Article

Quantification of uncertainty in the reliability of migration between habitat patches

Ali Muhammad Ali Rushdi, **Ahmad Kamal Hassan**
Department of Electrical and Computer Engineering, King Abdulaziz University, P. O. Box 80204, Jeddah21589, Saudi Arabia
E-mail: arushdi@kau.edu.sa, ahassan0057@stu.kau.edu.sa

Received 12 April 2016; *Accepted 21 May 2016*; *Published online 1 September 2016*

Abstract
This paper deals with the quantitative assessment of the unreliability of migration between habitat patches in terms of ecological corridor unreliabilities that are known only with uncertainty. The corridor unreliabilities are treated herein as log-normally distributed random variables, and hence the assessment becomes a doubly-stochastic one. The paper utilizes the multi-affine nature of the reliability function in deriving exact formulas for the mean and variance of the system unreliability, as well as accurate formulas for its third and fourth central moments. These formulas involve the nominal values of certain partial derivatives. The multi-affine nature is also beneficial in the development of an alternative method that involves powers rather than derivatives. These two analytical methods of moments agree reasonably with one another and with the results of Monte Carlo simulations. Several test cases are considered for typical ecology problems with corridor unreliabilities that have a significant variation in their error (range) factor. Numerical plots obtained yield plausible observations and useful insights.

Keywords habitat patch; ecological corridor; unreliability; uncertainty; methods of moments; derivatives and powers; Monte Carlo simulation.

Computational Ecology and Software
ISSN 2220-721X
URL: http://www.iaees.org/publications/journals/ces/online-version.asp
RSS: http://www.iaees.org/publications/journals/ces/rss.xml
E-mail: ces@iaees.org
Editor-in-Chief: WenJun Zhang
Publisher: International Academy of Ecology and Environmental Sciences

1 Introduction

An issue of crucial importance in the study of reliability of migration between ecological habitat patches is that any predictions of corridor unreliabilities will certainly involve relatively high uncertainties, which inflicts a significant uncertainty in the overall system unreliability (Rushdi and Hassan, 2015; 2016). This issue is handled in reliability engineering either by dealing with fuzzy rather than crisp probabilities (Tanaka et al., 1983; Weber, 1994) or by considering the pertinent probabilities as random variables (Rushdi, 1985). In this latter approach, the problem of uncertainty analysis is said to be doubly stochastic. In fact, the problem of uncertainty quantification of unreliability via the doubly stochastic approach has a very long history span

(Colombo and Jaarsma, 1980; Colombo, 1980; Ahmed et al., 1981; Jackson et al., 1982; Ahmed et al., 1982; Takaragi et al., 1982; Cox, 1982; Jackson, 1982; Keey and Smith, 1985; Chang et al., 1985; Laviron and Heising, 1985; Masera, 1987; Bobbio, 1990;Berleant, 1993;Soman, and Misra, 1994, 1996; Cho and Yum, 1997; Zhang and Schoenly, 1999; Zhang and Qi, 2004; Rao et al., 2007; Fisher et al., 2008; Cheng, 2009; Karanki et al., 2010; Ulmeanu, 2012; Pedroni and Zio, 2013; Zhang, 2012, 2016; Paul and Vignon-Davillier, 2014; Zhang et al., 2014; Paul and Rioux, 2015). However, we will restrict our attention herein to two methods which we call the Method of Moments via Derivatives (Rushdi, 1985; Rushdi and Kafrawi, 1988; Kafrawi and Rushdi, 1990; Rushdi and Ba-Rukab, 2005a, 2005b; Bamasak and Rushdi, 2016; Rushdi and Al-Qwasmi, 2017), henceforth referred to as MoM_D, and the Method of Moments via Powers (Soman and Misra, 1996), henceforth referred to as MoM_P. Both methods rely heavily on an insight from Rushdi (1983b) that a reliability/unreliability/availability/unavailability function is a multi-affine function, i.e., it is a straight-line relation in each of its arguments. This insight is, in essence, an expression of a basic feature of what is now known as the real or probability transform of a Boolean function (Papaioannou and Barrett, 1975; Kumar and Breuer, 1981; Rushdi, 1987a, 1987b; Heidtmann,1991;Jain, 1996; Rushdi and Ghaleb, 2015).

This paper deals with a model of landscape connectivity (Baranyi et al., 2011; Goodwin and Fahrig, 2002; Jordán et al., 2003; Jordán, 2003; Kindlmann and Burel, 2008; Naeem, 1998; Taylor et al., 1993; With and King, 1997; Vasas, et al., 2009) that includes the following elements

1. **Habitat patches**: Places where the local population of the pertinent species may reproduce and survive for a long term. A habitat patch is called a critical one if its conditions for survival of the species deteriorate significantly. To avoid local extinction in a critical habitat patch, the species is forced to migrate to more suitable habitat patches, called destination habitat patches.
2. **Stepping stone**: Relatively small places that help the migration of the local population of the pertinent species, but are not suitable for its long-term survival.
3. **Ecological corridor**: Physical areas which connect patches (habitats and stepping stones) and make migration possible for a given species between these patches. However, corridors are not expected to support long-term survival for the species.

The model is based on a few assumptions extracted from Jordán (2000) and Rushdi and Hassan (2015), namely

1. The analysis concerns one particular species, called the pertinent or concerned species. The analysis does not take into account any characteristic of the species.
2. The pertinent species is in danger of local extinction in a critical habitat patch. It escapes such extinction by migrating to a new habitat patch (one out of a few destination habitat patches) through imperfect corridors and perfect stepping stones.
3. Each of the corridors is in one of two states, either good (permeable)or failed (deleted or destroyed).
4. The migration system is also in one of two states, either successful or unsuccessful.
5. Destination habitat patches and stepping stones are not susceptible to failure.
6. Corridor states are statistically independent.

These assumptions are augmented herein by the “uncertainty” or “doubly stochastic” assumptions:

7. Corridor unreliabilities are random variables characterized by their probabilistic distributions or moments, and hence the probability of successful migration is also a random variable.
8. The unreliabilities of different corridors are statistically independent.

We added these last two assumptions, since we noted that uncertainty issues are frequently addressed and play an essential role in ecological studies and modelling, albeit in subareas other than that of unreliability (Bradshaw and Borchers, 2000; Williams, 2001; Regan, et al., 2002; Arbia et al., 2003; Heuvelink, 2003; Qin, et al., 2009; Lele and Taper, 2011; Chen, et al., 2013). The probability distribution to be employed herein will be the lognormal distribution. Though the lognormal distribution is not appropriate for certain ecological applications such as mean estimation and testing (Schmoyeri et al., 1996) or hypothesis testing (Williamson and Gaston, 2005), it is mainly the distribution of choice in quantifying uncertainty of reliability (Kafka and Polke, 1986; Masera, 1987; Rushdi and Kafrawy, 1988; Matsuoka and Kobayashi, 1997; Allella, 2001; Rushdi and Ba-Rukab, 2005b; Halpren et al., 2006).

The output of the current model is to quantify the probability of successful migration. This quantification can be a full one in terms of the entire probability distribution, but we will settle for a simple, albeit appropriate,quantification in terms of lower-order probabilistic moments. These moments include the first moment (the mean μ_1), the second moment (the variance μ_2), the third and fourth central moments μ_3 andμ_4, and are typically expressed in terms of the *dimensionless* coefficients of variation ($\rho = \mu_2^{1/2} / \mu_1$), skewness ($\gamma_1 = \mu_3/\mu_2^{3/2}$) and excess (kurtosis) ($\gamma_2 = \mu_4/\mu_2^2 - 3$) (Forbes et al., 2011). Table 1 lists the notation used throughout this paper.

Table 1 List of notation.

N	=	number of ecological corridors, $n \geq 0$.
T	=	Number of additive terms in the expression of$U(\boldsymbol{p})$
X_i	=	success of corridor i = indicator that the concerned species successfully migrates through corridor$i$= a switching random variable that takes only one of the two discrete values 0 and 1; ($X_i = 1$ iff corridor i is permeable, while $X_i = 0$ iff corridor i is failed).
$\bar{X}_i$	=	failure or deletion of corridor i = indicator variable for unsuccessful migration of the pertinent species through i, where $\bar{X}_i = 0$ iffcorridor i is good, while $\bar{X}_i = 1$ iff corridor i is deleted/destroyed. The success $X_i$and the failure $\bar{X}_i$ are complementary variables.
$\boldsymbol{X}$	=	a vector of n elements representing the successful species migration through corridor i, $\boldsymbol{X} = [X_1 X_2 \ldots X_n]^T$.
$S(\boldsymbol{X})$	=	indicator variable for the successful operation of the system (successful migration of the pertinent species), called system success.
$\bar{S}(\boldsymbol{X})$	=	indicator variable for system failure (unsuccessful migration of the pertinent species)
$Pr[...]$	=	probability of the event [...].
$E[...]$	=	expectation of the random variable [...].
$q_i,\ p_i$	=	reliability and unreliability of corridor i; Both q_i and p_i are real values in the closed real interval [0.0,1.0]. Here we follow the variable definition of Jordán (2000) and Rushdi and Hassan (2015; 2016) which is the opposite of the common practice in the reliability community.
q_i	=	$Pr[X_i = 1] = E[X_i] = 1.0 - p_i$.
$\boldsymbol{q}$	=	a vector of n elements representing the corridor reliabilities, $\boldsymbol{q} = [q_1\ q_2 \ldots q_{m-1}\ q_m\ q_{m+1} \ldots q_n]^T$.
$\boldsymbol{p}$	=	a vector of n elements representing the corridor unreliabilities = **1.0** – $\boldsymbol{q}$, where **1.0** is an n-tuple of real ones.
$\boldsymbol{q}\|q_m$	=	a vector of (n-1) elements obtained by omitting the mth element of vector $\boldsymbol{q}$, $\boldsymbol{q}\|q_m = [q_1\ q_2 \ldots q_{m-1}\ q_{m+1} \ldots q_n]^T$.
$\boldsymbol{q}\|j_m$	=	a vector of n elements obtained by setting the$m$th element of $\boldsymbol{q}$ to j, where j is either 0 or 1, $\boldsymbol{q}\|j_m = [q_1\ q_2 \ldots q_{m-1}\ j\ q_{m+1} \ldots q_n]^T$.
$R(\boldsymbol{q}),U(\boldsymbol{q})$	=	reliability and unreliability of the system. Both $R(\boldsymbol{q})$ and $U(\boldsymbol{q})$ are real values in the closed real

		interval [0.0,1.0].
$R(\boldsymbol{q})$	=	$Pr[S(\boldsymbol{X}) = 1] = E[S(\boldsymbol{X})]$.
$U(\boldsymbol{q})$	=	$Pr[\bar{S}(\boldsymbol{X}) = 1] = E[\bar{S}(\boldsymbol{X})] = 1.0 - R(\boldsymbol{q})$.
$\mathbf{v}_1$	=	mean value of $\mathbf{p}$: $\mathbf{v}_1 = [\nu_{11}, \nu_{21}, \ldots, \nu_{n1}]^T$
ν_{ij}	=	central moment j of p_i: $\nu_{ij} = E[(p_i - \nu_{i1})^j]$, j = 2, 3, 4
μ_1	=	mean value of U: $\mu_1 = E[U]$
μ_j	=	central moment j of U: $\mu_j = E[(U - \mu_1)^j]$, j = 2, 3, 4
σ	=	standard deviation of U: $\sigma = \mu_2^{1/2}$
ρ	=	coefficient of variation of U: $\rho = \mu_2^{1/2}/\mu_1 = \sigma/\mu_1$
γ_1	=	coefficient of skewness of U: $\gamma_1 = \mu_3/\mu_2^{3/2}$
γ_2	=	coefficient of excess (kurtosis) of U: $\gamma_2 = \mu_4/\mu_2^2 - 3$
M	=	median (50th percentile) of a log-normally distributed variable, replaces the nominal value of a deterministic variable.
F	=	error factor (range factor) of a log-normally distributed variable: F = 95th percentile/ 50th percentile = 50th percentile/ 5th percentile
λ, ξ	=	λ and ξ are mean and standard deviation of the natural logarithm of a log-normally distributed variable respectively. $\lambda = E[\ln(X)] = \ln(m)$; $\xi^2 = VAR[\ln(X)]$, $\xi = \ln(F)/1.645$.
$f_X(x)$	=	the probability density function (pdf) of the lognormal distribution $f_X(x) = \begin{cases} \exp\left(-\left(\frac{\ln(x) - \lambda}{\xi}\right)^2/2\right)/(\sqrt{2\pi}\xi x) & for\ x \geq 0 \\ 0 & for\ x < 0 \end{cases}$
$\nu_1, \nu_2, \nu_3, \nu_4$	=	mean, variance, third central moment and fourth central moment of the log-normally distributed variable: $\nu_1 = m * \exp(\xi^2/2)$; $\nu_2 = \nu_1^2 * (\exp(\xi^2) - 1)$; $\nu_3 = \nu_2^{3/2} * (\exp(\xi^2) - 1)^{1/2} (\exp(\xi^2) + 2)$; $\nu_4 = \nu_2^2 * (\exp(4\xi^2) + 2\exp(3\xi^2) + 3\exp(2\xi^2) - 3)$; For a deterministic variable F = 1, ξ = 0, ν_1 = m, and ν_2, ν_3, ν_4 = 0.

The organization of the remainder of this paper is as follows. Section 2 outlines methods for analyzing the unreliability of migration between habitat patches. Section 3 presents two analytical methods of moments for uncertainty analysis of unreliability, namely the Method of Moments via Derivatives (MoM_D), and the Method of Moments via Powers (MoM_P). Several improvements and enhancements are added to the exposition of these methods. Section 4 presents three demonstrative examples that serve as test cases for typical ecology problems. The two analytical methods of moments agree exactly with one-another and agree well with Monte Carlo simulation. Section 5 concludes the paper.

2 Analysis of Ecological Reliability

This section outlines methods for the reliability analysis of the failure of migration of a specific species from a critical habitat patch to certain destination habitat patches via imperfect corridors and perfect stepping stones (Rushdi and Hassan, 2015). The analysis starts by producing a switching or Boolean function for the indicator variable of migration failure in terms of the indicator variables of corridor failures. It is now necessary to move from Boolean domain to the probability domain so as to obtain the probability of unsuccessful migration as a function of corridor unreliabilities. Many algorithms are used for converting the switching (Boolean) expression for the indicator variable of migration failure into a Probability-Ready Expression (PRE), i.e., into an expression that is directly convertible, on a one-to-one basis, to a probability expression. It should be noted that, in a PRE, all ORed terms/(products) are disjoint, and all ANDed terms (sums) are statistically independent. The conversion from a PRE to a probability expression is trivially achieved by replacing Boolean variables by their expectations, AND operations by multiplications and OR operations by additions (Bennetts, 1975; 1982; Rushdi and Al-Khateeb, 1983; Rushdi and Abdulghani, 1993; Rushdi and Alturki, 2015). The algorithms for converting a general switching expression into a PRE can be classified as

a) Disjointness algorithms (Abraham, 1979; Dotson and Gobien, 1979; Rushdi, 1983a).
b) Algorithms based primarily on statistical independence (Rushdi and Goda, 1985; Rushdi and Abdulghani, 1993).
c) Boole-Shannon expansion or factoring algorithm (Rushdi, 1983a; Rushdi and Goda, 1985).

3 Uncertainty Analysis of Unreliability

In system reliability analysis, models such as reliability block diagrams, fault tree analysis, Markov chains and stochastic petri nets are built to predict the reliability of the system (Trivedi, 2002). The parameters in these models are usually obtained from the field data, data from systems with similar functionality, and even by expert guessing, and hence are bound to suffer from considerable uncertainty (Yin et al., 2001). The uncertainty problem pertaining to system unreliability has an analytic doubly-stochastic treatment via the method of moments. This method of moments utilizes the multi-affine nature (Rushdi, 1983b) of system unreliability as a function of the corridor unreliabilities. The method of moments has been extensively verified through comparison with results obtained via other methods including Monte Carlo simulations.

A. Method of Moments via Derivatives (MoM_D)

When the corridor unreliabilities (elements of **p**) are statistically independent, the basic results of the method of moments can summarized by the following closed- form formulas. The mean or expected value of system unreliability U is

$$\mu_1 = U(\nu_1) = U(\boldsymbol{p})]_{\boldsymbol{p}=\boldsymbol{\nu_1}}, \tag{1}$$

which is simply obtained by substituting the mean values ν_1 of corridor unreliabilities in the symbolic expression of system unreliability. The variance (measure of uncertainty) and the third and fourth central moments of corridor unreliability are given exactly by the finite series in un-truncated form as

$$\mu_2 = \sum_{1\le i\le n} C_i^2 \nu_{i2} + \sum\sum_{1\le i<j\le n} C_{ij}^2 \nu_{i2}\nu_{j2} + \sum\sum\sum_{1\le i<j<k\le n} C_{ijk}^2 \nu_{i2}\nu_{j2}\nu_{k2} + \cdots + C_{12\ldots n}^2 \nu_{12}\nu_{22}\ldots\nu_{n2}, \tag{2}$$

$$\mu_3 = \sum_{1\le i\le n} C_i^3 \nu_{i3} + \sum\sum_{1\le i<j\le n}[6C_iC_jC_{ij}\nu_{i2}\nu_{j2} + 3C_iC_{ij}^2\nu_{i3}\nu_{j2} + 3C_jC_{ij}^2\nu_{i2}\nu_{j3} + C_{ij}^3\nu_{i3}\nu_{j3}] + \sum\sum\sum_{1\le i<j<k\le n}\left[6\left\{C_{ij}C_{ik}C_{jk} + \left(C_iC_{jk} + C_jC_{ik} + C_kC_{ij}\right)C_{ijk}\right\}\nu_{i2}\nu_{j2}\nu_{k2} + \cdots\right] + \cdots C_{12\ldots n}^3\nu_{13}\nu_{23}\ldots\nu_{n3}, \tag{3}$$

$$\mu_4 = \sum_{1\le i\le n} C_i^4\nu_{i4} + \sum\sum_{1\le i<j\le n}[6C_i^2C_j^2\nu_{i2}\nu_{j2} + 12\, C_i^2C_jC_{ij}\nu_{i3}\nu_{j2} + 12\, C_iC_j^2C_{ij}\nu_{i2}\nu_{j3} + 6C_i^2C_{ij}^2\nu_{i4}\nu_{j2} + 6C_j^2C_{ij}^2\nu_{i2}\nu_{j4} + 12C_iC_jC_{ij}^2\nu_{i3}\nu_{j3} + \cdots] +$$

$$\sum\sum\sum_{1\le i<j<k\le n}\left[\left\{6\left(C_i^2C_{jk}^2+C_j^2C_{ik}^2+C_k^2C_{ij}^2\right)+\ 24C_iC_jC_kC_{ijk}\right\}\nu_{i2}\nu_{j2}\nu_{k2}+\cdots\right]+\cdots+C_{12\dots n}^4\nu_{14}\nu_{24}\dots\nu_{n4}. \quad (4)$$

The coefficients that appear in (1-4) are those in the finite Taylor expansion of the multi-affine function $U(\boldsymbol{p})$, namely

$$U(\boldsymbol{p})=U(\boldsymbol{\nu_1})+\ \sum_{1\le i\le n}C_i(p_i-\nu_{i1})+\ \sum\sum_{1\le i<j\le n}C_{ij}(p_i-\nu_{i1})(p_j-\nu_{j1})+\ \sum\sum\sum_{1\le i<j<k\le n}C_{ijk}(p_i-\nu_{i1})\left(p_j-\nu_{j1}\right)(p_k-\nu_{k1})+\cdots+\ C_{12\dots n}\prod_{1\le i\le n}(p_i-\nu_{i1}), \quad (5)$$

and are expressed by the partial derivatives

$$\begin{aligned} C_i &= (\partial U/\partial p_i)_{\boldsymbol{p}=\boldsymbol{\nu}_1} \\ C_{ij} &= (\partial^2 U/\partial p_i\partial p_j)_{\boldsymbol{p}=\boldsymbol{\nu}_1} \\ C_{ijk} &= (\partial^3 U/\partial p_i\partial p_j\partial p_k)_{\boldsymbol{p}=\boldsymbol{\nu}_1} \\ &\cdots\cdots \\ C_{12\dots n} &= (\partial^n U/\partial p_1\partial p_2\ \dots\ \partial p_n)_{\boldsymbol{p}=\boldsymbol{\nu}_1} \end{aligned} \quad (6)$$

Thanks to the multi-affine nature of $U(\boldsymbol{p})$, these partial derivatives reduce to differences of unreliabilities, namely

$$C_i = U(\boldsymbol{\nu_1}|1_i)-\ U(\boldsymbol{\nu_1}|0_i)$$

$$C_{ij} = U(\boldsymbol{\nu_1}|1_i,1_j)-\ U(\boldsymbol{\nu_1}|0_i,1_j)-U(\boldsymbol{\nu_1}|1_i,0_j)+U(\boldsymbol{\nu_1}|0_i,0_j)$$

$$C_{ijk} = U\left(\boldsymbol{\nu_1}\middle|1_i,1_j,1_k\right)-U\left(\boldsymbol{\nu_1}\middle|0_i,1_j,1_k\right)-U\left(\boldsymbol{\nu_1}\middle|1_i,0_j,1_k\right)+U\left(\boldsymbol{\nu_1}\middle|0_i,0_j,1_k\right)-\ U\left(\boldsymbol{\nu_1}\middle|1_i,1_j,0_k\right)+U(\boldsymbol{\nu_1}|0_i,1_j,0_k)+U(\boldsymbol{\nu_1}|1_i,0_j,0_k)-U(\boldsymbol{\nu_1}|0_i,0_j,0_k) \quad (7)$$

Since the above coefficients are differences of unreliabilities that take real values in [0.0, 1.0], they are bounded as follows (for all values of their subscripts)

$$\begin{gathered} -1.0\le C_i\le 1.0 \\ -2.0\le C_{ij}\le 2.0 \\ -4.0\le C_{ijk}\le 4.0 \\ -8.0\le C_{ijk}\le 8.0 \\ \cdots\cdots \\ -2^{n-1}\le C_{12\dots n}\le 2^{n-1} \end{gathered} \quad (8)$$

The above results mean that for all values of the subscripts $C_i^2\le 1$, $C_{ij}^2\le 4$, $C_{ijk}^2\le 16$,$C_{ijkl}^2\le 64$,$C_{12\dots n}^2\le 2^{2n-2}$. Therefore, an upper bound on the variance μ_2 is

$$\mu_2\le \sum_{1\le i\le n}\nu_{i2}+\ 4\sum\sum_{1\le i<j\le n}\nu_{i2}\nu_{j2}+16\sum\sum\sum_{1\le i<j<k\le n}\nu_{i2}\nu_{j2}\nu_{k2}+64\sum\sum\sum\sum_{1\le i<j<k<l\le n}\nu_{i2}\nu_{j2}\nu_{k2}\nu_{l2}+\cdots+\ 2^{2n-2}\prod_{1\le i\le n}\nu_{i2}. \quad (9)$$

If all corridors have the same value of their unreliability variances ($\nu_{i2}=\ \nu_2$ for $i=1,2,\dots,n$), then the above upper bound on μ_2 reduces to

$$\mu_2 \leq \binom{n}{1}\nu_2 + 4\binom{n}{2}\nu_2^2 + 16\binom{n}{3}\nu_2^3 + 64\binom{n}{4}\nu_2^4 + \cdots + 2^{2n-2}\binom{n}{n}\nu_2^n, \tag{10a}$$

$$\mu_2 \leq n\nu_2 + 2n(n-1)\nu_2^2 + \frac{8}{3}n(n-1)(n-2)\nu_2^3 + \frac{8}{3}n(n-1)(n-2)(n-3)\nu_2^4 + \cdots + 2^{2n-2}\nu_2^n. \tag{10b}$$

For n = 5

$$\mu_2 \leq 5\nu_2 + 40\nu_2^2 + 160\nu_2^3 + 320\nu_2^4 + 256\nu_2^5. \tag{11}$$

B. **METHOD OF MOMENTS VIA POWERS (MOM_P)**

Using the mean value of $U(\boldsymbol{p})$

$$\mu_1 = E[U(\boldsymbol{p})], \tag{12}$$

Soman and Misra (1996), and Ulmeanu (2012) utilized the multi-affine nature of $U(\boldsymbol{p})$ in a direct way for expressing the variance of system unreliability

$$\mu_2 = E[(U(\boldsymbol{p}) - \mu_1)^2] = E[U^2(\boldsymbol{p})] - \mu_1^2, \tag{13}$$

This method can be generalized to the third and fourth moment as follows

$$\mu_3 = E[(U(\boldsymbol{p}) - \mu_1)^3] = E[U^3(\boldsymbol{p})] - 3\mu_1\mu_2 - \mu_1^3, \tag{14}$$

$$\mu_4 = E[(U(\boldsymbol{p}) - \mu_1)^4] = E[U^4(\boldsymbol{p})] - 4\mu_1\mu_3 - 6\mu_1^2\mu_2 - \mu_1^4, \tag{15}$$

The above formulas require the raising of the symbolic expression of $U(\boldsymbol{p})$ to powers 2, 3, and 4 via the well-known identities

$$U^2(\boldsymbol{p}) = (\sum_{i=1}^{m} T_i)^2 = \sum_{i=1}^{m} T_i^2 + 2\sum\sum_{1\leq i<j\leq m} T_iT_j, \tag{16}$$

$$U^3(\boldsymbol{p}) = (\sum_{i=1}^{m} T_i)^3 = \sum_{i=1}^{m} T_i^3 + 3\sum\sum_{1\leq i\neq j\leq m} {T_i}^2T_j + 6\sum\sum\sum_{1\leq i<j<k\leq m} T_iT_jT_k, \tag{17}$$

$$U^4(\boldsymbol{p}) = (\sum_{i=1}^{m} T_i)^4 = \sum_{i=1}^{m} T_i^4 + 4\sum\sum_{1\leq i\neq j\leq m} {T_i}^3T_j + 6\sum\sum_{1\leq i<j\leq m} {T_i}^2{T_j}^2 + 12\sum\sum\sum_{1\leq i<j<k\leq m} {T_i}^2T_jT_k + 24\sum\sum\sum\sum_{1\leq i<j<k<l\leq m} T_iT_jT_kT_l. \tag{18}$$

Here, m is the number of additive terms in the expression of $U(\boldsymbol{p})$, T_i stands for one of the additive terms in the expression of $U(\boldsymbol{p})$, which is a product of certain unreliabilities p_r and reliabilities q_r. The evaluation of $E[U^2(\boldsymbol{p})]$ depends heavily on our assumption that the corridor unreliabilities are statistically independent. However, one should note that the unreliability p_r and reliability q_r of the *same* corridor are not statistically independent since $q_r = 1 - p_r$. Therefore, we produce an all-p formula for $U^2(\boldsymbol{p})$ by substituting $(1 - p_r)$ for each q_r. Now, the evaluation of $E[U^2(\boldsymbol{p})]$ in (12), $E[U^3(\boldsymbol{p})]$ in (13), and $E[U^4(\boldsymbol{p})]$ in (14) is straightforward, thanks to the statistical independence of corridor unreliabilities, and the fact that

$$E[p_r] = \nu_{r1}, \tag{19}$$

$$E[p_r^2] = \nu_{r2} + \nu_{r1}^2, \quad (20)$$

$$E[p_r^3] = \nu_{r3} + 3\nu_{r1}\nu_{r2} + \nu_{r1}^3, \quad (21)$$

$$E[p_r^4] = \nu_{r4} + 4\nu_{r1}\nu_{r3} + 6\nu_{r1}^2\nu_{r2} + \nu_{r1}^4. \quad (22)$$

Fig. 1 presents the scheme of going from the central moments of **p** to those of U via the moments about the mean of **p** and those of U. The figure also shows the numbers of the equations involved in each step.

4 Demonstrative Examples

This section presents three demonstrative examples involving two small networks (Fig. 2). Both networks initially appeared in Jordán (2000), and were later analyzed for non-identical corridors by Rushdi and Hassan (2015). Note that the two networks share the same construction and corridors 1-4 and 9, but network (b) is augmented by corridors 5-8.

EXAMPLE 1

The unreliability $U(\boldsymbol{p})$ of the network in Fig. 2 (a) is given in Rushdi and Hassan (2015) as.

$$U(\boldsymbol{p}) = p_3[p_1 + q_1 p_2(p_4 + q_4 p_9)]. \quad (23)$$

We assume that the corridor unreliabilities $p_i, i = 1 - 4$ and 9, are identically and log-normally distributed with F= 3 and m = 1.00E-02 (and hence with **λ** = 4.6051, **ξ** = 0.6678, ν1 = 0.0125, ν2 = 8.78E-05, ν3 = 2.61E-06, ν4 = 1.67E-07). We quantified the moments of the system unreliability as a random variable via (a) Method of Moments via Derivatives (MoM_D), (b) Method of Moments via Powers (MoM_P), and (c) Monte Carlo Simulations (10^6 samples). The results of the first four moments μ1, μ2, μ3, μ4 are shown for each of the aforementioned methods in Table 2, together with corresponding values of dimensionless coefficients ρ,γ_1, and γ_2. Table 2 shows that the results obtained for μ1, μ2 and ρ are in exact agreement (apart from minor rounding differences) for the two methods of moments, and these results are almost replicated by Monte Carlo simulation. Results for μ3, μ4,γ_1, and γ_2 are somewhat similar among the three methods. The results obtained by the Method of Moments via Powers seemmore reliable, while those obtained by the Method of Moments via Derivatives suffer from some truncations in (3) and (4), and while those obtained by the Monte Carlo simulation are limited by the quality and sample size of the simulation.

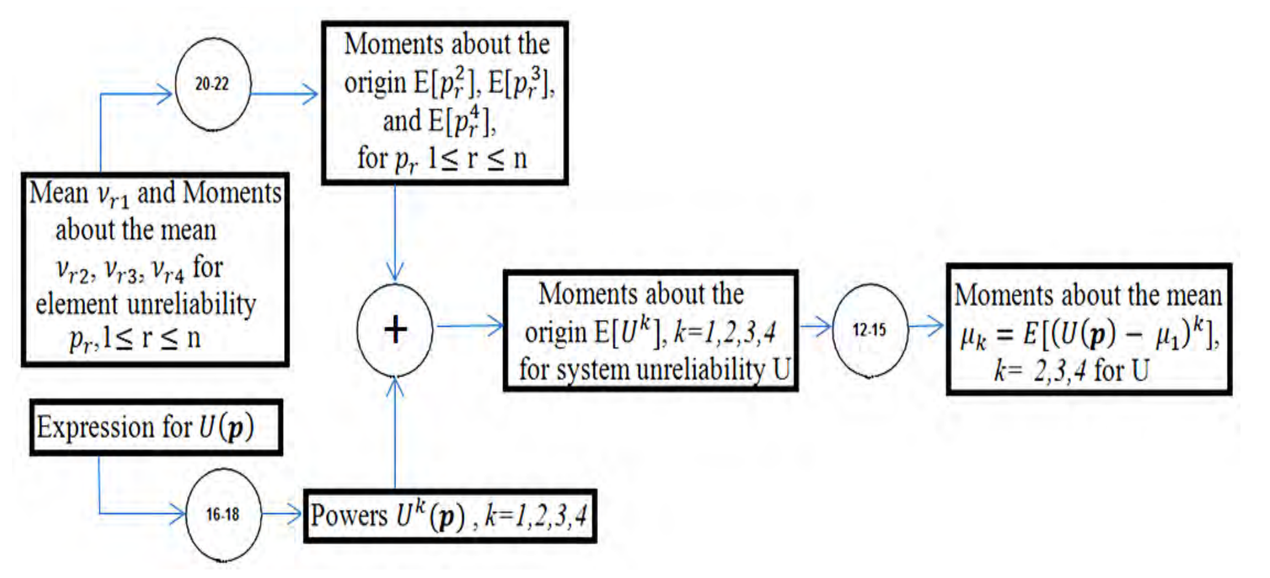

Fig. 1 Scheme of going from central moments of ***p*** to central moments of U.

Fig. 3 is a probability plot for the (supposedly) log-normally distributed pseudo-random number generated by our simulation. The figure is a log-log plot of the probability of the pseudo-random number versus the actual value of this number, and shows a minimal scattering of blue points around the exact black-dash zero-thickness straight line corresponding to a *true* lognormal distribution. The plot in Fig. 3 clearly indicates that the quality of the pseudo-random samples generated for use in the simulation is good, albeit of limited precision. Fig. 4 is a histogram for the 10^6 log-normally generated samples. This histogram gives an excellent approximation of the lognormal probability density function (pdf). This pdf is indistinguishable from zero at a value as low as 10^{-3} which is ≪1. In fact, out of 10^6 samples, not a single sample exceeded (or even approached) the value of 1.0 (the maximum number generated was 0.012246). This means that the tail of the log-normal distribution over the support (1.0, ∞) is definitely negligible.

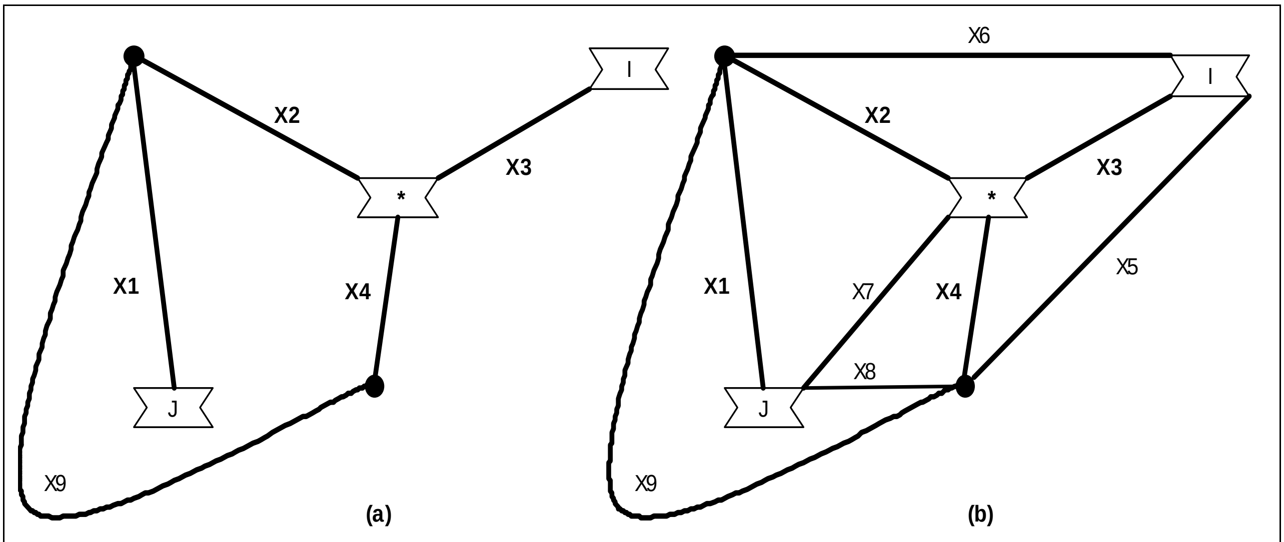

Fig. 2 Two small networks each representing migration from a critical habitat patch (*) to two destination patches I and J via imperfect corridors labelled by their successes *Xi, i=1,2,3,4,5,6,7,8,9*.

We pursue this example further to study the effect of varying the median m or the range factor F for each of the identical corridors on the second, third and fourth central moments μ2, μ3, μ4 obtained by the two methods of moments MoM_D and MoM_P. Fig. 5(a) plots the variance μ2 obtained via MoM_D and MoM_P versus the range or error factor F, and Fig. 5(b) reports similar plots albeit versus the median m. The two figures show indistinguishable plots indicating exact agreement between MoM_D and MoM_P when calculating μ2. Figs 5(c)-(f) report similar plots for μ3 and μ4 obtained by MoM_D and MoM_P versus F and m. The agreement between MoM_D and MoM_P is still good for μ3 but decreases for μ4, especially for larger values of F.

Table 2 Quantitative Assessment of system unreliabilities as a random variable by three methods.

	μ1	**μ2**	**μ3**	**μ4**	ρ	γ_1	γ_2
Method of Moments via Derivative (MoM_D)	1.6004e-04	3.5831e-08	5.5760e-11	6.2155e-14	1.1828	8.2211	45.4120
Method of Moments via Powers (MoM_P)	1.6004e-04	3.5838e-08	4.2156e-11	1.3425e-13	1.1829	6.2137	101.5308
Monte Carlo Simulation (10^6 samples)	1.6002e-04	3.5771e-08	3.5247e-11	9.5546e-14	1.1819	5.2098	71.6692

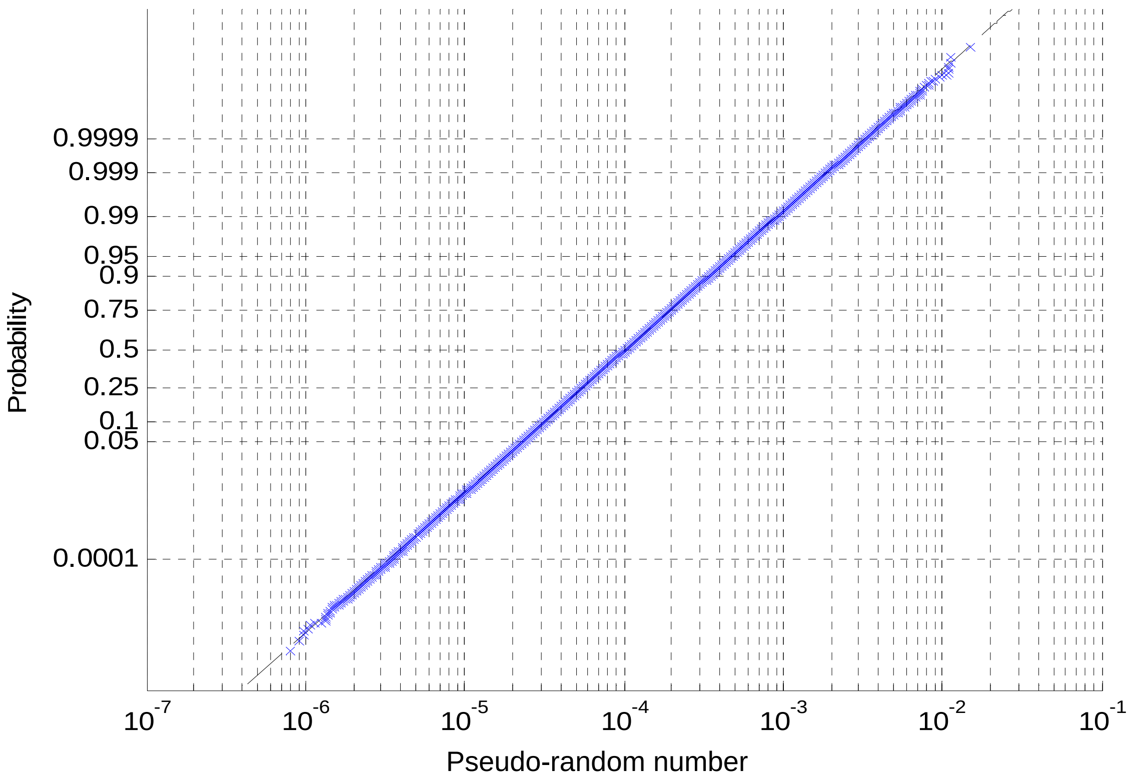

Fig. 3 Probability plot for the (supposedly) lognormal distribution used in the simulation.

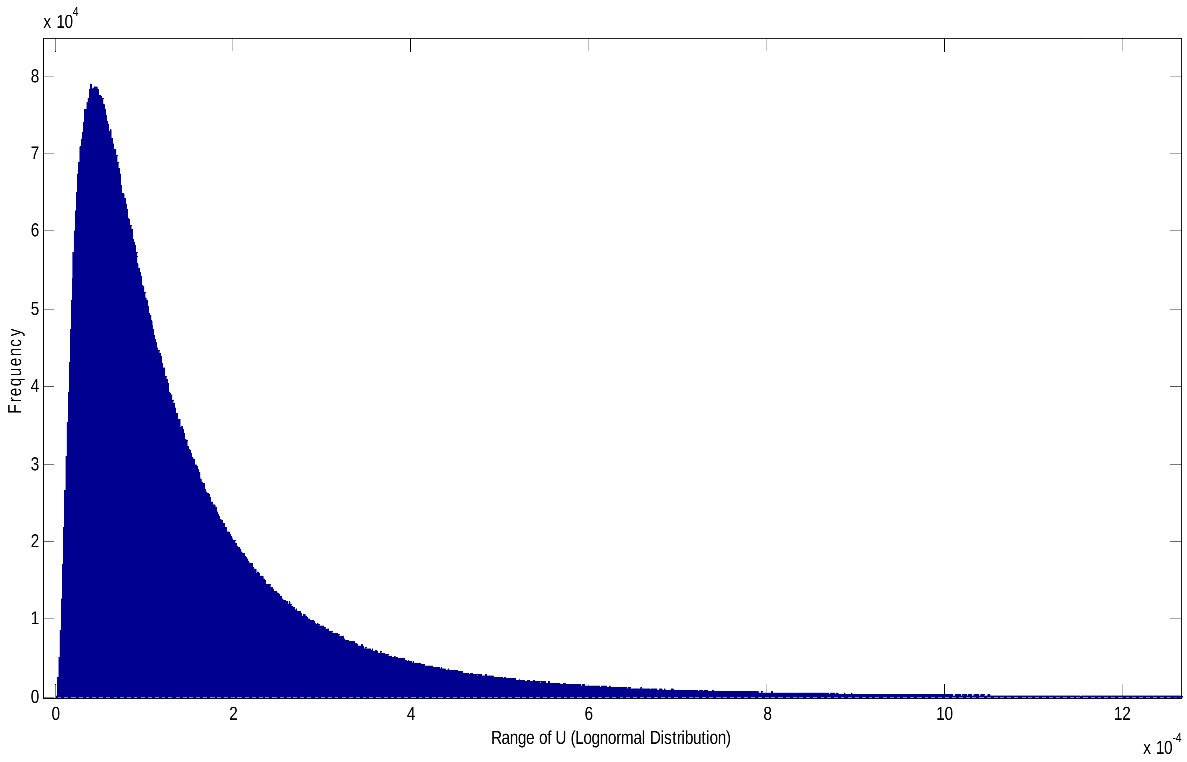

Fig. 4 A histogram (of the 10^6 generated samples) representing the log-normal pdf in Example 1.

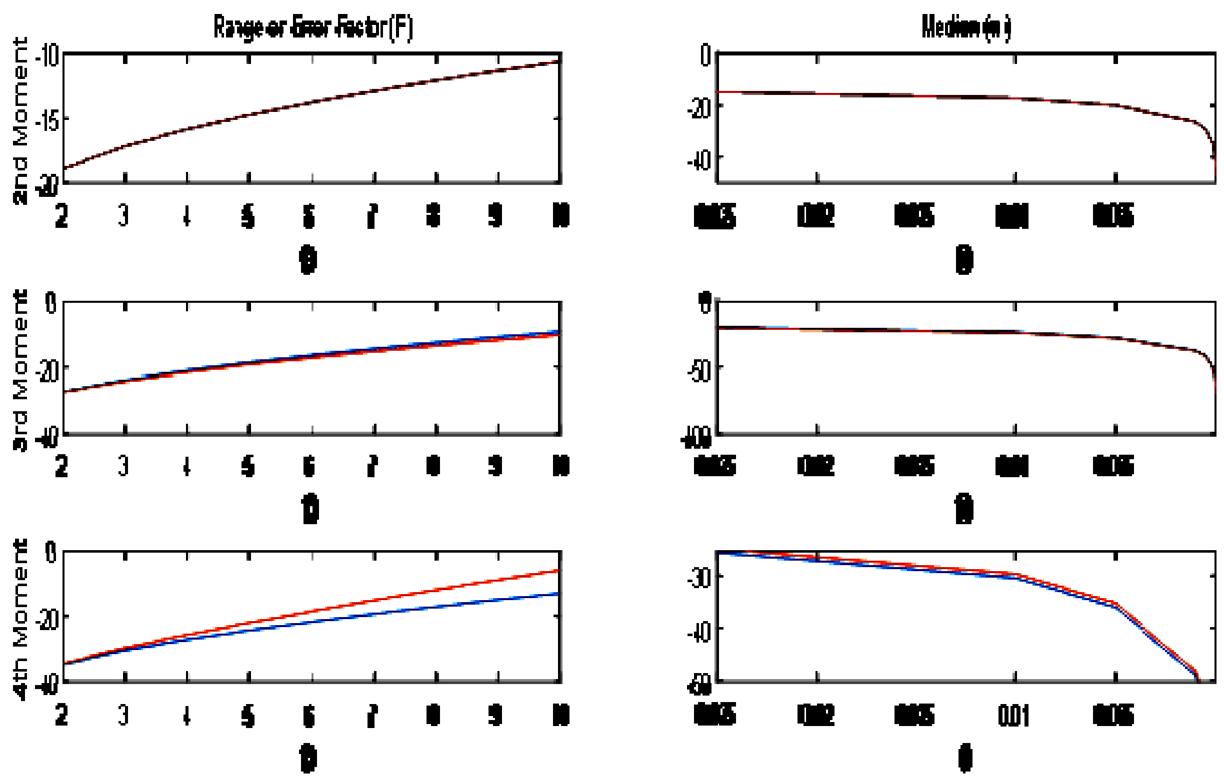

Fig. 5 Plots of the variance μ2, the third central moment μ3, and the fourth central moment μ4 (on logarithmic scales) versus either the range factor F or the median m (on linear scales), as obtained by MoM_D (red-line) and MoM_P (blue-line).

EXAMPLE 2

We reconsider the unreliability (23) in Example1 with *non-identical*corridor unreliabilities, still log-normally distributed, but with medians m_i and Error Factors F_i as shown in Table 3. For convenience, the first four moments of each $p_i$are also shown in Table 3.

Table 3 Median m_i and Error factors F_i for corridor unreliabilities in Example 2.

Corridor No. i	Median m_i	Error Factor F_i	ν_{i1}	ν_{i2}	ν_{i3}	ν_{i4}
1	2.00E-02	2	0.0219	9.2814e-05	2.2590e-06	1.1514e-07
2	1.00E-02	3	0.0125	8.7803e-05	2.6090e-06	1.6729e-07
3	8.00E-03	4	0.0114	1.3468e-04	6.3599e-06	6.9909e-07
4	6.00E-03	5	0.0097	1.5044e-04	9.7775e-06	1.6808e-06
9	4.00E-03	6	0.0072	1.1923e-04	9.0358e-06	2.1520e-06

Table 4 reports computational results similar to those in Table 2 obtained by MoM_D and MoM_P. Again, the results of the two MoM methods agree exactly for μ2 and approximately for μ3 and μ4.

Table 4 Quantitative Assessment of system unreliabilities as a random variable for non-identical corridors by three methods.

	μ1	**μ2**	**μ3**	**μ4**	$\boldsymbol{\rho}$	$\boldsymbol{\gamma_1}$	$\boldsymbol{\gamma_2}$
Method of Moments via Derivative (MoM_D)	2.5175e-04	9.0154e-08	1.5439e-10	2.2218e-13	1.1927	5.7034	24.3364
Method of Moments via Powers (MoM_P)	2.5175e-04	9.0154e-08	1.5440e-10	7.2044e-13	1.1927	5.7040	85.6399
Monte Carlo Simulation (10^6 samples)	2.5172e-04	9.0118e-08	1.4033e-10	5.7610e-13	1.1926	5.1873	67.9381

EXAMPLE 3

The unreliability $U(\boldsymbol{p})$ of the network in Fig. 2(b) is given by Rushdi and Hassan (2015)

$$U(\boldsymbol{p}) = \ p_3p_7[p_2p_4 + (p_2q_4p_5p_8 + q_2p_4p_6p_1)p_9 + (p_2q_4q_9 + q_2p_4q_9 + q_2q_4)p_5p_6p_8p_1]. \quad (24)$$

For simplicity, we assume that there is uncertainty in a single corridor unreliability, say that of corridor number 1. We let p_1 be lognormally distributed with F_1 = 3 and m_i = 1.00E-02 (similar to corridor unreliabilities in Example 1), and we let each other p_i $(2 \le i \le 9)$ be deterministic (F=1) and equal to m_1= 1.00E-02. Equations (2)-(4) now reduce to

$$\mu_2 = C_1^2\nu_{12} \quad . \quad (25)$$

$$\mu_3 = C_1^3\nu_{13} \quad . \quad (26)$$

$$\mu_4 = C_1^4\nu_{14} \quad . \quad (27)$$

This means MoM_D simplifies considerably, while MoM_P is still involved and hence not used herein. The numerical results are shown in Table 5. It demonstrates an agreement between MoM_D and Monte Carlo Simulation, that is best for μ1 followed by μ2.

Table 5 Quantitative Assessment of system unreliabilities as a random variable for uncertainties pertaining to p_1 only.

	μ1	μ2	μ3	μ4	ρ	γ_1	γ_2
Method of Moments via Derivative	2.4412e-08	3.1462e-23	5.5963e-34	2.1480e-44	2.2977e-04	3.1711	18.6992
Monte Carlo Simulations (10^6 samples)	2.4412e-08	3.1507e-23	4.7378e-34	2.0841e-44	2.2993e-04	2.6790	17.9947

4 Conclusion

This paper dealt with the quantification of uncertainty when studying the probability that a specific species migrates successfully from a critical habitat patch to destination habitat patches via imperfect ecological corridors whose unreliabilities are random variables. The uncertainty issue is studied herein in a general setting encompassing the variance μ2 (the typical measure of uncertainty), the third central moment (a measure of skewness or lack of symmetry), and the fourth central moment (a measure of peakedness). The paper utilized and enhanced two methods of moments, namely, the Method of Moments via Derivatives (MoM_D) and the Method of Moments via Powers (MoM_P). The two methods are similar and originate from a utilization of a multi-affine nature of the unreliability function. MoM_D is more effective when only a few corridor unreliabilities have uncertainties. The two analytical methods of moments agree reasonably with one another and with the results of Monte Carlo Simulation.

References

Ahmed S, Clark,RE, Metcalf DR. 1982. A method for propagating uncertainty in probabilistic risk assessment. Nuclear Technology, 59(2): 238-245

Ahmed S, Metcalf DR, Pegram JW. 1981. Uncertainty propagation in probabilistic risk assessment: a comparative study. Nuclear Engineering and Design, 68(1): 1-3

Allella F, Chiodo E, Lauria D, Pagano M. 2001. Negative Log-Gamma distribution for data uncertainty modelling in reliability analysis of complex systems-Methodology and robustness. International Journal of Quality and Reliability Management, 18(3): 307-323

Arbia G, Griffith, DA, Haining RP. 2003. Spatial error propagation when computing linear combinations of spectral bands: The case of vegetation indices. Environmental and Ecological Statistics, 10(3): 375-396

Bamasak SM, Rushdi AMA. 2016. Uncertainty analysis of fault-Tree models for power system protection. Journal of Qassim University: Engineering and Computer Sciences, 8(1): 65-80

Baranyi G, Saura S, Podani J, Jordán F. 2011, Contribution of habitat patches to network connectivity: Redundancy and uniqueness of topological indices. Ecological Indicators, 11(5): 1301-1310

Bennetts RG. 1975. On the analysis of fault trees. IEEE Transactions on Reliability, R-24(3): 175-185

Bennetts RG. 1982, Analysis of reliability block diagrams by Boolean techniques. IEEE Transactions on Reliability, R-31(2): 159-166

Berleant D. 1993. Automatically verified reasoning with both intervals and probability density functions. Interval Computations, 2: 48-70

Bobbio A. 1990. Dependability analysis of fault-tolerant systems: a literature survey. Microprocessing and Microprogramming, 29(1): 1-13

Bradshaw GA, Borchers JG. 2000. Uncertainty as information: Narrowing the science-policy gap. Conservation Ecology, 4(1): 7

Chang SH, Park JY, Kim MK. 1985. The Monte-Carlo method without sorting for uncertainty propagation analysis in PRA. Reliability Engineering, 10(4): 233-243

Chen S, Chen B, Fath BD. 2013. Ecological risk assessment on the system scale: a review of state-of-the-art models and future perspectives. Ecological Modelling, 250: 25-33

Cheng D. 2009. Uncertainty analysis of large risk assessment models with applications to the railway safety and standards board safety risk model. PhD thesis. University of Strathclyde, UK

Cho JG, Yum BJ. 1997. Development and evaluation of an uncertainty importance measure in fault tree analysis. Reliability Engineering and System Safety, 57(2): 143-157

Colombo AG. 1980. Uncertainty propagation in fault-tree analysis. In: Synthesis and analysis methods for safety and reliability studies, 95-103, Springer, USA

Colombo AG, Jaarsma, RJ. 1980. A powerful numerical method to combine random variables. IEEE Transactions on Reliability, 2: 126-129

Cox DC. 1982. An analytic method for uncertainty analysis of non-linear outputs functions, with applications to fault tree analysis. IEEE Transactions on Reliability, 31: 465-468.

Dotson W, Gobien J. 1979. A new analysis technique for probabilistic graphs. IEEE Transactions on Circuits and Systems, 26(10): 855-865

Fisher JB, Tu KP, Baldocchi DD. 2008. Global estimates of the land–atmosphere water flux based on monthly AVHRR and ISLSCP-II data, validated at 16 FLUXNET sites. Remote Sensing of Environment, 112(3): 901-919

Forbes C, Evans,M, Hastings N, Peacock B. 2011. Statistical Distributions. John Wiley & Sons, New York, NY, USA

Goodwin BJ, Fahrig L. 2002. How does landscape structure influence landscape connectivity. Oikos, 99(3): 552-570

Halpern BS, Regan HM, Possingham HP, McCarthy MA. 2006. Accounting for uncertainty in marine reserve design. Ecology Letters, 9(1): 2-11

Heidtmann KD. 1991. Arithmetic spectrum applied to fault detection for combinational networks. IEEE Transactions on Computers, 40(3): 320-324

Heuvelink GBM. 2003. Comment to “Spatial error propagation when computing linear combinations of spectral bands: The case of vegetation indices”, by G. Arbia, DA Griffith and RP Haining. Environmental and Ecological Statistics, 10(3): 397-398

Jackson PS. 1982. A second-order moments method for uncertainty analysis. IEEE Transactions on Reliability, 31(4): 382-384

Jackson PS, Hockenbury RW, Yeater ML. 1982. Uncertainty analysis of system reliability and availability assessment, Nuclear Engineering and Design, 68(1): 5-29

Jain J. 1996. Arithmetic transform of Boolean functions. In: Representations of Discrete Functions (Sasao T, Fujita M, eds). 133-161, Kluwer Academic Publishers, Dordrecht, Netherlands

Jordán F. 2000. A reliability-theory approach to corridor design. Ecological Modelling, 128(2): 211-220

Jordán F, Báldi A, Orci KM, Racz I, Varga Z. 2003. Characterizing the importance of habitat patches and corridors in maintaining the landscape connectivity of a *Pholidoptera transsylvanica* (Orthoptera) metapopulation. Landscape Ecology, 18(1): 83-92

Jordán F. 2003. Quantifying landscape connectivity: Key patches and key corridors. In: Ecosystems and Sustainable Development IV (Tiezzi E, Brebbia CA, Usó JL, eds). 883-892, WIT Press, Southampton, UK

Kafka P, Polke H. 1986. Treatment of uncertainties in reliability models. Nuclear Engineering and Design, 93(2): 203-214

Kafrawy KF, Rushdi AM. 1990. Uncertainty analysis of fault trees with statistically correlated failure data. Microelectronics and Reliability, 30: 157–175

Karanki DR, Jadhav PA, Chandrakar A, Srividya A, Verma AK. 2010. Uncertainty analysis in PSA with correlated input parameters. International Journal of Systems Assurance Engineering and Management, 1(1): 66-71

Keey RB, Smith CH. 1985. The propagation of uncertainties in failure events. Reliability Engineering, 10(2): 105-127

Kindlmann P, Burel F. 2008. Connectivity measures: a review. Landscape Ecology, 23(8): 879-890

Kumar SK, Breuer MA. 1981. Probabilistic aspects of Boolean switching functions via a new transform. Journal of the ACM (JACM), 28(3): 502-520

Laviron A, Heising CD. 1985. Error transmission in large complex fault trees using the ESCAF method. Reliability Engineering, 12(3): 181-192

Lele,SR, Taper ML. 2011. Statistical Inference in Ecology. In International Encyclopedia of Statistical Science. 1430-1433, Springer Berlin Heidelberg, Germany

Masera M. 1987. Uncertainty propagation in fault tree analyses using lognormal distributions. IEEE Transactions on Reliability, 36(1): 145-149

Matsuoka T, Kobayashi M. 1997. The GO-FLOW reliability analysis methodology—analysis of common cause failures with uncertainty. Nuclear Engineering and Design, 175(3): 205-214

Naeem S. 1998. Species redundancy and ecosystem reliability. Conservation Biology, 12(1): 39-45

Papaioannou SG, Barrett WA. 1975. The real transform of a Boolean function and its applications. Computers and Electrical Engineering, 2(2): 215-224

Paul S, Rioux L. 2015. Over 20 years of research into cybersecurity and safety engineering: a short bibliography. Safety and Security Engineering, VI: 151-335

Paul S, Vignon-Davillier R. 2014. Unifying traditional risk assessment approaches with attack trees. Journal of Information Security and Applications, 19(3): 165-181

Pedroni N, Zio E. 2013. Uncertainty analysis in fault tree models with dependent basic events. Risk Analysis, 33(6): 1146-1173

Qin X, Huang G, Chen B, Zhang B. 2009. An interval-parameter waste-load-allocation model for river water quality management under uncertainty. Environmental Management, 43(6): 999-1012

Rao KD, Kushwaha HS, Verma AK, Srividya A. 2007. Quantification of epistemic and aleatory uncertainties in level-1 probabilistic safety assessment studies. Reliability Engineering & System Safety, 92(7): 947-956

Regan HM, Colyvan M, Burgman MA. 2002, A taxonomy and treatment of uncertainty for ecology and conservation biology. Ecological Applications, 12(2): 618-628

Rushdi AM. 1983a, Symbolic reliability analysis with the aid of variable-entered Karnaugh maps. IEEE Transactions on Reliability, R-32(2): 134-139

Rushdi AM. 1983b, How to hand-check a symbolic reliability expression. IEEE Transactions on Reliability, R-32(5): 402-408

Rushdi AM. 1985. Uncertainty analysis of fault-tree outputs. IEEE Transactions on Reliability, R-34(5): 458-462

Rushdi AM. 1987a. On computing the syndrome of a switching function. Microelectronics and Reliability,

27(4): 703-716
Rushdi AM., 1987b. On computing the spectral coefficients of a switching function. Microelectronics and Reliability, 27(6): 965-979
Rushdi AM., Al-Khateeb DL. 1983. A review of methods for system reliability analysis: A Karnaugh-map perspective. Proceedings of the First Saudi Engineering Conference, Jeddah, Saudi Arabia, 1: 57-95
Rushdi AM., AbdulGhani AA. 1993. A comparison between reliability analyses based primarily on disjointness or statistical independence, Microelectronics and Reliability, 33: 965-978
Rushdi AM. A., Al-Qwasmi MA. 2017. Utilization of basic-event statistical independence in simplifying the uncertainty analysis of fault trees. Journal of King Abdulaziz University: Computing and Information Technology, 6 (1)
Rushdi AMA, Alturki AM. 2015. Reliability of coherent threshold systems. Journal of Applied Sciences, 15(3): 431-443
Rushdi AM, Ba-Rukab OM. 2005a. A doubly-stochastic fault-tree assessment of the probabilities of security breaches in computer systems. Proceedings of the Second Saudi Science Conference, Part Four: Computer, Mathematics, and Statistics. 1-17, Jeddah, Saudi Arabia
Rushdi AM, Ba-Rukab OM. 2005b, Fault-tree modelling of computer system security. International Journal of Computer Mathematics, 82 (7): 805-819
Rushdi AM, Ghaleb FAM. 2015. The Walsh spectrum and the real Transform of a switching function: A review with a Karnaugh-map perspective. Journal of Qassim University: Engineering and Computer Sciences, 7(2): 73-112
Rushdi AM, Goda AS. 1985, Symbolic reliability analysis via Shannon's expansion and statistical independence. Microelectronics and Reliability, 25(6): 1041-1053
Rushdi AMA, Hassan AK. 2015. Reliability of migration between habitat patches with heterogeneous ecological corridors. Ecological Modelling, 304: 1-10
Rushdi AMA, Hassan AK. 2016. An exposition of system reliability analysis with an ecological perspective. Ecological Indicators, 63: 282-295
Rushdi AM, Kafrawy KF. 1988. Uncertainty propagation in fault-tree analysis using an exact method of moments. Microelectronics and Reliability, 28: 945-965
Schmoyeri RL, Beauchamp JJ, Brandt CC, Hoffman Jr FO. 1996. Difficulties with the lognormal model in mean estimation and testing. Environmental and Ecological Statistics, 3(1): 81-97
Soman KP, Misra KB. 1994. A simple procedure of computing variance sensitivity coefficients of top events in a fault tree. Microelectronics Reliability, 34(5): 929-934
Soman KP, Misra KB. 1996. A simple method of determining moments of a top event. International Journal of Quality & Reliability Management, 13(5): 50-60
Takaragi K, Sasaki R, Shingai S. 1982. An improved moment-matching algorithm for evaluating top-event probability bounds. IEEE Transactions on Reliability, 31(1): 45-48
Tanaka H, Fan LT, Lai FS, Toguchi D. 1983. Fault-tree analysis by fuzzy probability. IEEE Transactions on Reliability, R-32: 453-457
Taylor PD, Fahrig L, Henein K, Merriam G. 1993. Connectivity is a vital element of landscape structure. Oikos, 68: 571-572
Tischendorf L, Fahrig L. 2000. On the usage and measurement of landscape connectivity. Oikos, 90(1): 7-19
Trivedi KS. 2002. Probability and Statistics with Reliability Queuing and Computer Science Applications (2nd edn). Prentice-Hall, NJ, USA
Ulmeanu AP. 2012. Analytical method to determine uncertainty propagation in fault trees by means of binary

decision diagrams. IEEE Transactions on Reliability, 61(1): 84-94

Vasas V, Magura T, Jordán F, Tóthmérész B. 2009. Graph theory in action: evaluating planned highway tracks based on connectivity measures. Landscape Ecology, 24(5): 581-586

Weber DP. 1994. Fuzzy fault tree analysis. Proceedings of the 3rd IEEE Conference on Fuzzy Systems, 3: 1899-1904

William, BK. 2001. Uncertainty, learning, and the optimal management of wildlife. Environmental and Ecological Statistics, 8(3): 269-288

Williamson M, Gaston KJ. 2005. The lognormal distribution is not an appropriate null hypothesis for the species–abundance distribution. Journal of Animal Ecology, 74(3): 409-422

With, KA, King AW. 1997. The use and misuse of neutral landscape models in ecology. Oikos, 79: 219-229

Zhang WJ. 2012. Computational Ecology: Graphs, Networks and Agent-based Modeling. World Scientific, Singapore

Zhang WJ. 2016. Selforganizology: The Science of Self-Organization. World Scientific, Singapore

Zhang WJ, Qi YH. 2004. Between-habitat movement of rice arthropods and its ecological role. IRRN, 29(2): 43-45

Zhang WJ, Qi YH, Zhang ZG. 2014. Two-dimensional ordered cluster analysis of component groups in self-organization. Selforganizology, 1(2): 62-77

Zhang WJ, Schoenly KG. 1999. IRRI Biodiversity Software Series. III. BOUNDARY: A Program for Detecting Boundaries in Ecological Landscapes. IRRI Technical Bulletin No. 3. International Rice Research Institute, Manila, Philippines