Article

Effect of automobiles on global warming: A modeling study

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Abstract

Global warming threatens our environment as well as basic human needs. In the present scenario, increasing demand and excessive use of automobiles have increased the level of carbon dioxide emission in the environment, providing a significant contribution to increase the global warming. This paper deals with the modeling of the effect of automobiles on global warming. For this, three nonlinearly interacting variables namely; density of human population, density of automobiles and the concentration of carbon dioxide have been taken into account. In the modeling process, it is assumed that the density of automobiles increases in proportion to human population following a logistic growth. The model is analyzed using stability theory of ordinary differential equations. Local and global stability conditions are established to study the feasibility of the model system. It is shown that with increase in human population, the demand for automobiles increases which has significant effect on global warming increase.

Keywords mathematical model; automobile; carbon dioxide; stability.

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1 Introduction

The major factors responsible for increasing the level of carbon dioxide in the atmosphere are mainly due to man made activities. The human activities, mainly burning of fossil fuels, excessive use of automobiles, land use changes etc., are responsible for the observed rise in the concentration of atmospheric CO_2. It is estimated that approximately 90% contribution is due to the emissions from fossil fuel combustion and the rest is due to agriculture, deforestation, land use changes (EPA, 2014; IPCC, 2014).

Automobiles are serious global threat for increase in global warming due to emission of carbon dioxide gas. Over the past century, there are many factors which produce an effect of automobile emissions on global warming such as fuel used, road gradient, increasing vehicle age, payload, cold starts and maintenance condition of the vehicle. Automobiles are one of the significant sources of various gases and particulate matters. The black smoke released into the air by vehicles is the common form of pollution that we routinely face. Carbon dioxide emitted from automobiles is one of the largest contributors to the global warming (Engr and Thomas, 2015; Shaw, 2015; Sundar and Naresh, 2015; Opare, 2016). Due to rapid growth of human population, density of automobiles is increasing atrociously.

Due to increase in Earth's average temperature, various new diseases are occurring frequently, since the bacteria can survive better in such conducive situations and multiply faster. Further, many undesirable consequences, such as economical loss, loss of biodiversity, drought, flood, etc., may arise due to global warming (Douglas and Selden, 1995; Robinson et al., 2007; Gao et al., 2012; Wu et al., 2012; Ferrarini, 2012; Wu and Zhang, 2012; Zhang and Liu, 2012; Zhang and Zhang, 2012; Rai and Rai, 2013a, b).

The concentration of greenhouse gases continues to increase due to increasing demand and excessive use of automobiles which can affect the environment in several ways. These greenhouse gases contribute to global warming and affect the respiratory, reproductive, immune and neurological systems of animals. These gases are also main contributors to acid rain, which changes the pH of waterways and soils. Carbon dioxide (CO_2) is naturally present in the atmosphere which is the primary greenhouse gas emitted through human activities. In 2012, CO_2 accounted for about 82% of all U.S. greenhouse gas emissions from human activities. Human activities are changing the carbon cycle-both by adding more CO_2 to the atmosphere and by influencing the ability of natural sinks, like forests, to remove CO_2 from the atmosphere (EIA-2012).

From the above, it may be pointed out that the continuous increase in population and the associated use of automobiles have significant impact on increasing the global warming burden. Therefore, in this study our main aim is to explore the role of automobiles on global warming. For this, we propose a model consisting of three nonlinearly interacting variables namely; density of human population, density of automobiles and the concentration of carbon dioxide.

2 Model

To develop a mathematical model, we consider the density of human population as $N(t)$, $A(t)$ be the density of automobiles and C(t) be the concentration of CO_2 in the atmosphere. In the modeling process, the human population density is assumed to follow logistic growth with intrinsic growth rate r and carrying capacity K. It is assumed that the emission of carbon dioxide affects the growth of human population and influences the carrying capacity, with depletion rate constants r_1 and r_2 due to CO_2. It is natural to assume that as the population increases, the demand of automobiles also increases. Thus, we assume that the density of automobiles is directly proportional to the density of human population with a rate λ with its natural depletion rate coefficient λ_0. Let Q be the constant rate of emission of carbon dioxide from various sources into the atmosphere which is further enhanced by the emission from automobiles with a rate δ. The constant δ_0 represents natural depletion rate coefficient of carbon dioxide.

In view of the above assumptions, the dynamics of the system is assumed to be governed by the following system of nonlinear differential equations,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - r_1 NC - r_2 N^2 C \tag{1}$$

$$\frac{dA}{dt} = \lambda N - \lambda_0 A \tag{2}$$

$$\frac{dC}{dt} = Q + \delta A - \delta_0 C \tag{3}$$

with $N(0) > 0, A(0) > 0,\ C(0) > 0$. Here all the parameters are assumed to be positive.

Lemma The region of attraction in the positive octant is given by the set,

$$\Omega = \left\{ (N, A, C) \in R_+^3 : 0 \le N \le K, 0 \le A \le \frac{\lambda K}{\lambda_0}, 0 \le C \le \frac{1}{\delta_0}\left(Q + \frac{\delta \lambda K}{\lambda_0} \right) \right\} \tag{4}$$

Remark 1 From equation (1), it is noted that in presence of carbon dioxide, the intrinsic growth rate of human population at any time $t > 0$ is $r - r_1 C > 0$.

3 Equilibrium and Stability Analysis

The model system (1) – (3) has only one non-trivial equilibrium $E^*(N^*, A^*, C^*)$.

The solution of $E^*(N^*, A^*, C^*)$ is given by the following set of algebraic equations,

$$r\left(1 - \frac{N}{K}\right) - r_1 C - r_2 NC = 0 \tag{5}$$

$$A = \frac{\lambda}{\lambda_0} N \tag{6}$$

$$C = \frac{1}{\delta_0}\left(Q + \frac{\delta \lambda}{\lambda_0} N \right) \tag{7}$$

Using equations (5) and (6) in equation (7), we get,

$$r_2 \delta_0 C^2 + \left(\frac{\lambda}{\lambda_0} r_1 \delta + \frac{r\delta_0}{K} - r_2 Q \right) C - \left(\frac{Q}{K} r + \frac{\lambda}{\lambda_0} r\delta \right) = 0 \tag{8}$$

From which, we get,

$$C = \frac{ -\left(\frac{\lambda}{\lambda_0} r_1 \delta + \frac{r\delta_0}{K} - r_2 Q \right) + \sqrt{ \left(\frac{\lambda}{\lambda_0} r_1 \delta + \frac{r\delta_0}{K} - r_2 Q \right)^2 + 4 r_2 \delta_0 \left(\frac{Q}{K} r + \frac{\lambda}{\lambda_0} r\delta \right) } }{2 r_2 \delta_0} = C^* \text{ (let)}$$

Using this value of $C (= C^*)$, we can find $N = N^*$ and $A = A^*$ from equations (5) and (6) respectively.

Differentiating (8), with respect to λ and noting remark (1), we get

$$\frac{dC}{d\lambda} = \frac{ \frac{\delta}{\lambda_0}(r - r_1 C) C }{ r_2 \delta_0 C^2 + \frac{Q}{K} r + \frac{\lambda}{\lambda_0} r\delta } > 0 \tag{9}$$

This implies that C increases as λ increases. Similarly, we can show easily that $\frac{dA}{d\lambda} > 0$, $\frac{dN}{d\lambda} < 0$ and $\frac{dN}{d\delta} < 0$. This analysis reveals that the number density of automobiles increases as human population increases and consequently the concentration of carbon dioxide in the atmosphere also increases which in turn may decrease the growth of population density.

4 Stability Analysis

4.1 Local stability of the equilibrium $E^*(N^*, A^*, C^*)$

To present local stability behaviour of the model system (1) – (3) corresponding to the equilibrium $E^*(N^*, A^*, C^*)$, we compute the following Jacobian matrix,

$$J(E^*) = \begin{bmatrix} -\left(\frac{r}{K} + r_2 C^*\right)N^* & 0 & -(r_1 + r_2 N^*)N^* \\ \lambda & -\lambda_0 & 0 \\ 0 & \delta & -\delta_0 \end{bmatrix}$$

Eigenvalues of the Jacobian matrix $J(E^*)$ are given by the following characteristic equation,

$$x^3 + A_1 x^2 + A_2 x + A_3 = 0$$

where,

$$A_1 = \left(\frac{r}{K} + r_2 C^*\right)N^* + \lambda_0 + \delta_0 > 0$$

$$A_2 = \left(\frac{r}{K} + r_2 C^*\right)N^* \lambda_0 + \lambda_0 \delta_0 + \left(\frac{r}{K} + r_2 C^*\right)N^* \delta_0$$

$$A_3 = \left(\frac{r}{K} + r_2 C^*\right)N^* \lambda_0 \delta_0 + (r_1 + r_2 N^*)N^* \lambda \delta$$

It may be easily noted that $A_1 A_2 - A_3 > 0$ provided the following condition is satisfied,

$$(r_1 + r_2 N^*) < \frac{\lambda_0 \delta_0}{\lambda \delta}\left(\frac{r}{K} + r_2 C^*\right) \tag{10}$$

Therefore, by Routh-Hurwitz criteria, eigenvalues of the Jacobian matrix corresponding to the equilibrium $E^*(N^*, A^*, C^*)$ are all negative or have negative real part. Hence equilibrium E^* is locally asymptotically stable.

Theorem 1 The equilibrium E^* is locally asymptotically stable provided the condition (10) is satisfied.

4.2 Nonlinear stability of the equilibrium $E^*(N^*, A^*, C^*)$

To establish nonlinear stability, we consider the following positive definite function

$$V = m_1\left(N - N^* - N^* \ln\frac{N}{N^*}\right) + \frac{1}{2}m_2\left(A - A^*\right)^2 + \frac{1}{2}m_3\left(C - C^*\right)^2 \quad (11)$$

where $m_i\,(i = 1,2,3)$ are positive constants to be chosen appropriately.

Differentiating (11) with respect to t and using the model system (1) – (3), we get

$$\frac{dV}{dt} = -m_1\left(\frac{r}{K} + r_2 C\right)\left(N - N^*\right)^2 - m_2\lambda_0 (A - A^*)^2 - m_3\delta_0 (C - C^*)^2 + m_2\lambda(N - N^*)(A - A^*)$$

$$+ m_3\delta(A - A^*)(C - C^*) - m_1\left(r_1 + r_2 N^*\right)(N - N^*)(C - C^*)$$

Choosing $m_1 = 1$, $\frac{dV}{dt}$ will be negative definite if the following condition is satisfied.

$$(r_1 + r_2 N^*) < \frac{\lambda_0\delta_0}{\lambda\delta}\left(\frac{r}{K}\right) \quad (12)$$

Theorem 2 The equilibrium E^* is nonlinearly asymptotically stable within the region of attraction Ω, provided the condition (12) is satisfied.

5 Numerical Simulation

In this paper, we propose a three-dimensional nonlinear model to study the effect of automobiles on global warming which is analyzed using stability theory of differential equations. The model incorporates three nonlinearly interacting variables viz the density of human population, the density of automobiles and the concentration of carbon dioxide. The equilibrium analysis of the model reveals that the density of automobiles increases as human population increases leading to increase the concentration of carbon dioxide in the atmosphere. The model has only one non-trivial unique equilibrium point which is locally asymptotically stable under condition (10). Moreover, this equilibrium is also found to be nonlinearly asymptotically stable inside the region of attraction Ω under condition (12).

To validate the analytical findings, we perform numerical simulation of the model system (1) – (3) with respect to $E^\bullet$ for different values of parameters. For that the system (1) – (3) is integrated numerically with the help of MAPLE 7 by considering the following set of parameters values,

$Q = 1, r = 2, K = 1000, r_1 = 0.000005, r_2 = 0.0000006, \lambda = 0.4, \lambda_0 = 0.2,\ \delta = 0.01, \delta_0 = 0.3$

The equilibrium E^* so obtained is,

$N^* = 979.6542863, A^* = 1959.308573, C^* = 68.64361908$

Eigenvalues of the Jacobian matrix corresponding to E^* are -2.000415548, -0.2153863006 and -0.2838549336. Since all the eigenvalues are negative, the interior equilibrium E^* is locally asymptotically stable.

To see the nonlinear stability behaviour, for the above set of parameters, computer generated graph in $N - A$ is shown in Fig. 1, where the trajectories with different initial starts approaches the equilibrium E^*. It depicts that E^* is nonlinearly stable.

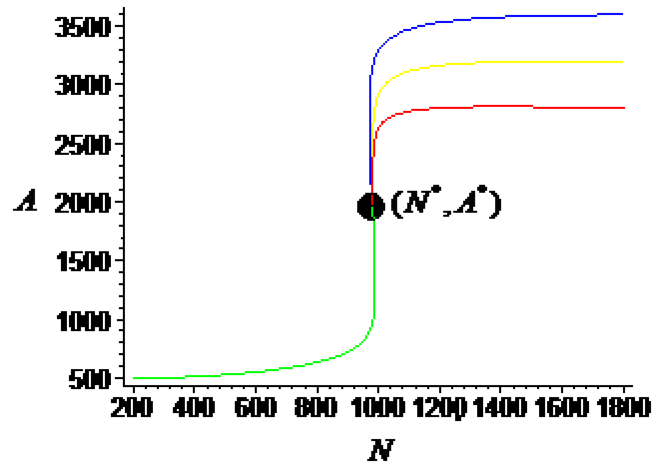

Fig. 1 Nonlinear stability in $N - A$ plane.

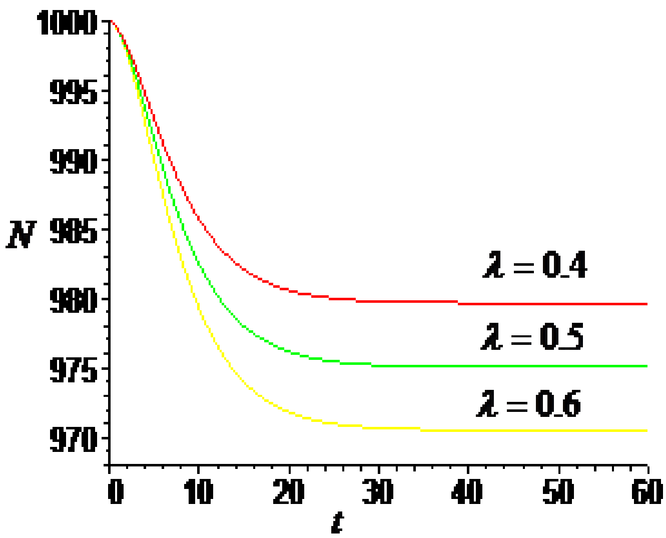

Fig. 2 Variation of N with time t for different values of λ .

In Fig. 2, the variation of density of human population with time '*t*' is shown for different values of λ, the growth rate of automobiles. From the figure, it is clear that the density of human population decreases as the growth rate of automobiles increases. In Fig. 3, the variation of concentration of carbon dioxide with time '*t*' is shown for different values of growth rate of automobiles. From this figure, it is seen that the concentration of carbon dioxide increases as the growth rate of automobiles increases. Thus, the increase in the growth rate of automobiles increases the level of carbon dioxide in the atmosphere which in turn adversely affects the human population.

As discussed in introduction, demand of automobiles increases as the population increases which in turn increase the concentration of carbon dioxide in the atmosphere. The effect of automobiles on human population and carbon dioxide concentration is plotted in Figs. 4 and 5 respectively. Thus, in Figs. 4 and 5, the variation of density of human population (N) and the concentration of carbon dioxide (C) with time '*t*' is shown for different values of δ, the emission rate of carbon dioxide due to automobiles, respectively. From these figures, it is observed that the density of human population decreases and the concentration of carbon dioxide increases with increase in the emission rate of carbon dioxide from automobiles. This elevated level of carbon dioxide has significant effect on global warming.

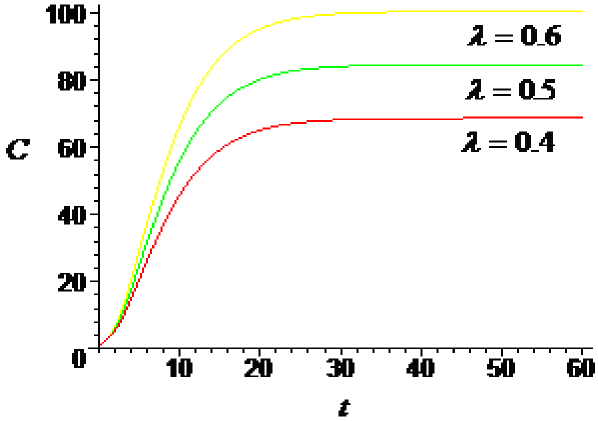

Fig. 3 Variation of C with time t for different values of λ .

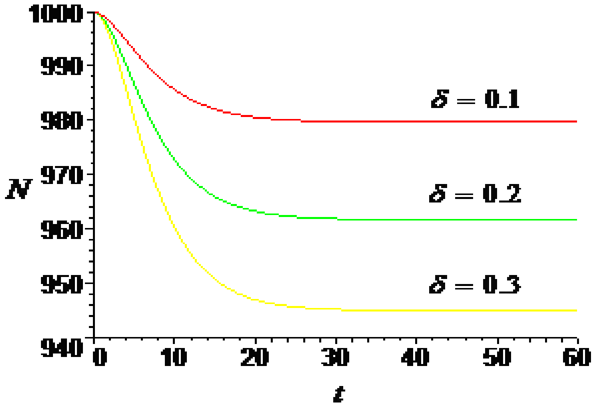

Fig. 4 Variation of N with time t for different values of δ .

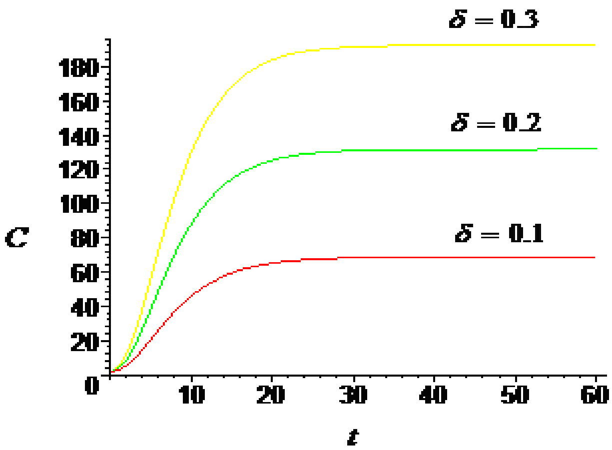

Fig. 5 Variation of C with time t for different values of δ .

6 Conclusion

In this paper, an attempt has been made to comprehend the effect of automobiles on global warming. It is assumed that the model system consists of three nonlinearly dependent variables namely, density of human population, density of automobiles and the concentration of carbon dioxide. The proposed model is analysed using stability theory of ordinary differential equations and numerical simulations. It is shown, analytically and numerically, that the concentration of carbon dioxide increases as the density of automobiles increases. This elevated level of carbon dioxide in the atmosphere due to emissions from automobiles may significantly contribute to increase the global warming. The burden of global warming could be reduced significantly, if the emission from automobiles is restricted by way of enforcing stringent pollution norms and by reducing the unnecessary load of automobiles.

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