

Article

How to calculate statistical power for vegetation research

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Abstract

Calculation of statistical power is important for proper interpretation of research results. Statistical Power depends on the selected significance level, sample size and effect size. Selection of an appropriate formula for calculating power of a test is dependent on the study design, type and statistical distribution of data and the statistical test. In this paper, several formulas are presented with examples for calculating power for one sample mean test and one sample proportion test, comparing between two independent groups and two paired groups, correlation analysis, simple and multiple linear regression, simple and multiple logistic regression, contingency tables and analysis of variance (ANOVA).

Keywords effect size; statistical power; sample size; significance level; vegetation.

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1 Introduction

One of the most important stages of vegetation studies is calculation of statistical power. The power of a statistical test is the probability that it will yield statistically significant results. Power of a test is defined as the probability of correctly rejecting the null hypothesis when it is false, and it is equal to $1 - \beta$. Type II error (β) is the probability of accepting the null hypothesis when it is false. The power of a test depends upon three parameters; sample size, significance level and effect size.

In vegetation studies, sample size is the required number of samples (e.g., number of plots, points, transects, plant individuals, seed and fruit traps, etc.) taken for estimating and comparing vegetation characteristics such as cover, biomass, density, production, height, diameter, richness, nutrients, rooting depth and interactions. The larger the sample size, other things (significance level, effect size) being equal, the larger the power. Significance level (α) or Type I error is the probability of rejecting the null hypothesis, when it is true. The lower the value of significance level, the lower the power of the test (Cohen, 1988). The most common significance levels used in vegetation studies are 10%, 5% and 1% (Federal register Vol. 44, No 50; 1979; Stauffer, 1983; Mosley et al., 1989; Hofmann and Ries, 1990).

Effect size is the minimum detectable difference between the groups being compared. The term “effect size”

was defined by Cohen (1988) as the degree to which the phenomenon is present in the population. Some examples of the effect size include the difference between sample mean and true mean in a one sample t-test, the difference between two proportions in a comparison of two independent proportions, the correlation coefficient and the slope of a linear regression model. The larger the effect size, the greater the statistical power. Standard deviation is another important parameter that is usually used for calculating effect size, sample size and power (Nedorezov, 2014; Zhang, 2017).

The statistical power can be calculated before and after data collection. The power can be calculated before data collection using previous studies to obtain the required sample size. When the power is calculated after data collection, it can be used to verify whether a non-significant result is due to lack of relationship between the groups or due to lack of statistical power. An ideal study is the one with a high power. This means that the study has a high chance of detecting a difference between groups if it exists (Suresh and Chandrashekar, 2012). Common values for power in vegetation studies are 0.8 and 0.9 or higher depending on the study (Peek et al., 2003; Moffet, 2009; Conser and Connor, 2009).

Different research designs and statistical tests need different methods of power calculations, and one single formula cannot be used for all research designs (Charan and Biswas, 2013). In this paper several formulas are explained for calculating statistical power with examples according to the study designs and statistical tests. The power formulas presented in this paper can be used for testing the difference from a constant, comparing two means based on paired and unpaired groups, comparing two independent proportions, correlation analysis, simple and multiple linear regression, simple and multiple logistic regression, contingency tables and analysis of variance (ANOVA).

The most frequently used software for power calculations are G* Power (Erdfelder et al., 1996; Faul et al., 2007), PS (Dupont and Plummer, 1997) and PASS (StataCorp, L. P. 2013). For a review of statistical power analysis software see Thomas and Krebs (1997).

The formulas and examples presented in this paper are for calculating the power based on a two sided-test, and the smaller of the two terms of formulas is negligible and therefore is ignored (except for ANOVA, multiple linear regression and Chi-square). For one-sided tests, the approximations obtained from the formulas are exact, with t_a used instead of $t_{a/2}$, and Z_α used instead of $Z_{\alpha/2}$ in the formulas ($Z_\alpha=1.645$ at 5% and $Z_\alpha= 2.326$ at 1% significance level).

2 Methods

2.1 Power calculation for one-sample mean test

Suppose the objective is to test the difference of the mean of a quantitative variable (e.g., biomass, density, production) from a constant using one sample t-test to see if the difference is significant. Then the following formula is used to calculate the power of the test

$$1 - \beta = T_{n-1} \left[\left(\sqrt{n} \times \frac{d}{S} \right) - t_{\frac{\alpha}{2}, n-1} \right] + T_{n-1} \left[\left(-\sqrt{N} \times \frac{d}{S} \right) - t_{\frac{\alpha}{2}, n-1} \right]$$

$T_{n-1}(x)$ is the cumulative probability distribution for a t statistic with $N-1$ degrees of freedom. $t_{\alpha/2, n-1}$ is the value of t -distribution based on the significance level (α) and the degrees of freedom ($N-1$), N is sample size, S is the standard deviation of the quantitative variable, d is the difference between the sample mean and the constant. The effect size is defined as d/s .

Example

Suppose a researcher wishes to determine the accuracy of density estimates by quadrat. To do this, total

number of plants is counted to be 2000 in a 100 m^2 sampling area and the true density is calculated as $2000/100 = 2\text{ m}^{-2}$. This density is assumed to be the true density. The density is also estimated using 20 located quadrats as “number of plants/quadrat area” in the sampling area and the mean density is 2.3 m^{-2} with the standard deviation of 0.45. The power for detecting the difference between the estimated density and the actual density at 5% significance level is calculated as follows

For $N=20$, df is $20-1=19$ and at $\alpha=0.05$, $t_{\frac{\alpha}{2}, n-1}$ is 2.093

$$\left(\sqrt{20} \times \frac{2.3 - 2}{0.45}\right) - 2.093 = 0.888$$

From t-table, in row $df=19$, the value of t is 0.861 for power $(1-\beta)=0.8$ and the value of t is 1.066 for power $(1-\beta)=0.85$. The power for $t = 0.888$ is calculated using interpolation as

$$0.8 + \frac{0.888 - 0.861}{1.066 - 0.861}(0.85 - 0.8) = 0.807$$

and

$$\left(-\sqrt{20} \times \frac{2.3 - 2}{0.45}\right) - 2.093 = -4.94$$

From t-table in row $df=19$, for $t = -4.94$, the power is 0.00005. Then the power of the test is $0.807 + 0.00005 = 0.80705$. The smaller of the two terms is negligible and can be ignored. Therefore the power can be calculated as

$$t_{\beta} = \left(\sqrt{N} \times \frac{|d|}{S}\right) - t_{\frac{\alpha}{2}, n-1}$$

The power can be calculated using the value of t_{β} from t-distribution table. The above-equation can be used to calculate the power for a one-sided test with $t_{\alpha/2}$ replaced by t_{α} . The above power formula can be used to test the difference of mean plant biomass, production, density, plant height, seed production, litter, rooting depth and nutrients from a constant.

2.2 Power calculation for one sample proportion test

Suppose the objective is to test the difference of plant cover proportion estimated by points from a constant (e.g., cover proportion measured by transect or quadrat) to detect if there is any significant difference. Then the following equation should be used for calculating the power of the test

$$Z_{\beta} = \left[\left| 2\arcsin\sqrt{P_1} - 2\arcsin\sqrt{P_0} \right| \times \sqrt{N} \right] - Z_{\alpha/2}$$

The power is calculated using the value of Z_{β} from the table of standard normal probabilities. $Z_{\alpha/2}$ is standard normal variate for significance level ($Z_{\alpha/2}=1.96$ at 5% and $Z_{\alpha/2}=2.576$ at 1% Type I error), P_1 is the expected proportion in population, P_0 is the tested proportion (constant) and N is the size of sample. The effect size is defined as $p_1 - p_0$. For a one-sided test, Z_{α} is used instead of $Z_{\alpha/2}$ in the equation ($Z_{\alpha}=1.645$ at 5% and $Z_{\alpha}=2.326$ at 1% significance level).

Example

The cover of plants in a large sampling plot is measured to be 40% by line transects (based on total length of intercepted plants divided by the transect length). This cover is assumed to be the true cover. The cover proportion is also estimated to be 46% by 460 located points in the sampling plot (as % of points contacted

with plants). The statistical power for testing the difference between the estimated cover by points and the true cover at 5% significance level is

$$Z_{\beta} = \left[\left| 2\arcsin\sqrt{0.46} - 2\arcsin\sqrt{0.4} \right| \times \sqrt{460} \right] - 1.96 = 0.641$$

From the z-table the power for the value of $Z_{\beta} = 0.641$ is 0.73. The above power equation can be used to test the difference of proportion (such as proportion of quadrats occupied by a plant species in a population, proportion of plants associated with fungi, bacteria species or epiphytes, proportion of germinating seeds in a plant species, proportion of plants having a disease in a population) from a constant.

2.3 Power calculation for comparing two independent samples with quantitative data

Suppose a researcher wishes to determine the effects of grazing on the biomass of plants by comparing the mean biomass of plants between the grazed and ungrazed (exclosure) plots. In such case, independent sample t-test can be used to test if the difference between the two means is significant. The power is then calculated using the following equation if equal variances are assumed

$$t_{\beta} = \frac{|d|}{S \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} - t_{\frac{\alpha}{2}, n_1+n_2-2}$$

The power is calculated using the value of t_{β} from t-distribution table based on the degrees of freedom $n_1 + n_2 - 2$, $t_{\alpha/2, n_1+n_2-2}$ is the value of t-distribution based on the significance level and the degrees of freedom ($n_1 + n_2 - 2$) and S is the standard deviation calculated as follows

$$S = \sqrt{\frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}}$$

S_1 and S_2 are the standard deviation of group 1 and 2 respectively, and n_1 and n_2 are respectively the size of group 1 and 2. d is the difference between the two means ($\bar{x}_1 - \bar{x}_2$). The effect size in this equation can be defined as d/S (Cohen, 1988).

Example

Suppose the mean biomass of plants in 15 ungrazed (exclosure) plots is 80 g with standard deviation of 14.5, and the mean biomass of plants in 15 grazed plots is 60 g with standard deviation of 13.5. Then, the power for testing the difference between the two means at 0.01 significance level is calculated as follows

$$S = \sqrt{\frac{14.5^2(15 - 1) + 13.5^2(15 - 1)}{15 + 15 - 2}} = 14$$

$t_{\alpha/2, n_1+n_2-2}$ for $\alpha=0.01$ and $df= 15+15-2=28$ is 2.763

$$t_{\beta} = \frac{|80 - 60|}{14 \times \sqrt{\frac{1}{15} + \frac{1}{15}}} - 2.763 = 1.15$$

From t-table, in row $df= 28$, t_{β} is 1.056 for power $(1 - \beta)=0.85$ and t_{β} is 1.313 for power $(1 - \beta)=0.90$. Then the power for $t_{\beta} = 1.15$ is calculated using interpolation as

$$0.85 + \frac{1.15 - 1.056}{1.313 - 1.056} (0.90 - 0.85) = 0.87$$

If equal variances are not assumed then the power is calculated using

$$t_{\beta} = \frac{|d|}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} - t_{\frac{\alpha}{2}, \nu} \quad \text{and the degrees of freedom } (\nu) \text{ is calculated as } \nu = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{(\frac{s_1^2}{n_1})^2}{n_1-1} + \frac{(\frac{s_2^2}{n_2})^2}{n_2-1}}$$

The above-mentioned power equation can be used to calculate the statistical power for comparing the attributes such as density, biomass, height, litter, seed production and nutrients between the plants which are and are not associated with other organisms (e.g., fungus, epiphytes), the plants which are and are not exposed to pollutants, the plants which are and are not manured and between the plants in two stands.

2.4 Sample size calculation for comparing two independent proportions

A researcher wants to determine the effects of gibberellins and cytokinin hormones on percentage germination of seeds of a plant species. To do this, two groups of seeds were selected; one group was treated with gibberellins and one group with cytokinin. The proportions of germinated seeds in both groups were compared to detect any significant difference. Then, the following formula is used to calculate power when two groups are equal

$$Z_{\beta} = \frac{|P_1 - P_2|}{\sqrt{\frac{2p(1-p)}{N}}} - Z_{\alpha/2}$$

The power is calculated using the value of Z_{β} from the table of standard normal probabilities. $Z_{\alpha/2}$ is standard normal variate for significance level ($Z_{\alpha/2}=1.96$ at $\alpha =5\%$ and $Z_{\alpha/2}= 2.576$ at $\alpha =1\%$). N is sample size, p_1 and p_2 are respectively the proportion of events in group 1 and 2. p is the average proportion of the events = (proportion of the event in group1 + proportion of the event in group2)/2. The effect size can be defined as $p_1 - p_2$, p_2/p_1 or the odds ratio.

Example

Suppose 80% of 35 seeds exposed to gibberellins and 40% of 35 seeds exposed to cytokinin were germinated within a specified time. The power for detecting the difference between the two proportions at $\alpha=0.05$ is calculated as follows

$$P = (0.8 + 0.4)/2 = 0.6$$

$$Z_{\beta} = \frac{|0.8 - 0.4|}{\sqrt{\frac{2 \times 0.6(1-0.6)}{35}}} - 1.96 = 1.46$$

The power for the value of $Z_{\beta} = 1.46$ is 0.926 from z-table. In the above example if $N=10$, then the power at $\alpha=0.05$ is

$$Z_{\beta} = \frac{|0.8 - 0.4|}{\sqrt{\frac{2 \times 0.6(1-0.6)}{10}}} - 1.96 = -0.134$$

The power for the value of $Z_{\beta} = -0.134$ is 0.447 from z-table.

If the size of two groups are unequal ($n_1 \neq n_2$), the power is calculated as

$$Z_{\beta} = \frac{|P_1 - P_2|}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} - Z_{\alpha/2}$$

The above equation can be used to calculate the required power for comparing the effects of factors such as hormones, water etc, on seed germination.

2.5 Power calculation for comparing two paired (dependent) samples with quantitative data

Suppose a researcher wishes to compare the differences in biomass of plants in fixed plots before and one year after the fire using paired sample t-test to see if the differences are significant. Then the following formula is used to calculate the power of the test

$$t_{\beta} = \frac{|d|}{\frac{S}{\sqrt{N}}} - t_{\frac{\alpha}{2}, n-1}$$

The power is calculated using the value of t_{β} from t-distribution table based on the degrees of freedom ($N-1$). $t_{\alpha/2, n-1}$ is the value of t-distribution based on the significance level and the degrees of freedom ($N - 1$), N is sample size, S is the standard deviation of the differences, and d is the difference between two means before and after disturbances or treatments. The effect size can be defined as d/S .

Example

Suppose the difference in the mean biomass of plants in 10 fixed plots before and one year after the fire is 18 g and the standard deviation of the differences is 13.5. Then, the power for testing the difference between the two means at 1% significance level is calculated as follows

$t_{\alpha/2, n-1}$ at $\alpha = 0.01$ and $df = 10-1=9$ is 3.25

$$t_{\beta} = \frac{|18|}{\frac{13.5}{\sqrt{10}}} - 3.25 = 0.966$$

From t-table, in row $df=9$, t_{β} is 0.883 for power $(1-\beta)=0.8$ and t_{β} is 1.1 for power $(1-\beta)=0.85$. Then the power for $t_{\beta} = 0.966$ is calculated using interpolation as

$$0.80 + \frac{0.966 - 0.883}{1.1 - 0.883} (0.85 - 0.80) = 0.82$$

In the above example if $N=6$, then the power at 1% significance level is calculated as follows

$t_{\alpha/2, n-1}$ at $\alpha = 0.01$ and $df = 6-1=5$ is 4.032

$$t_{\beta} = \frac{|18|}{\frac{13.5}{\sqrt{6}}} - 4.032 = -0.766$$

From t-table, in row $df=5$, t_{β} is -0.727 for power $(1-\beta)=0.25$ and t_{β} is -0.92 for power $(1-\beta)=0.20$. Then the power for $t_{\beta} = -0.766$ is

$$0.25 - \frac{-0.766 - (-0.727)}{-0.92 - (-0.727)} (0.25 - 0.20) = 0.24$$

The above equation can be used to calculate the power to interpret the differences in vegetation attributes

such as biomass, production, density, rooting depth, seed production and height of plants in fixed plots before and after disturbances (such as grazing, fire, drought) and treatments (such as manures, watering, leaf pruning, growth hormones)

2.6 Power calculation for simple linear regression model and correlation

The relationship between a vegetation attribute as dependent variable (e.g., biomass, cover, density) and an environmental variable as independent variable (e.g., precipitation, temperature, soil salinity, soil water content) can be determined using a simple linear regression and correlation. The power equation for a simple linear regression model is calculated as follows (Dupont and Plummer, 1998)

$$t_{\beta} = \frac{\sqrt{\frac{N}{\frac{s_y^2}{\lambda^2 s_x^2} - 1}}}{t_{\alpha/2, n-2}}$$

The power is calculated using the value of t_{β} from t-distribution table based on the degrees of freedom ($N-2$). $t_{\alpha/2, n-2}$ is obtained from t-table at a given significance level with $N-2$ degrees of freedom. S_x is standard deviation of the independent variable, S_y is standard deviation of the dependent variable and λ is the slope of regression line. The slope of the linear regression line can be used as the effect size. The relationship between correlation coefficient (r), S_x and S_y is $r = (\lambda S_x)/S_y$.

Example

Suppose a researcher wishes to determine the relationship between a plant biomass and soil EC through fitting a simple linear regression. The data of the plant biomass and soil EC were collected from 30 quadrats and a simple linear regression was made. The standard deviation of the biomass (S_y) was 3.2, the standard deviation of soil EC (S_x) was 2 and the slope of regression line (λ) was 0.8. The power for the regression model at $\alpha=0.05$ is calculated as follows

For $N = 30$, df is $30-2=28$, and at 0.05 significance level, $t_{\alpha/2}$ is 2.048

$$t_{\beta} = \frac{\sqrt{\frac{30}{\frac{3.2^2}{0.8^2 \times 2^2} - 1}}}{2.048} = 1.11$$

From t-table, in row $df=28$, t_{β} is 1.056 for power $(1-\beta)=0.85$ and t_{β} is 1.313 for power $(1-\beta)=0.90$. Then the power for $t_{\beta} = 1.11$ is calculated using interpolation as

$$0.85 + \frac{1.11 - 1.056}{1.313 - 1.056} (0.90 - 0.85) = 0.862$$

Based on the data, $r = (0.8 \times 2)/3.2 = 0.5$, and therefore power can also be calculated based on r .

2.7 Power calculation for correlation analysis

The power for testing the difference of Pearson correlation coefficient (r) from the constant value of $r = 0$ is calculated as follows

$$Z_{\beta} = \left(\frac{1}{2} \times \ln \left(\frac{1+r}{1-r} \right) \times \sqrt{n-3} \right) - Z_{\alpha/2}$$

$Z_{\alpha/2}$ is standard normal variate for significance level ($Z_{\alpha/2}=1.96$ at $\alpha =5\%$ and $Z_{\alpha/2}= 2.576$ at $\alpha =1\%$), n is sample size and r is the Pearson correlation coefficient. When the difference is tested between the Pearson r and zero, r is the effect size (Cohen, 1988; Benzer, 2017).

Example

Suppose the Pearson correlation coefficient between a plant biomass and soil Nitrogen is 0.6 using the data of 20 plots. Then the power for detecting the difference of the correlation coefficient from zero at 5% significance level is obtained as follows

$$Z_{\beta} = \left(\frac{1}{2} \times \ln\left(\frac{1+0.6}{1-0.6}\right) \times \sqrt{20-3}\right) - 1.96 = 0.90$$

The power for the value of $Z_{\beta} = 0.90$ is 0.816 from z-table. The power for Pearson correlation can also be calculated using t-distribution as follows

$$t_{\beta} = r/\sqrt{(1-r^2)/(n-2)} - t_{\frac{\alpha}{2}, n-2}.$$

2.8 Power calculation for multiple linear regression model

In a multiple linear regression model, the relationship between a dependent variable (Y) and a number of independent variables ($x_1, x_2 \dots x_n$) is modeled. Suppose a researcher wants to determine the relationship between a quantitative attribute of vegetation (e.g., biomass, cover, density) and a number of predictors (e.g., precipitation, temperature, humidity, elevation) using a multiple linear regression model. The statistical power for multiple linear regression model is calculated using the following formula (Laubscher, 1960; Cohen, 1988)

$$Z_{\beta} = \frac{\sqrt{2(u+\lambda) - \frac{u+2\lambda}{u+\lambda}} - \sqrt{(2v-1) \frac{uF_c}{v}}}{\sqrt{\frac{uF_c}{v} + \frac{u+2\lambda}{u+\lambda}}}$$

The power is calculated based on Z_{β} value from Z- table. u is the number of predictors or degrees of freedom of the numerator of the F ratio. λ is the noncentrality parameter and is calculated as $\lambda = f^2 N$ where N is the total sample size, f^2 is the effect size and is calculated as $f^2 = R_{Y.X}^2 / (1 - R_{Y.X}^2)$, $R_{Y.X}^2$ is the squared multiple correlation between Y and X ($x_1, x_2 \dots x_n$), v is the denominator (error) degrees of freedom ($v = N - u - 1$) and F_c is the critical F -value obtained from F-distribution table using the given significance level (α), numerator degrees of freedom (u) and denominator degrees of freedom (v).

Example

Suppose a researcher wishes to determine the relationship between the biomass of a plant species and three climatic variables ($u = 3$) including precipitation, temperature and humidity. A multiple linear regression model was made using the data of plant biomass and the three variables obtained from 27 plots. $R_{Y.X}^2$ of the model was 0.30. The power of the regression model at $\alpha=0.05$ is calculated as follows

numerator df (u) = 3, error df (v) = 27-3-1=23,

$$f^2 = 0.3 / (1 - 0.3) = 0.4285$$

$$\lambda = 0.4285 \times 27 = 11.57$$

The critical F -value at $\alpha=0.05$, numerator $df = 3$, and error $df = 23$ is 3.03 from F-table.

$$Z_{\beta} = \frac{\sqrt{2(3 + 11.57) - \frac{3+2(11.57)}{3+11.57}} - \sqrt{(2(23) - 1) \frac{3(3.03)}{23}}}{\sqrt{\frac{3(3.03)}{23} + \frac{3+2(11.57)}{3+11.57}}} = 0.704$$

The power obtained for $Z_{\beta} = 0.704$ from Z-table is 0.758. The statistical power for multiple linear regression model can also be calculated as a function of λ , u and v using Cohen's power tables (Cohen, 1988, p. 416-423). A part of the Cohen's table for obtaining the power at 5% significance level is presented below (Fig. 1).

u	v	λ														
		2	4	6	8	10	12	14	16	18	20	24	28	32	36	40
1	20	27	48	64	77	85	91	95	97	98	99	*				
	60	29	50	67	79	88	92	96	98	99	99	*				
	120	29	51	68	80	88	93	96	98	99	99	*				
	∞	29	52	69	81	89	93	96	98	99	99	*				
2	20	20	36	52	65	75	83	88	92	95	97	99	*			
	60	22	40	56	69	79	87	91	95	97	98	*				
	120	22	41	57	71	80	87	92	95	97	98	*				
	∞	23	42	58	72	82	88	93	96	97	99	*				
3	20	17	30	44	56	67	75	82	87	91	94	97	99	*		
	60	19	34	49	62	73	81	87	92	95	97	98	*			
	120	19	35	50	64	75	83	89	93	95	97	99	*			
	∞	19	36	52	65	76	84	90	93	96	98	99	*			
4	20	15	26	38	49	60	69	76	83	87	91	95	98	99	*	
	60	17	30	44	57	68	77	83	89	92	95	98	99	*		
	120	17	31	46	58	70	78	85	90	93	96	98	99	*		
	∞	17	32	47	60	72	80	87	91	94	96	99	*			

Fig. 1 Power of the F test as a function of λ , u and v at $\alpha=0.05$ (Cohen, 1988).

For each of the values of u , power entries for the following four values of v are provided 20, 60, 120 and ∞ . Power is found for a given λ using interpolation as

$$Power = Power_L + \frac{\frac{1}{v_L} - \frac{1}{v}}{\frac{1}{v_L} - \frac{1}{v_u}} (Power_U - Power_L)$$

v_L and v_u are the lower and upper value of v and, $Power_L$ and $Power_u$ are their respective values.

Example

For the above example, the power of the multiple regression model at $\alpha= 0.05$ is calculated as follows: In Cohen's table for $\alpha=0.05$ (the above table), at block $u=3$, for $v = 20$, power at $\lambda=10$ is 0.67 and at $\lambda=12$ is 0.75, linear interpolation finds the power at $v=20$ for $\lambda=11.57$ to be 0.73. Similarly interpolation at $v=60$ between $\lambda=10$ (power =0.73) and 12 (power=0.81) finds the power for $\lambda =11.57$ to be 0.79. Finally interpolation between power 0.73 (for $v=20$) and 0.79 (for $v=60$) using the above formula gives

$$Power = 0.73 + \frac{\frac{1}{20} - \frac{1}{23}}{\frac{1}{20} - \frac{1}{60}} (0.79 - 0.73) = 0.75$$

2.9 Power calculation for contingency tables

A contingency table is used for detecting association or independence between two or more traits. The power for a contingency table can be calculated using the following equation (Milligan 1979)

$$Z_{\beta} = - \frac{\left(\frac{\chi_c^2}{v+\lambda}\right)^{\frac{1}{3}} - \left[1 - \frac{2(v+2\lambda)}{9(v+\lambda)^2}\right]}{\sqrt{\frac{2(v+2\lambda)}{9(v+\lambda)^2}}}$$

The power is calculated based on Z_{β} value from Z- table. v is degrees of freedom and is calculated for a contingency table with r rows and c columns as $v = (r - 1)(k - 1)$. χ_c^2 is the critical value of the chi-square obtained from chi-square distribution table based on a specified significance level and degrees of freedom (v). λ is the noncentrality parameter (calculated χ^2). The effect size is defined as $w = \sqrt{\chi^2/N}$, where N is the total sample size.

Example

Suppose the chi-square calculated from the data of a 2×4 table is 6. Then, the power at $\alpha=0.01$ is calculated as follows

$$\lambda = 6$$

$$v = (2-1)(4-1)=3, \text{ table } \chi_c^2 \text{ for } \alpha=0.01 \text{ and } df = 3 \text{ is } 11.3,$$

$$Z_{\beta} = - \frac{\left(\frac{11.3}{3+6}\right)^{\frac{1}{3}} - \left[1 - \frac{2(3+2(6))}{9(3+6)^2}\right]}{\sqrt{\frac{2(3+2(6))}{9(3+6)^2}}} = -0.59$$

The power obtained for $Z_{\beta} = -0.59$ from Z-table is 0.277.

For contingency tables, the power can also be calculated as a function of sample size (N), effect size (w), degrees of freedom (v) and significance level (α) using Cohen's power tables (Cohen, 1988, p.228-248). A part of the Cohen's table for calculating the power of χ^2 at $\alpha=0.01$ and $v = 3$ is presented below (Fig. 2).

Example

Suppose the chi-square calculated from the data of a 2×4 table is 6 based on a sample of size (N) =30. Then, the power at $\alpha=0.01$ is calculated as follows

$w = \sqrt{(6/30)} = 0.447$, $v = (2-1)(4-1)=3$. According to the Cohen's table at $\alpha=0.01$ and $v = 3$ (the above table), at row $N=30$, we find power for column $w=0.4$ to be 0.22 and for $w=0.5$ to be 0.38. Linear Interpolation yields the power

$$0.22 + \frac{0.447 - 0.4}{0.5 - 0.4} (0.38 - 0.22) = 0.29$$

N	w								
	.10	.20	.30	.40	.50	.60	.70	.80	.90
25	01	03	08	16	30	48	66	82	92
30	02	04	10	22	38	59	77	90	96
35	02	05	12	26	46	68	85	95	99
40	02	05	14	31	54	76	91	97	99
45	02	06	17	36	61	82	94	99	"
50	02	07	19	42	68	87	97	99	
60	02	08	25	52	78	94	99	"	
70	02	10	31	61	86	97	*		
80	03	12	36	69	91	99			
90	03	14	42	76	95	99			
100	03	16	48	82	97	*			
120	04	22	59	90	99				
140	05	26	68	95	"				
160	05	31	76	97					
180	06	36	82	99					
200	07	42	87	99					
250	09	54	95	"					
300	11	65	98						
350	14	74	99						
400	16	82	"						
500	22	91							
600	29	96							
700	35	98							
800	42	99							
900	48	*							
1000	54								

Fig. 2 Power of χ^2 test at $\alpha=0.01$ and $\nu =3$ (Cohen, 1988).

2.10 Power calculation for 2x2 contingency table

A 2x2 contingency table is used for detecting association or independence between two traits. The power for a 2x2 contingency table can be calculated using the formula in the previous section. For a 2x2 table, the degrees of freedom is 1 and the critical chi square value at $\alpha=0.05$ is 3.84 and at $\alpha=0.01$ is 6.63. The statistical power for a 2x2 contingency table can also be calculated using the following equation:

$$Z_\beta = (\sqrt{N} \times w) - Z_{\alpha/2}$$

The power is calculated based on Z_β value from Z- table. $Z_{\alpha/2}$ is standard normal variate for significance level ($Z_{\alpha/2}=1.96$ at $\alpha =5\%$ and $Z_{\alpha/2}= 2.576$ at $\alpha =1\%$). N is sample size, w is the effect size and is calculated as $w = \sqrt{\frac{\chi^2}{N}}$, where χ^2 is the chi - square value calculated using the data of 2x2 table.

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i}$$

o_i and e_i are the observed and expected value for cell i respectively.

Example

Suppose the objective is to determine the association between two plant species. The data were obtained using located quadrats and summarized in a 2x2 table as follows

		Species B		
		Present	Absent	
Species A	Present	14	2	16
	Absent	4	10	14
		18	12	N=30

14 quadrats (cell a) contain both species A and B. In 2 quadrats (cell b) species A is present but not B. In 4

quadrats (cell *c*) species B is present but not A. In 10 quadrats (cell *d*) neither species A nor B are found. The expected value for each cell of the table is calculated as follows

$$E(a) = \frac{16 \times 18}{30} = 9.6$$

$$E(b) = \frac{16 \times 12}{30} = 6.4$$

$$E(c) = \frac{14 \times 18}{30} = 8.4$$

$$E(d) = \frac{14 \times 12}{30} = 5.6$$

then

$$\chi^2 = \frac{(14 - 9.6)^2}{9.6} + \frac{(2 - 6.4)^2}{6.4} + \frac{(4 - 8.4)^2}{8.4} + \frac{(10 - 5.6)^2}{5.6} = 10.8$$

$$w = \sqrt{\frac{10.8}{30}} = 0.6$$

The power at 0.05 significance level is $Z_\beta = (\sqrt{30} \times 0.6) - 1.96 = 1.326$

The power obtained for $Z_\beta = 1.326$ from Z-table is 0.907

2.11 Power calculation for analysis of variance (ANOVA)

a) k groups with equal sizes

ANOVA is used to test whether there is a significant difference between the means of two or more groups. The statistical power for ANOVA can be calculated using the following equation (Laubscher 1960; Cohen 1988).

$$Z_\beta = \frac{\sqrt{2(u + \lambda) - \frac{u+2\lambda}{u+\lambda}} - \sqrt{(2v - 1) \frac{u F_c}{v}}}{\sqrt{\frac{u F_c}{v} + \frac{u+2\lambda}{u+\lambda}}}$$

The power is calculated based on Z_β value from Z- table. u is numerator degrees of freedom ($u = k - 1$) and k is the number of groups, λ is the noncentrality parameter and is calculated as $\lambda = f^2 n(u + 1) = f^2 N$.

n is the average sample size per group ($n = (N/k)$ and N is the total sample size, v is the denominator (error) degrees of freedom ($v = N - k$), F_c is the critical F -value obtained from F-distribution table using the given significance level (α), numerator degrees of freedom (u) and denominator (error) degrees of freedom (v). The

effect size is defined as $f = \frac{\delta_m}{\delta}$, where δ_m is the standard deviation of means calculated as $\delta_m =$

$\sqrt{\frac{\sum_{i=1}^k (m_i - m)^2}{k}}$. In this equation, m_i is the mean of group i , m is the mean of the means of the groups with equal sizes, k is the number of groups and δ is the within-population standard deviation and is calculated as

$\delta = \sqrt{\frac{\sum \delta_i^2}{k}}$, where δ_i is the standard deviation for each group.

Example

Assume a researcher wishes to compare the estimates of density from three methods ($k=3$) obtained with four replications (the following table) using F-test. Then the power at 5% significance level is calculated as follows

Method			
	1	2	3
	15	17	17
	16	18	17
	15	15	15
	12	14	19
m_i	14.5	16	17
δ_i^2	2.25	2.5	2

$$m = \frac{14.5 + 16 + 17}{3} = 15.8$$

$$\delta_m = \sqrt{\frac{(14.5 - 15.8)^2 + (16 - 15.8)^2 + (17 - 15.8)^2}{3}} = 1.027$$

$$\delta = \sqrt{\frac{2.25 + 2.5 + 2}{3}} = 1.50$$

$$f = \frac{1.027}{1.50} = 0.684$$

$$n=12/3=4, u=3-1=2,$$

$$\lambda = 0.684^2 \times 4 \times (2 + 1) = 5.614$$

The critical F -value at $\alpha=0.05$, numerator $df(u)=3-1=2$, and error $df(v)=12-3=9$ is 4.26 from F-table.

$$Z_\beta = \frac{\sqrt{2(2 + 5.614) - \frac{2+2(5.614)}{2+(5.614)}} - \sqrt{(2(9) - 1) \frac{2(4.26)}{9}}}{\sqrt{\frac{2(4.26)}{9} + \frac{2+2(5.614)}{2+5.614}}} = -0.21$$

The power obtained for $Z_\beta = -0.21$ from Z-table is 0.42.

The statistical power for ANOVA can also be obtained using Cohen's power tables (Cohen, 1988, p. 289-354).

A part of the Cohen's table for obtaining sample size for ANOVA at $\alpha=0.05$ and $u=2$ is presented below (Fig.3).

n	F _c	f											
		.05	.10	.15	.20	.25	.30	.35	.40	.50	.60	.70	.80
2	9.552	05	05	06	06	07	07	08	08	10	12	15	18
3	5.143	05	05	06	07	08	09	10	12	17	22	29	37
4	4.256	05	06	06	08	09	11	14	17	24	33	44	54
5	3.885	05	06	07	09	11	14	17	22	32	44	56	69
6	3.682	05	06	07	10	13	16	21	26	39	53	67	79
7	3.555	05	06	08	11	14	19	25	31	46	62	76	87
8	3.467	05	06	08	12	16	22	28	36	53	69	83	92
9	3.403	05	07	09	13	18	24	32	40	59	75	88	95
10	3.354	05	07	10	14	20	27	35	45	64	81	91	97
11	3.316	05	07	10	15	21	30	39	49	69	85	94	98
12	3.285	06	07	11	16	23	32	42	53	74	88	96	99
13	3.260	06	08	11	17	25	35	46	57	77	91	97	99
14	3.238	06	08	12	18	27	38	49	61	81	93	98	*
15	3.220	06	08	13	20	29	40	52	64	84	95	99	
16	3.205	06	08	13	21	31	43	55	67	86	96	99	
17	3.191	06	09	14	22	33	45	58	70	89	97	99	
18	3.179	06	09	14	23	34	48	61	73	90	98	*	
19	3.168	06	09	15	24	36	50	64	76	92	99		

Fig. 3 Power of F test at $\alpha=0.05$ and $u=2$ (Cohen, 1988).**Example**

For the above example the power at 5% significance level is calculated as follows:

In Cohen's table for $\alpha=0.05$ and $u=2$ (the above table), at row=4 ($n=N/k=12/3=4$) the power for column $f=0.6$ is 0.33 and for $f=0.7$ is 0.44. Then the power for $f=0.684$ is obtained using interpolation

$$0.33 + \frac{0.684 - 0.6}{0.7 - 0.6} (0.44 - 0.33) = 0.429$$

b) k groups with unequal sizes

When the compared groups are of unequal sizes, m , δ_m and δ are calculated as follows

$$m = \frac{\sum n_i m_i}{n_1 + n_2 + \dots + n_k}$$

n_i is the size of group i .

$$\delta_m = \sqrt{\frac{\sum n_i (m_i - m)^2}{n_1 + n_2 + \dots + n_k}}$$

$$\delta = \sqrt{\frac{\sum n_i \delta_i^2}{n_1 + n_2 + \dots + n_k}}$$

Suppose the objective is to compare the estimates of biomass from three methods with unequal replications (the table below). Then the power at 5% significance level is calculated as follows

Method			
	A	B	C
	12	14	19
	14	18	17
	16	15	15
		15	19
		17	15
		17	17
m_i	14	16	17
δ_i^2	2.67	2	2.67
n_i	3	6	6

$$m = \frac{3(14) + 6(16) + 6(17)}{3 + 6 + 6} = 16$$

$$\delta_m = \sqrt{\frac{3(14 - 16)^2 + 6(16 - 16)^2 + 6(17 - 16)^2}{15}} = 1.095$$

$$\delta = \sqrt{\frac{3(2.67) + 6(2) + 6(2.67)}{15}} = 1.55$$

$$f = \frac{1.095}{1.55} = 0.707$$

$$n = 15/3 = 5, u = 3 - 1 = 2, v = 15 - 3 = 12,$$

$$\lambda = 0.707^2 \times 5 \times (2 + 1) = 7.497$$

The critical F -value at $\alpha = 0.05$, numerator $df(u) = 3 - 1 = 2$, and error $df(v) = 15 - 3 = 12$ is 3.89 from F -table.

$$Z_\beta = \frac{\sqrt{2(2 + 7.497) - \frac{2 + 2(7.497)}{2 + 7.497}} - \sqrt{(2(12) - 1) \frac{2(3.89)}{12}}}{\sqrt{\frac{2(3.89)}{12} + \frac{2 + 2(7.497)}{2 + 7.497}}} = 0.182$$

The power obtained for $Z_\beta 0.182$ from Z -table is 0.571.

The statistical power for ANOVA can also be obtained using Cohen's tables (Cohen 1988, p. 289-354).

The average sample size per group (n) is $15/3 = 5$. In Cohen's table for $\alpha = 0.05$ and $u = 2$ (the above table), at row = 5, the power for column $f = 0.7$ is 0.56 and for $f = 0.8$ is 0.69. Then the power for $f = 0.707$ is obtained using interpolation

$$0.56 + \frac{0.707 - 0.7}{0.8 - 0.7} (0.69 - 0.56) = 0.573$$

2.12 Power calculation for simple logistic regression model

A simple (univariate) logistic regression model describes the relationship between a binary response variable ($Y = 1$ and $Y = 0$) and an independent variable such as climatic, soil or topographic factors. The following

formula is used to calculate power for a simple logistic regression model (Hsieh, 1989).

$$Z_{\beta} = \sqrt{N P_1(1 - P_1)\beta_*^2} - Z_{\alpha/2}$$

The power is obtained from z-table using the value of Z_{β} . $Z_{\alpha/2}$ is the normal standard variate for significance level, $Z_{\alpha/2}=1.96$ at 5% and 2.576 at 1% significance level, β_* is the tested effect size calculated as $\beta_* = \ln(\text{ODDS ratio}) = \ln\left[\frac{\left(\frac{p_2}{1-p_2}\right)}{\left(\frac{p_1}{1-p_1}\right)}\right]$ and p_1 and p_2 are respectively the probability of the event at the mean of x , (\bar{x}) and one standard deviation (s) above the mean ($\bar{x} + s$). If the data of independent variable are not standardized, then

$$p_1 = \frac{e^{\beta_0 + \beta_1(\bar{x})}}{1 + e^{\beta_0 + \beta_1(\bar{x})}}$$

and

$$p_2 = \frac{e^{\beta_0 + \beta_1(\bar{x} + s)}}{1 + e^{\beta_0 + \beta_1(\bar{x} + s)}}$$

If the data of independent variable (x) is standardized using $\frac{x - \bar{x}}{s}$, then the mean of standardized values will be zero and the standard deviation will be 1. In this case

$$p_1 = \frac{e^{\beta_0}}{1 + e^{\beta_0}}$$

and

$$p_2 = \frac{e^{\beta_0 + \beta_1}}{1 + e^{\beta_0 + \beta_1}}$$

Example

Suppose a researcher wishes to determine the probability of presence of a plant species based on annual mean temperature using a simple logistic regression. The data of presence ($y = 1$) and absence ($y = 0$) of the plant species and the annual temperature (x) was obtained from 60 sampling plots, and a logistic regression was made. If constant (β_0) of the model is -3.72, coefficient of regression (β_1) is 0.238, the mean of temperature (\bar{x}) in the sampling plots is 15.72 and the standard deviation of temperature is 3.55, what is the power of the regression model at 0.05 significance level?

Since the data of x are not standardized,

$$p_1 = \frac{e^{-3.72 + 0.238(15.72)}}{1 + e^{-3.72 + 0.238(15.72)}} = \frac{e^{0.021}}{1 + e^{0.021}} = 0.505$$

and

$$p_2 = \frac{e^{-3.72 + 0.238(15.72 + 3.55)}}{1 + e^{-3.72 + 0.238(15.72 + 3.55)}} = \frac{e^{0.866}}{1 + e^{0.866}} = 0.704$$

$$\beta_* = \ln\left[\frac{\left(\frac{0.704}{1-0.704}\right)}{\left(\frac{0.505}{1-0.505}\right)}\right] = 0.845$$

then the power is $Z_\beta = \sqrt{60 \times 0.505(1 - 0.505) 0.845^2} - 1.96 = 1.312$

The power for $Z_\beta = 1.312$ obtained from z-table is 0.905. If the data of temperature are standardized to have zero mean and unit standard deviation, the following parameters are obtained for the logistic regression model.

$$\beta_0=0.021, \beta_1=0.845$$

then

$$p_1 = \frac{e^{0.021}}{1+e^{0.021}} = 0.505$$

$$p_2 = \frac{e^{0.021+0.845}}{1 + e^{0.021+0.845}} = \frac{e^{0.866}}{1 + e^{0.866}} = 0.704$$

$$\beta_* = \ln \left[\frac{\left(\frac{0.704}{1-0.704} \right)}{\left(\frac{0.505}{1-0.505} \right)} \right] = 0.845.$$

Therefore when values are standardized, $\beta_1 = \ln(\text{ODDS ratio}) = \beta_*$

The power is calculated as before.

2.13 Power calculation for multiple logistic regression model

The relationship between a binary variable ($y = 1, y = 0$) and a set of variables (x_1, x_2, \dots, x_p) can be modeled using a multiple logistic regression. The power for multiple logistic regression model can be obtained using the following formula (Hsieh, 1989).

$$Z_\beta = \sqrt{N P_1(1 - P_1) \beta_*^2(1 - \rho_{1,23\dots P})} - Z_{\alpha/2}$$

The power is obtained from z-table using the value of Z_β .

N is the sample size, $Z_{\alpha/2}$, β_* and p_1 were explained for a simple logistic regression in the previous section. p_1 and β_* are calculated for the logistic regression using one predictor in the model. $\rho_{1,23\dots P}$ also known as R^2 is the squared multiple correlation coefficient and is equal to the proportion of the variance of x_1 explained by the regression relationship with x_2, \dots, x_p (Hsieh, 1989; Hsieh et al., 1998).

Example

Suppose the data of the occurrence of a plant species and the annual temperature and precipitation were obtained from 68 sampling units, and a simple logistic regression was fitted between the occurrence of the plant species and the annual temperature. If the standardized coefficient of the simple logistic model based on temperature is $\beta_0=0.038$ and $\beta_1=0.872$, and the squared multiple coefficient of correlation (R^2) between the temperature and precipitation is 0.10. What is power of the model for $\alpha = 0.05$ ($Z_{\alpha/2} = 1.96$) for detecting the effect of temperature while controlling for the effects of precipitation?

$$p_1 = 0.509$$

$$Z_\beta = \sqrt{68 \times 0.509(1 - 0.509) \times 0.872^2(1 - 0.1)} - 1.96 = 1.45$$

The power for $Z_\beta = 1.45$ is 0.926 from z-table.

We can also determine the power for a simple logistic regression model with precipitation and approximate it

for a multiple case. If the standardized coefficients of the simple logistic model based on only precipitation are $\beta_0 = -0.004$ and $\beta_1 = -0.762$, and $R^2 = 0.1$, the power is

$$p_1 = 0.499$$

$$Z_\beta = \sqrt{68 \times 0.499(1 - 0.499) \times (-0.762)^2(1 - 0.1)} - 1.96 = 1.02$$

The power for $Z_\beta = 1.02$ is 0.846 from z-table. Generally, sample size for a multiple logistic regression is calculated by specifying the probability of an event at the mean value of all the covariates (predictors), and the odds ratio of the event corresponding to an increase of one standard deviation from the mean value of the specific covariate, given the mean values of the remaining covariates (Hsieh, 1989). Therefore the power for a multiple logistic regression can also be calculated without using squared multiple coefficient of correlation (R^2) as explained in the following examples.

Example

Suppose a multiple logistic regression model was made based on the presence/absence data of a plant species and two predictors including annual temperature (x_1) and precipitation (x_2) that were used in the previous example. What is the power of the multiple logistic regression model based on the following specifications at $\alpha = 0.05$?

$$N=68$$

$$\text{mean of temperature } (\bar{x}_1) = 15.744$$

$$\text{standard deviation of temperature } (s_1) = 5.06$$

$$\text{mean of precipitation } (\bar{x}_2) = 191.76$$

$$\text{standard deviation of precipitation } (s_2) = 58.62$$

unstandardized coefficients of the multiple logistic regression model:

$$\beta_0 = -0.169, \quad \beta_1 \text{ (for temperature)} = 0.162, \quad \beta_2 \text{ (for precipitation)} = -0.0123$$

The power of the model for determining the effect of temperature while controlling for the effects of precipitation is calculated as follows:

$$p_1 = \frac{e^{-0.169+0.162(15.744)-0.0123(191.76)}}{1 + e^{-0.169+0.162(15.744)-0.0123(191.76)}} = \frac{e^{0.023}}{1 + e^{0.023}} = 0.506$$

and

$$p_2 = \frac{e^{-0.169+0.162(15.744+5.06)-0.0123(191.76)}}{1 + e^{-0.169+0.162(15.744+5.06)-0.0123(191.76)}} = \frac{e^{0.843}}{1 + e^{0.843}} = 0.699$$

$$\beta_* = \ln \left[\frac{\left(\frac{0.699}{1-0.699} \right)}{\left(\frac{0.506}{1-0.506} \right)} \right] = 0.82$$

$$Z_\beta = \sqrt{68 \times 0.506(1 - 0.506) 0.82^2} - 1.96 = 1.42$$

The power for $Z_\beta = 1.42$ obtained from z-table is 0.922. The power of the model for detecting the effect of precipitation while controlling for the effects of temperature is calculated as

$$p_1 = 0.506$$

and

$$p_2 = \frac{e^{-0.169+0.162(15.744)-0.0123(191.76+58.62)}}{1 + e^{-0.169+0.162(15.744)-0.0123(191.76+58.62)}} = \frac{e^{-0.7}}{1 + e^{-0.7}} = 0.332$$

$$\beta_* = \ln \left[\frac{\left(\frac{0.332}{1-0.332} \right)}{\left(\frac{0.506}{1-0.506} \right)} \right] = -0.723$$

$$Z_\beta = \sqrt{68 \times 0.506(1 - 0.506) (-0.723)^2} - 1.96 = 1.02$$

The power for $Z_\beta = 1.02$ obtained from z-table is 0.846. The powers calculated in this way are close to those calculated in the previous section. If the data are standardized using $\frac{x-\bar{x}}{s}$ to have mean = 0 and variance = 1,

the standardized coefficients of the multiple logistic regression model will be as follows:

$$\beta_0 = 0.023, \quad \beta_1 \text{ (for temperature)} = 0.82, \quad \beta_2 \text{ (for precipitation)} = -0.723$$

In this case $p_1 = 0.506$ using $\beta_0 = 0.023$, and for calculating the power based on the effect of temperature $\beta_* = \beta_1$ and based on the effect of precipitation $\beta_* = \beta_2$.

3 Results and Discussion

The statistical power in vegetation studies is reported to be 39.8 ± 10.2 (mean \pm SE) for detecting a medium effect size (Jennions and Moller, 2003). This is far lower than the general recommendation of a power of 80% (Cohen, 1988). Most ecologists still report non-significant results without indicating the statistical power of the test. A non-significant result in a study may be due to a low power (inadequate sample size) and not because there is no significant effect or difference (Steidl et al., 1997; Jennions and Moller, 2003). Therefore the non-significant outcome may be inconclusive in a study designed with a low power. An appropriate power can be achieved for a study by selecting an adequate sample size and minimum detectable effect size before data collection. The effect size can also be obtained using a preliminary sampling based on a small sample. However most researchers do not take into consideration the power of research in design stage and calculate it after data collection and obtaining results. One solution to improve the power of a test after analysis is to use a one-tailed hypothesis test if possible. One-tailed (directional) test has more statistical power than two-tailed test to detect an effect and is recommended to be used when the effect in one direction can be explained (Ruxton and Neuhauser, 2010). For example a one-tailed test can be used to test the hypothesis that plant density and biomass is greater in ungrazed stands than in grazed stands or percentage of germinated seeds is higher under gibberellin treatment than control condition or plant production increases with increasing precipitation. Another solution for increasing the power of a test is to select a higher significance level (e.g., 5% instead of 1%). 5% significance level is an acceptable and reasonable significance level for accepting/rejecting a null hypothesis and also provides a higher power than significance levels of less than 5%. The power also depends on the statistical method itself. For example, the power of paired sample t-test and one sample t-test is higher than that of independent two sample t-test (Jennions and Moller, 2003). It must be noted that the decision for selection of a powerful statistical test must be made at the sampling design stage and before data collection.

The statistical methods presented in this paper assume a normal distribution of vegetation data (except the Chi-square test which is a non-parametric statistic also called a distribution free test). Although some studies have indicated that vegetation characteristics such as biomass, density, production, plant height and seed production are normally distributed (Naylor 1980; Cleary and Priadjati, 2005; Rosner and Rose, 2006; Hoch and Korner, 2009; Giannini et al., 2018). Some others have revealed that distribution of these characteristics is not normal (Windle and Franz, 1979; Visscher et al., 2006; Millington et al., 2011; Mascaro and Schnitzer, 2011; Murphy et al., 2018; Bisi et al., 2018). Therefore it is recommended to test the distribution of vegetation data before selection of a statistical test using Kolmogorov–Smirnov test or Shapiro-Wilk test. If the data are

not normally distributed, transformation methods such as log, square root, and angular transformation can be used to normalize the distribution of data (Bonham, 1989). However non-parametric tests may present more efficient results if data are not normally distributed. Non-parametric tests make no assumptions about distribution of data. Some non-parametric tests include sign test and Wilcoxon signed rank test for paired samples and one sample case, Mann–Whitney U test for two independent samples, Kruskal–Wallis test for one-way ANOVA, Spearman rank coefficient for correlation and nonparametric regression methods. It must be noted that non-parametric tests are usually less powerful than parametric tests and require a larger sample size to achieve the same level of power as a parametric test. The power analysis for non-parametric tests is beyond the scope of this paper but can be done using G*Power software. In conclusion, power analysis is more valuable in the design or planning stages of research than after data collection. However if power is calculated after data collection and testing the hypothesis, presenting confidence interval for an observed effect size, significance level and sample size along with the estimated power can provide additional information about the null hypothesis that are not rejected.

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