaller than that of the longitudinal velocity. Based on the above discussion, the Eqs (11)-(13) become

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_0^2 u = 0, \tag{14}$$

$$\frac{\partial p}{\partial y} = 0. \tag{15}$$

The Eq. (15) imply that $p = p(x),$ then Eq. (14) becomes

$$\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\mu} B_0^2 u = \frac{1}{\mu}\frac{dp}{dx}. \tag{16}$$

When $B_0 = 0,$ that is in the absence of magnetic field, Eq. (16) is identical to that which has been seen in the study of the (Middleman, 1977).

4 Solution of the Problem

We define the dimensionless parameters as

$$x = \frac{\bar{x}}{L},\ u = \frac{\bar{u}}{U},\ y = \frac{\bar{y}}{H_0},\ p = \frac{\bar{p} H_0^2}{\mu U L},\ h = \frac{\bar{h}}{H_0}, \beta = \frac{\gamma}{H_0} \tag{17}$$

then eq. (16) in non-dimensional form after dropping "$-$" sign becomes

$$\frac{\partial^2 u}{\partial y^2} - M^2 u = \frac{dp}{dx} \tag{18}$$

where $M = \sqrt{\frac{\sigma}{\mu}} B_0 H_0$, and the boundary conditions in the dimensionless form becomes

$$\left.\begin{array}{l} u = 1 \text{ at } y = 0, \\ u + \beta \dfrac{du}{dy} = 0 \text{ at } y = h(x) \end{array}\right\}. \tag{19}$$

where $h(x) = \kappa + (1-\kappa)x$ and $\kappa = \frac{H_1}{H_0}$. The solution of the Eq. (18) using the boundary conditions become

$$u=\frac{1}{\left(M\beta\cosh\left(Mh\right)+M^{2}\sinh\left(Mh\right)\right)}\left[\begin{array}{l}\left(M^{2}+\frac{dp}{dx}\right)\left\{\begin{array}{l}\cosh(My)\cosh(Mh)+\sinh M(h-y)\\-M\beta\sinh(My)\sinh(Mh)\end{array}\right\}\\+\frac{dp}{dx}\left\{1-\left(M\beta\cosh(Mh)+\sinh(Mh)\right)\right\}\end{array}\right]$$

(20)

Therefore, u is an implicit function of x and an explicit function of y and, through $p(x)$ and $h(x)$.

In non-dimensional form, the volumetric flow rate Q per unit width is

$$\frac{Q}{W}=\int_{0}^{h(x)}udy \tag{21}$$

where $Q=\frac{\bar{Q}}{UH_0}$, and the quantity $\bar{Q}$ is the dimensional flow rate per unit width. Thus, the equation (21) gives

$$Q=\frac{1}{(M\beta\cosh(Mh)+M^{2}\sinh(Mh))}\left[\begin{array}{l}\left(M^{2}+\frac{dp}{dx}\right)\left\{\begin{array}{l}\frac{\sinh(2Mh)}{2}\left(\frac{1}{M}+\beta\right)+\frac{1}{M}(\cosh(Mh)-1)\\+\beta\sinh(Mh)\end{array}\right\}\\+\frac{dp}{dx}\left\{1-\left(\sinh(Mh)+M\beta\cosh(Mh)\right)\right\}h\end{array}\right]$$

(22)

If we suppose that the sheet is leaving the blade with a a speed U and a thickness H, then we have $Q=UH.$ This gives

$$\frac{H}{H_0}=\frac{\bar{Q}}{UH_0}=Q \tag{23}$$

Let $\frac{H}{H_0}$; λ, then Q ; λ, Now, the equation (22) for $\frac{dp}{dx}$

$$\frac{dp}{dx}=\left[\frac{-M^{2}(e^{(2Mh)}(M^{2}\beta\lambda-M^{2}\lambda-1)+2e^{(Mh)}+M^{2}\beta\lambda+M\beta-M\lambda-1)}{e^{(2Mh)}(M(h\beta+h-\beta)-2)+h(M(M\beta-1))+4e^{(Mh)}-2+M\beta}\right]$$

(24)

It is not possible to solve Eq. (24) exactly, so the numerical techniques has been applied to obtain the results for pressure. Since there are two unknown parameters in the equation, coating thickness *H* and the constant of integration, so two boundary conditions are required on *p*. Also from the physics of the problem, the pressure is zero at the entrance and also at the detachment point. Therefore, the boundary conditions for Eq. (24) can be written in dimensionless form as

$$\left.\begin{array}{l} p = 0 \text{ at } x = 0, \\ p = 0 \text{ at } x = 1, \end{array}\right\} \tag{25}$$

And the load can be calculated from the equation

$$L = \int_0^1 p dx \tag{26}$$

and the numeric results of Eq.(26) are given in the table 1.

5 Results and Discussion

In this paper, the process of blade coating for an incompressible MHD Newtonian fluid along with the slip condition has been analyzed. We made use of the lubrication approximation theory for the simplification of the equations of the motion. The exact solutions for pressure gradient, velocity profile and the volumetric flow rate are obtained. Moreover, the numerical solutions for the pressure distribution and load are carried out. Some of the results are shown graphically while other are given in the tabulated form. The effect of *M* (the Hartman number) and the slip parameter β on the velocity field are depicted in Figs. 2-5 and 6-8 respectively, at different positions of the blade coating process fixing $\kappa = 2$. It is clear that with an increase in *M*, the velocity decreases. One can also see that the motion of the fluid is fully developed near and after the detachment point, which shows that the velocity of the fluid and the plane is the same, this further means that the final coating of the fluid has taken place. As expected, It has also been observed that the velocity curve is reduced by the Hartman number *M*. Since *M* is due to Lorentz force, which is opposing the flow, therefore, the velocity becomes a decreasing function of the Hartman number *M*. It has been observed that the slip velocity increases with an increase in β and consequently because of the slip condition, the fluid velocity decreases; the pulling of the stretching wall can only be partially transmitted to the fluid. Moreover, the substantial effects have also been noted that the slip parameter β has on the solutions.

The graphs of the pressure gradient keeping the value of κ fixed and for various values of the slip parameter β and *M* respectively have been shown in the Figs. 9 and 10. One can observe that the pressure gradient decreases with an increase in the values of *M*. On the other hand, the pressure gradient first decreases and then increases monotonically with an increasing values of β. The graphs for the pressure distribution are presented in the Figs. 11 and 12 for the different values of *M* and β respectively, keeping κ fixed. One can observe that the pressure decreases with an increase in the values of *M*, while with an increase in the values of β the pressure increases. So we can really control pressure gradient and pressure distribution by the influence of MHD and the slip parameter.

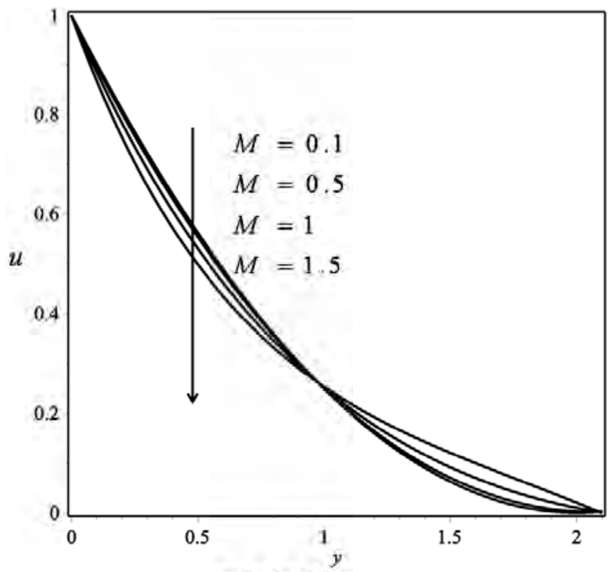

Fig 2: Velocity profile for different values of M at x=0

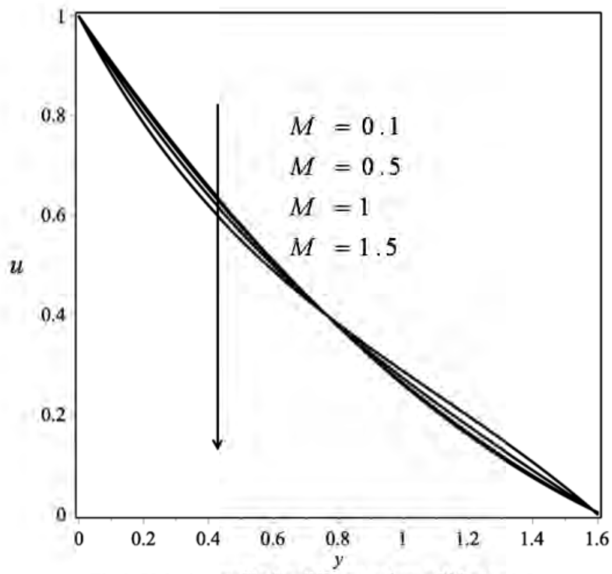

Fig 3: Velocity profile for different values of M at x=0.5

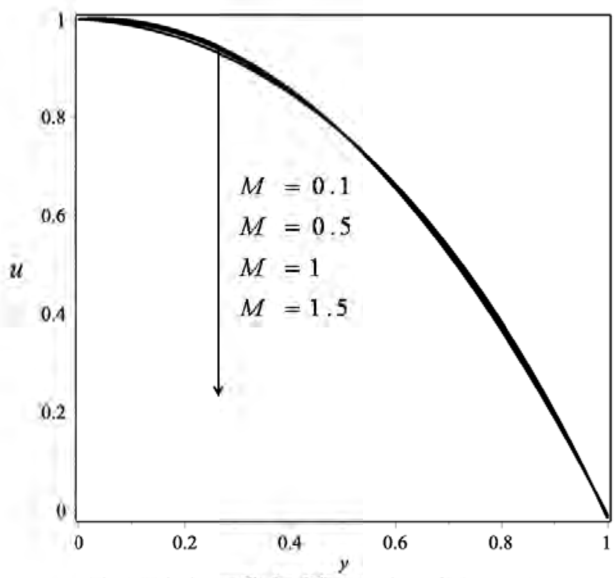

Fig 4: Velocity profile for different values of M at x=1.1

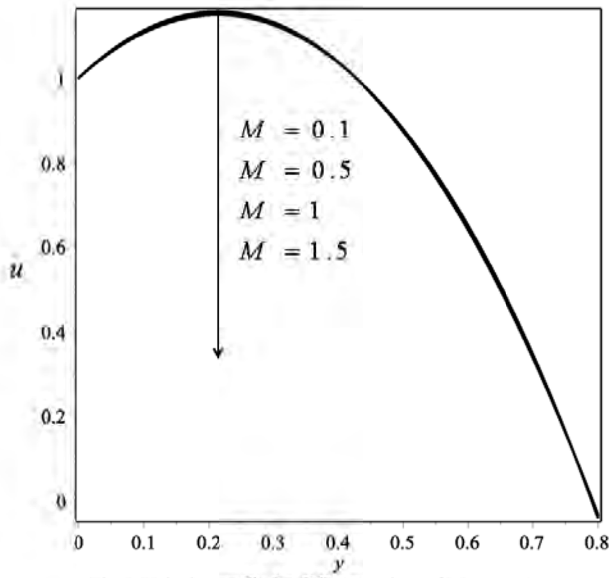

Fig 5: Velocity profile for different values of M at x=1.3

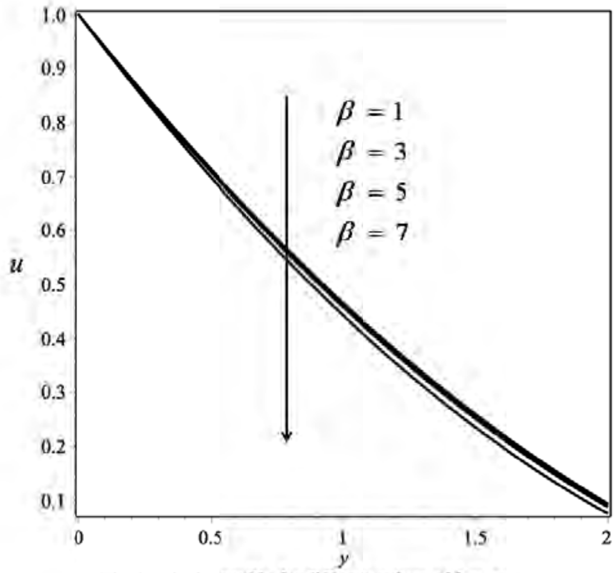

Fig 6: velocity profile for different values of β at x=0

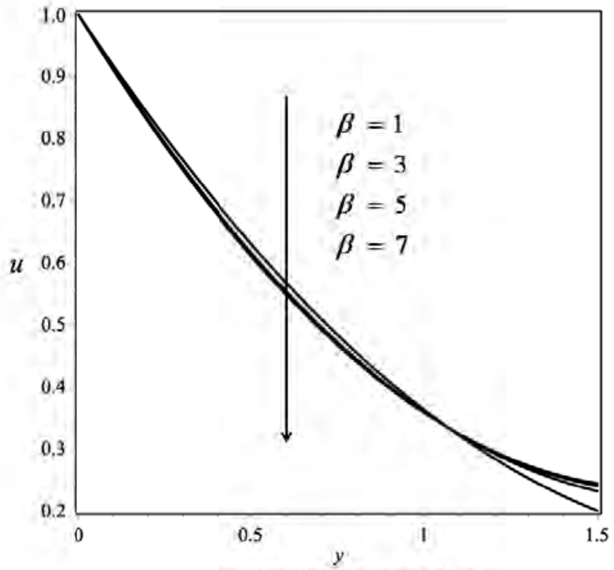

Fig 7: velocity profile for different values of β at x=0.8

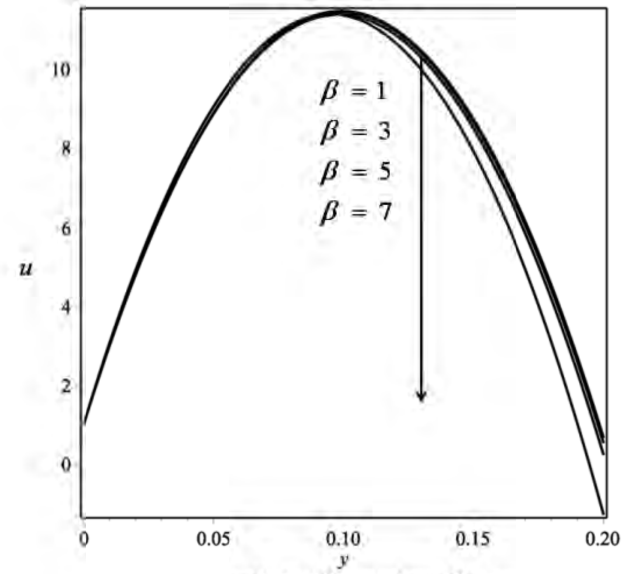

Fig 8: velocity profile for different values of β at x=1.3

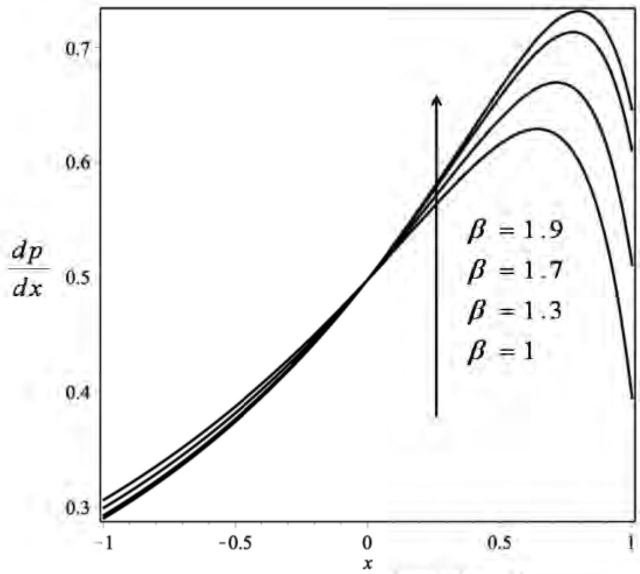

Fig 9: Pressure gradient for different values of β

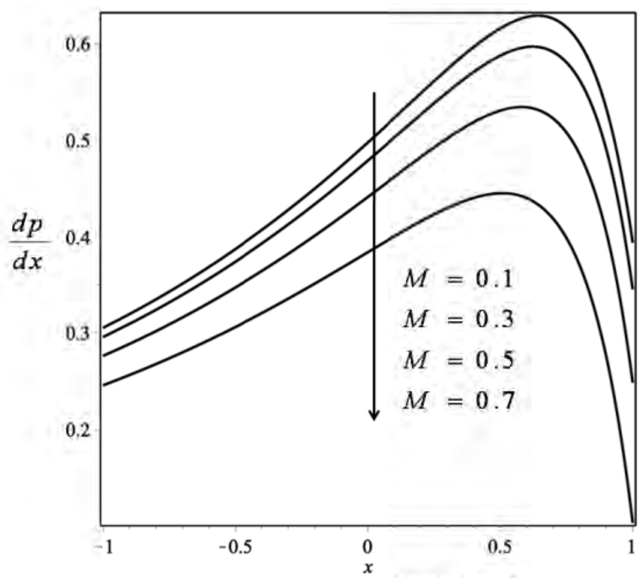

Fig 10: Pressure gradient for different values of M

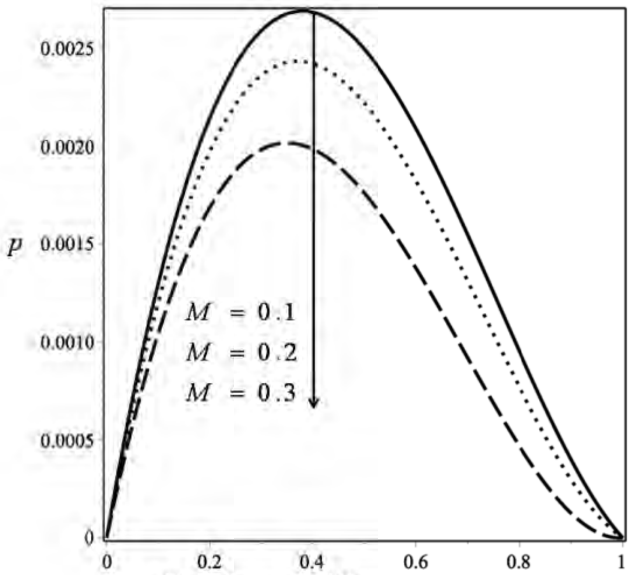

Fig 11: Pressure for different values of M

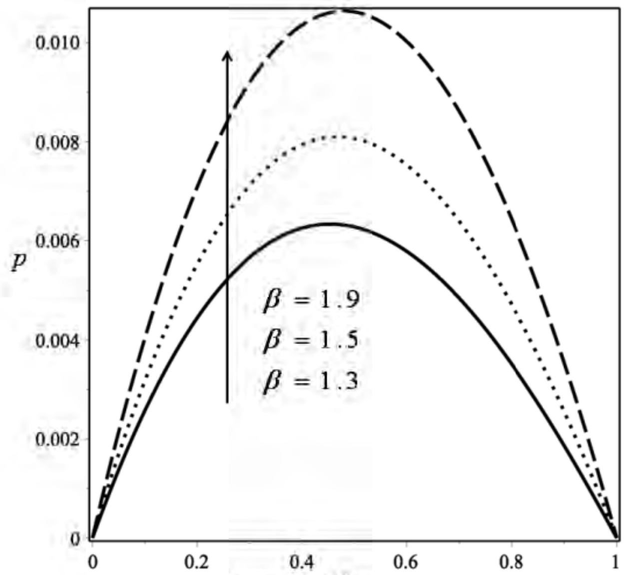

Fig 12: Pressure for different values of β

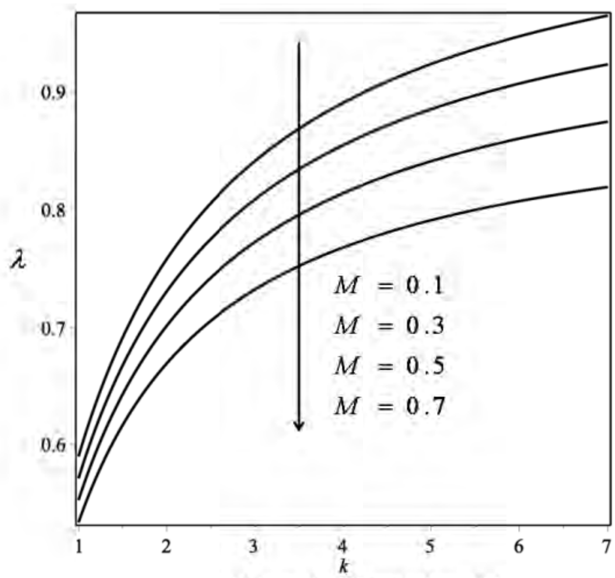

Fig 13: Film thickness for different values of M

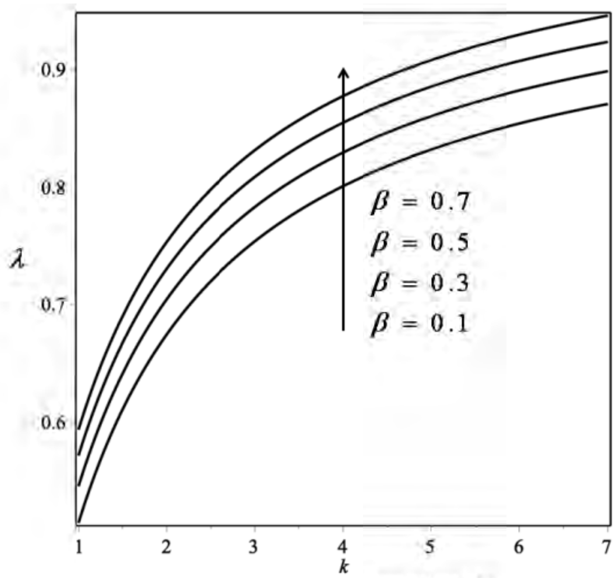

Fig 14: Film thickness for different values of β

The effect of the Hartman number *M* on the load at two different values of κ (κ=2,κ=5) and for two different values of β (β=1,β=2) are shown in Table 1. It is worth to observe that the magnitude of the load distribution increases by increasing the value of Hartmann number, while by increasing the values of the slip parameter the load distribution decreases. The dimensionless coating thickness is shown in the Tables 2 and 3 for different values of β and *M* respectively. It is clear that the film thickness decreases if there is an increase in the values of *M*, whereas the film thickness increases if there is an increase in the values of β. We have also shown it graphically in Figs. 13 and 14, so we can really control and get the desired film thickness by the influence of MHD and the slip parameter.

Table 1 Calculation of load for different values of M.

	$\beta = 1$	$\beta = 1$	$\beta = 2$	$\beta = 2$
M	$\kappa = 5$	$\kappa = 2$	$\kappa = 5$	$\kappa = 2$
0.1	0.641797	0.176973	0.270944	0.001378
0.3	0.685807	0.1953097	0.316042	0.005031
0.5	0.773375	0.2318789	0.405557	0.042525
0.7	0.903765	0.286488	0.538422	0.098430
0.9	1.076164	0.358880	0.7135329	0.17240
1	1.177879	0.40165	0.816627	0.216039
3	5.311508	2.130921	1.960711	3.33527
5	5.475115	13.40968	13.07893	5.310530
7	10.44352	25.4921	25.16537	10.28065
9	41.56278	17.04547	41.23729	16.88300

Table 2 Coating thickness for different values of M

M	λ
0.1	0.772163
0.3	0.733843
0.5	0.697706
0.7	0.661857
0.9	0.625671
1	0.60749
3	0.32187
5	0.19945
7	0.14285
9	0.111109

Table 3 Coating thickness for different values of β

β	λ
0.1	0.625617
0.3	0.656007
0.5	0.6821403
0.7	0.704852
0.9	0.724773
1	0.73384
3	0.844112
5	0.894415
7	0.923211
9	0.941863

6 Conclusion

In this paper the behavior of the Newtonian fluid with MHD is analyzed in the blade coater geometry, in which the flow is occurring between the rigid blade and the moving web. The slip between the fluid and the plate is also taken into account. Lubrication approximation is used to develop governing equation for the MHD Newtonian fluid along with the slip in the narrow channel. A combination of both exact method and numerical method were used for the investigation of the number and the nature of the existing steady solutions.

The outcomes of the present work are summarized as follows:

- The velocity decreases when the Harman number and the slip parameter increase.
- The increasing values of *M*, reduce the coating thickness.
- The increasing values of β, increase the coating thickness.
- The slip parameter β and the magnetic parameter *M* provide us with a mechanism to control velocity, pressure distributions and the coating thickness.
- The presented results are more generalized than that of (Middleman, 1977). Thus, as a special case of the problem, the Middleman results can be recovered from these generalized results.

Our work provides a quick reference to the research community and the scientist who are working on the blade coating processes, and this can be utilized for the production purposes.

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