Article

Impact of human activities on forest resources and wildlife population

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Abstract
Nowadays, management and regulation of natural resources like agriculture, fisheries, forestry and wildlife is one of the popular topics in research. The evolution of humankind is largely dependent on the quality of the environment and the resources it provides; but numerous human-induced factors, and climate change may drastically change the conditions of human sustainability. A wide range of human activities on forestland contribute to climatic change, prominent among these are, deforestation, desertification, industrialization, urbanization and other socio-economic activities. In this paper, attempts have been made to trace the causes and consequences of these human activities on the depletion of forestry resources. A nonlinear mathematical model is proposed and analyzed. In modeling process, we assume that the growth rate of wildlife population wholly depend on forestry biomass. It is depleted by human activities. Local and global stability analysis of the mathematical model along with the persistence of the system is checked using theory of nonlinear ordinary differential equations. Analytical results obtained are justified numerically through numerical simulation. Important parameters are investigated and variation of variables with change in these parameters is determined.

Keywords forestry biomass; depletion; human population; human activities; stability; persistence.

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1 Introduction

Human activities often referred to as land-use, such as forest exploitation, mining, tourism, hunting, fishing, agriculture, have concluded in the loss of forest resources, such as deforestation and forest degeneration (Zhang, 2007). These activities have accumulated the process of exploitation of forests to meet human needs and its resources which has resulted in the reduction of global forest area and substantial reduction in carbon storage area. It has been estimated that 20% of forest area has reduced because of the activities mentioned above; further reports have indicated that many plants and animal species are currently endangered (Maelle and Oghenerobor, 2012; Ghoddousi et al., 2021; Thom et al., 2017; Anand and Radhakrishna, 2017). Virtually all human activities can impact wildlife populations either positively or negatively. Those activities that are likely to have hostile effects can be divided into those that function primarily by altering the physical environment in a relatively permanent way and those that cause changes to an animal’s behavior. Activities that shift the physical environment change the amount or the suitability of habitat for a species. Widespread and large-scale examples include activities that directly change the structure and composition of the landscape, such agriculture, forestry, livestock grazing, and unregulated off-road vehicle use. Perhaps less obvious in their ecological impacts are those non-consumptive human activities that do not appreciably change the physical environment but nonetheless can affect wildlife adversely. Examples consist of recreational activities such as hiking, wildlife viewing, and boating—all common activities for visitors in parks. (Steidl et al, 2006). Although there are human activities that affect the population of wildlife spices, such as construction of building and roads, or vegetation destruction resulting from overuse of particular areas, most wildlife-related impacts away from these areas likely result from limited recreational pursuits of visitors. We focus the remainder of our discussion on these types of activities.

Forests provide habitats for wildlife species and help remove dangerous greenhouse gases from the atmosphere to reduce global warming. It is not surprising, then, that deforestation of forest resources can have serious consequence on biodiversity and survival of wildlife species. Repeatedly, scientists have shown that deforestation can spell doom for the species that rely upon forest habitats, for example, many type of birds, various animal species,the Eurasian red squirrel in UK forests (Trivedi, 2001). Another example of Singapore where 95% of the forest area has been lost since the British colonization in 1819 for human use and a comparison of historical and modern checklists of species indicates that 26% of plant species, 34% of bird species, and 42% of mammal species that were originally recorded on this island are now extinct (Brook et al., 2003). The depletion of forest biomass by human population and industrialization has been investigated both theoretically as well as experimentally by many researchers (Dubey and Narayanan, 2010; Agarwal and Devi, 2012; Chaudhary et al., 2013). Agarwal and Pathak (2015) have proposed and investigated a mathematical model for conservation of forestry biomass and wildlife population. In this paper, they have assumed that the growth rate of wildlife conservation is proportional to the depletion of forestry biomass by considering the affect of illegal trade in forestry biomass and wildlife species. Pathak et al. (2017) have proposed and investigated a mathematical model and shown the impact of population and industrialization on the forestry biomass and wildlife species. In the present paper, we have proposed a model to study the affect of human activities on forest resources and related disturbance on the growth rate of wildlife population living in the forest.

2 Mathematical Model

Let F be the cumulative biomass density of forest resources, W be the density of wildlife species, N be the density of human population and H represents density of human activities. It is assumed that cumulative biomass density follows logistic model with intrinsic growth rate s with carrying capacity L, and s_0 be the competion coefficent.It is assumed further that the growth rate of biomass density by the density of wildlife population,human population and human activities with rates α, β and γ respectively. Keeping in view of these considerations, the non-linear model is proposed as follows

$$\frac{dF}{dt} = sF - s_0 \frac{F^2}{L} - \alpha FW - \beta FN - \gamma F^2 H,$$
$$\frac{dW}{dt} = \theta_1 \alpha FW - \delta_1 W - \delta_0 W^2 - \nu_1 NW - \nu_2 HW,$$
$$\frac{dN}{dt} = rN - r_0 \frac{N^2}{K} + \theta_2 \beta FN - \sigma NW,$$
$$\frac{dH}{dt} = \lambda N + \theta_3 \gamma F^2 H - \mu_0 H.$$

(1)

where $F(0) \geq 0, W(0) \geq 0, N(0) \geq 0, H(0) \geq 0.$

In model (1) $s, s_0, \alpha, \beta, \gamma, \theta_1, \delta_1, \delta_0, \nu_1, \nu_2, r, r_0, \theta_2, \sigma, \lambda, \theta_3$ and μ_0 are positive constants. The growth rate of wildlife depends on the commutative density of biomass; it is represented by $\theta_1 \alpha FW$, where θ_1 is growth rate coefficient for wildlife species. The constants δ_0, ν_1, ν_2 and δ_1 are depletion coefficient of wildlife species due to intra specific interaction, human population, human activities and natural death respectively. The density of human population follows logistic model with intrinsic growth rate r with carrying capacity K, and r_0 be the competion coefficent. The constants θ_2 is coefficient of growth rate for human population due to forestry biomass and the constant σ its depletion rate due to interaction between human population and wildlife species (For example, A 65-year-old man was killed in a tiger attack near Katarniaghat range of the Dudhwa Tiger Reserve (DTR) on 19 April 2020, Hindustan Times, Bareilly). The constants λ, θ_3 and μ_0 are growth rate and depletion rate of human activities which have negative affect on the environment respectively.

3 Boundedness of the System

In the following lemma, we state the bounds of the various variables which would be needed in our study. The set $\Omega = \{(F, W, N, H) : 0 \leq F \leq F_m, 0 \leq W \leq W_m, 0 \leq N \leq N_m \, and \, 0 \leq H \leq H_m\}$, is the region of attraction for all solutions initiating in the interior of the positive octant,

where $F_m = \frac{sL}{s_0}, W_m = \frac{\theta_1 \alpha F_m}{\delta_0}, N_m = \frac{(r + \theta_2 \beta F_m)k}{r_0} \; and \; H_m = \frac{\lambda N_m}{\mu_0 - \theta_3 \gamma F_m^2},$ with condition $\mu_0 > \theta_3 \gamma F_m^2.$

Proof. Let (F, W, N, H) be solution with positive initial values (F_0, W_0, N_0, H_0). From system (1), we get

$$\frac{dF}{dt} \leq sF - \frac{s_0 F^2}{L}.$$

(2)

According to comparison principle, it follows that

$$F_m = \frac{sL}{s_0}.$$

(3)

Now from the system (1), we get

$$\frac{dW}{dt} \le \theta_1 \alpha F_m W - \delta_0 W^2,$$

(4)

According to comparison principle again, we get

$$W_m = \frac{\theta_1 \alpha F_m}{\delta_0}.$$

(5)

Again from the system (1),we have

$$\frac{dN}{dt} \le \left(r + \theta_2 \beta F_m\right) N - \frac{r_0}{k} N^2.$$

(6)

According to comparison principle again, we get

$$N_m = \frac{\left(r + \theta_2 \beta F_m\right) K}{r_0}$$

(7)

From the system (1), we have

$$\frac{dH}{dt} \le \lambda N_m - \left(\mu_0 - \theta_3 \gamma F_m^2\right) H.$$

(8)

According to comparison principle again, we get

$$H_m = \frac{\lambda N_m}{\mu_0 - \theta_3 \gamma F_m^2}.$$

(9)

With condition $\mu_0 > \theta_3 \gamma F_m^2$.

This completes the proof of lemma.

4 Equilibrium Analysis

The system (21) has eight nonnegative equilibrium points namely; $E_0(0,0,0,0)$, $E_1\left(\frac{sL}{s_0},0,0,0\right)$, $E_2\left(0,0,\frac{rK}{r_0},\frac{\lambda r K}{r_0 \mu_0}\right)$, $E_3\left(\overline{F},\overline{W},0,0\right)$, $E_4\left(\overline{\overline{F}},0,0,\overline{\overline{H}}\right)$ $E_5\left(\tilde{F},0,\tilde{N},\tilde{H}\right)$, $E_6\left(\hat{F},\hat{W},0,\hat{H}\right)$ and interior equilibrium point $E_7\left(F^*,W^*,N^*,H^*\right)$. The existence of equilibrium points $E_0(0,0,0,0)$,

$E_1\left(\frac{sL}{s_0},0,0,0\right)$ and $E_2\left(0,0,\frac{rK}{r_0},\frac{\lambda\, r\, K}{r_0\mu_0}\right)$ are obvious. We will show the existence of $E_3\left(\overline{F},\overline{W},0,0\right)$, $E_4\left(\overline{\overline{F}},0,0,\overline{\overline{H}}\right)$, $E_5\left(\tilde{F},0,\tilde{N},\tilde{H}\right)$, $E_6\left(\hat{F},\hat{W},0,\hat{H}\right)$ and interior equilibrium point $E_7\left(F^*,W^*,N^*,H^*\right)$ as follows:

Existence of $E_3\left(\overline{F},\overline{W},0,0\right)$,

Here $\overline{F}$ and $\overline{W}$ are the positive solutions of the following equations:

$$s-\frac{s_0\overline{F}}{L}-\alpha\overline{W}=0, \tag{10}$$

$$\theta_1\alpha\,\overline{F}-\delta_1-\delta_0\overline{W}=0. \tag{11}$$

From equations (10), and (11), we get

$$\overline{F}=\frac{(s\delta_0+\alpha\delta_1)L}{s_0\delta_0+\alpha^2L\theta_1},\overline{W}=\frac{\theta_1\alpha L(s\delta_0+\alpha\delta_1)-\delta_1\left(s_0\delta_0+\alpha^2\theta_1L\right)}{\delta_0\left(s_0\delta_0+\alpha^2\theta_1L\right)}. \tag{12}$$

The value of $\overline{W}$ is always positive if the following condition is satisfied

$$\theta_1\alpha L(s\delta_0+\alpha\delta_1)>\delta_1\left(s_0\delta_0+\alpha^2\theta_1L\right). \tag{13}$$

Existence of $E_4\left(\overline{\overline{F}},0,0,\overline{\overline{H}}\right)$

Here $\overline{\overline{F}}$ and $\overline{\overline{H}}$ are the positive solutions of the following equations:

$$s-\frac{s_0\overline{\overline{F}}}{L}-\gamma\,\overline{\overline{F}}\,\overline{\overline{H}}=0, \tag{14}$$

$$\theta_3\gamma\overline{\overline{F^2}}\,\overline{\overline{H}}-\mu_0\overline{\overline{H}}=0. \tag{15}$$

From the equations (14) and (15),we get the values of $\overline{\overline{F}}$ and $\overline{\overline{H}}$ as follows

$$\overline{\overline{F}}=\pm\sqrt{\frac{\mu_0}{\theta_3\,\gamma}},\overline{\overline{H}}=s\sqrt{\frac{\theta_3}{\gamma\,\mu_0}}-\frac{s_0}{L\gamma}. \tag{16}$$

Existence of $E_5\left(\tilde{F},0,\tilde{N},\tilde{H}\right)$

Here $\tilde{F}$, $\tilde{N}$ and $\tilde{H}$ are the positive solutions of the following equations:

$$s-\frac{s_0\tilde{F}}{L}-\beta\,\tilde{N}-\gamma\,\tilde{F}\,\tilde{H}=0, \tag{17}$$

$$r-\frac{r_0\tilde{N}}{K}+\theta_2\beta\,\tilde{F}=0, \tag{18}$$

$$\lambda\,\tilde{N}+\theta_3\gamma\,\tilde{F}^2\tilde{H}-\mu_0\tilde{H}=0. \tag{19}$$

From the equations (18) and (19),we get the values of $\tilde{N}$ and $\tilde{H}$ as follows

$$\tilde{N}=\frac{\left(r+\theta_2\beta\,\tilde{F}\right)K}{r_0},\tilde{H}=\frac{\lambda\,K\,r+\theta_2\beta\,\lambda\,K\,\tilde{F}}{r_0\left(\mu_0-\theta_3\gamma\,\tilde{F}^2\right)}. \tag{20}$$

Putting the values of $\tilde{N}$ and $\tilde{H}$ in the equation (17), we get following equation

$$P_1\left(\tilde{F}\right)=A_1\tilde{F}^3+A_2\tilde{F}^2+A_3\tilde{F}+A_4. \tag{21}$$

Where

$$A_1=\left(s_0r_0+\beta^2K\theta_2L\right)\theta_3\gamma, A_2=-\left\{\left(sr_0-\beta Kr\right)L\,\theta_3\gamma+\gamma\theta_2\beta\lambda LK\right\},$$

$$A_3=-\left\{\left(s_0r_0+\beta^2K\theta_2L\right)\mu_0+\gamma\lambda LKr\right\}, A_4=-\left(\beta Kr-sr_0\right)\mu_0L.$$

From (21), we have

$$P_1(0)=-\left(\beta\,K\,r-s\,r_0\right)\mu_0L<0. \tag{22}$$

$$P_1\left(L^*\right)=A_1L^{*3}+A_2L^{*2}+A_3L^*+A_4>0. \tag{23}$$

Thus there exists a $\tilde{F}$, $0<\tilde{F}<F_m$, such that $P_1(\tilde{F})=0$. Now, the sufficient condition for the uniqueness of E_5 is $P_1^{'}\left(\tilde{F}\right)>0$. For this we find $P_1^{'}\left(\tilde{F}\right)>0$ from (22) as follows.

$$P_1^{'}\left(\tilde{F}\right)=3A_1\tilde{F}^2+2A_2\tilde{F}+A_3>0.$$

(24)

This completes the existence of E_5.

Existence of $E_6\left(\widehat{F},\widehat{W},0,\widehat{H}\right)$

Here $\widehat{F}$, $\widehat{W}$ and $\widehat{H}$ are the positive solutions of the following equations:

$$s-\frac{s_0\widehat{F}}{L}-\alpha\widehat{W}-\gamma\widehat{F}\widehat{H}=0, \tag{25}$$

$$\theta_1\alpha\widehat{F}-\delta_1-\delta_0\widehat{W}-\nu_2\widehat{H}=0, \tag{26}$$

$$\theta_3\gamma\widehat{F}^2-\mu_0=0. \tag{27}$$

After solving equations (25), (26) and (27), we get

$$\widehat{F}=\sqrt{\frac{\mu_0}{\theta_3\gamma}},\quad \widehat{W}=\frac{\theta_1\alpha}{\delta_0}\sqrt{\frac{\mu_0}{\theta_3\gamma}}-\frac{\delta_1}{\delta_0}-\frac{\nu_2}{\delta_0}\widehat{H}, \tag{28}$$

$$\widehat{H}=\frac{\delta_0\left[\frac{s_0}{L}+\frac{\theta_1\alpha^2}{\delta_0}\right]\sqrt{\mu_0}-(\delta_0 s+\delta_1\alpha)\sqrt{\theta_3\gamma}}{\gamma_2\alpha\sqrt{\theta_3\gamma}-\gamma\sqrt{\mu_0}\delta_0}. \tag{29}$$

The value of $\widehat{H}$ is always positive if following conditions are satisfied

$$\delta_0\left[\frac{s_0}{L}+\frac{\theta_1\alpha^2}{\delta_0}\right]\sqrt{\mu_0}>(\delta_0 s+\delta_1\alpha)\sqrt{\theta_3\gamma}, \tag{30}$$

$$\nu_2\alpha\sqrt{\theta_3\gamma}>\gamma\sqrt{\mu_0}\delta_0. \tag{31}$$

This completes the existence of E_6.

Existence of $E_7\left(F^*,W^*,N^*,H^*\right)$

Here F^*,W^*,N^*,H^* are the positive solutions of the system of algebraic equations given below

$$s-\frac{s_0F^*}{L}-\alpha W^*-\beta N^*-\gamma F^*H^*=0,$$

(32)

$$\theta_1\alpha F^* - \delta_1 - \delta_0 W^* - \nu_1 N^* - \nu_2 H^* = 0,$$

(33)

$$r - \frac{r_0 N^*}{K} + \theta_2\beta F^* - \sigma W^* = 0,$$

(34)

$$\lambda N^* + \theta_3\gamma F^{*2} H^* - \mu_0 H^* = 0.$$

(35)

From equations (33), (34) and (35), we have following expressions

$$N^* = M_1 H^* - M_2 F^{*2} H^*, W^* = \frac{r}{\sigma} + M_3 H^* + M_4 F^* + M_5 F^{*2} H^*, H^* = \frac{M_9 - M_7 F^*}{M_8 F^{*2} - M_6}.$$

(36)

With condition $\sqrt{\frac{M_6}{M_8}} < F^* < \frac{M_9}{M_7}$.

Putting the values of N^*, W^* and H^* in the equation (33), we get following equation

$$P_2(F^*) = p_1 F^{*3} + p_2 F^{*2} + p_3 F^* + p_4 = 0.$$

(37)

Where

$$p_1 = \delta_0 M_8 + M_7^2, p_2 = M_6 M_8 - M_9 M_7 + M_8 M_7,$$

$$p_3 = M_7 M_9 - \delta_0 M_6 - M_8 M_9, p_4 = -\left(M_6^2 + M_9^2\right), M_1 = \frac{\mu_0}{\lambda}, M_2 = \frac{\theta_3\gamma}{\lambda},$$

$$M_3 = -\frac{r_0}{K\sigma} M_1, M_4 = -\frac{\theta_2\beta}{\sigma}, M_5 = \frac{r_0}{K\sigma} M_2, M_6 = \delta_0 M_3 + \nu_1 M_1 + \nu_2, M_7 = \theta_1\alpha - \delta_0 M_4,$$

$$M_8 = \nu_1 M_2 - \delta_0 M_5, M_9 = \delta_1 + \frac{\delta_0 r}{\sigma}.$$

From (37), we have

$$P_2(0) = -\left(M_6^2 + M_9^2\right) < 0.$$

(38)

$$P_2(L^*) = p_1 L^{*3} + p_2 L^{*2} + p_3 L^* + p_4 > 0.$$

(39)

Thus there exists a F^*, $0 < F^* < L^*$, such that $V(B^*) = 0$.

Now, the sufficient condition for the uniqueness of E_7 is $P_2^{'}(F^*)>0$.For this we find $P_2^{'}(F^*)>0$ from (37) as follows.

$$P_2^{'}(F^*)=3p_1F^{*2}+2p_2F^*+p_3>0. \quad (40)$$

This completes the existence of E_7.

4.1 Local stability

To discuss the local stability of system (1) as follows,

$$V(E)=\begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} \\ e_{21} & e_{22} & e_{23} & e_{24} \\ e_{31} & e_{32} & e_{33} & 0 \\ e_{41} & 0 & e_{43} & e_{44} \end{bmatrix}.$$

Where the entries in the matrix are

$$e_{11}=s-\frac{2s_0F}{L}-\alpha W-\beta N-2\gamma FH, e_{12}=-\beta F, e_{13}=-\beta F, e_{14}=-\gamma F^2, \quad (41)$$

$$e_{21}=\theta_1\alpha W, e_{22}=\theta_1\alpha F-2\delta_0W-\delta_1-\nu_1N-\nu_2H, e_{23}=-\nu_1W, e_{24}=-\nu_2W, e_{31}=\theta_2\beta N, \quad (42)$$

$$e_{32}=-\sigma N, e_{33}=r-\frac{2r_0N}{K}+\theta_2\beta F-\sigma W, e_{41}=2\theta_3\gamma FH, e_{43}=\lambda, e_{44}=\theta_3\gamma F^2-\mu_0. \quad (43)$$

Accordingly, the linear stability analysis about the equilibrium points $E_i, i=0,1,2,3,4,5,6,7$ gives the following results:

1.The equilibrium point E_0 is unstable manifold in $F-N$ plane.

2.The equilibrium point E_1 is unstable manifold in $W-N$ plane and equilibrium point E_2 is unstable manifold in $F-W-N$ plane

3.The equilibrium point E_3 is unstable manifold in $F-W-N$ plane and the equilibrium point E_4 is unstable manifold in $F-W$ plane.

4.The equilibrium point E_5 is unstable manifold in $W-$ direction and the equilibrium point E_6 is unstable manifold in $N-$ direction.

The stability behaviour of equilibrium point E_7 is not obvious. We first linearize model (2.1) by substituting

$$F=F^*+f, W=W^*+w, N=N^*+n, H=H^*+h.$$

Where f,w,n,h are small perturbations around equilibrium point E_7. We get following linearize system

$$\frac{df}{dt}=-\left(\frac{s_0F^*}{L}+\gamma\,F^*H^*\right)f-\alpha\,F^*w-\alpha\,F^*n-\gamma\,F^{*2}h,$$
$$\frac{dw}{dt}=\theta_1\alpha\,f\,W^*-\delta_0W^*w-\nu_1nW^*-\nu_2hW^*,$$
$$\frac{dn}{dt}=-\frac{r_0}{K}N^*n+\theta_2\beta\,f\,N^*-\sigma\,N^*w,$$
$$\frac{dh}{dt}=\lambda\,n+2\theta_3\gamma\,F^*H^*f+\theta_3\gamma\,F^{*2}h-\mu_0h. \tag{44}$$

Then we consider the following positive definite function:

$$U=\frac{1}{2}\left(f-f^*\right)^2+\frac{1}{2}\left(w-w^*\right)^2+\frac{1}{2}\left(n-n^*\right)^2+\frac{1}{2}\left(h-h^*\right)^2. \tag{45}$$

Differentiating U with respect to time t along the linearized system (1), we get

$$\frac{dU}{dt}=\left(f-f^*\right)\frac{df}{dt}+\left(w-w^*\right)\frac{dw}{dt}+\left(n-n^*\right)\frac{dn}{dt}+\left(h-h^*\right)\frac{dh}{dt}. \tag{46}$$

$$\begin{aligned}\frac{dU}{dt}=&-\frac{a_{11}}{3}\left(f-f^*\right)^2+a_{21}\left(w-w^*\right)\left(f-f^*\right)-\frac{a_{22}}{3}\left(w-w^*\right)^2-\frac{a_{11}}{3}\left(f-f^*\right)^2\\&+a_{31}\left(n-n^*\right)\left(f-f^*\right)-\frac{a_{33}}{3}\left(n-n^*\right)^2-\frac{a_{11}}{3}\left(f-f^*\right)^2+a_{41}\left(h-h^*\right)\left(f-f^*\right)\\&-\frac{a_{44}}{3}\left(h-h^*\right)^2-\frac{a_{22}}{3}\left(w-w^*\right)^2+a_{23}\left(n-n^*\right)\left(w-w^*\right)-\frac{a_{33}}{3}\left(n-n^*\right)^2-\frac{a_{22}}{3}\left(w-w^*\right)^2\\&+a_{24}\left(w-w^*\right)\left(h-h^*\right)-\frac{a_{44}}{3}\left(h-h^*\right)^2-\frac{a_{33}}{3}\left(n-n^*\right)^2+a_{34}\left(n-n^*\right)\left(h-h^*\right)-\frac{a_{44}}{3}\left(h-h^*\right)^2.\end{aligned} \tag{47}$$

Where

$$a_{11}=\frac{s_0F^*}{L}+\gamma\,F^*H^*,a_{22}=\delta_0W^*,a_{33}=\frac{r_0N^*}{K},a_{44}=\mu_0-\theta_3\gamma\,F^*,a_{21}=\theta_1\alpha\,N^*-\alpha\,F^*,a_{31}=\theta_2\beta\,N^*-\alpha\,F^*,$$
$$a_{41}=-\gamma\,F^{*2}+2\theta_3\gamma\,F^*H^*,a_{23}=-\nu_1W^*-\sigma\,N^*,a_{24}=-\nu_2W^*,a_{34}=\lambda.$$

Then for $\frac{dU}{dt}$ to be negative definite, the following inequalities must hold:

$$a_{21}^2<\frac{4}{9}a_{22}a_{11}, \tag{48}$$

$$a_{31}^2<\frac{4}{9}a_{33}a_{11}, \tag{49}$$

$$a_{41}^2<\frac{4}{9}a_{44}a_{11}, \tag{50}$$

$$a_{23}^2<\frac{4}{9}a_{22}a_{33},$$

$$a_{24}^2 < \frac{4}{9} a_{22} a_{44}, \tag{51}$$

$$a_{34}^2 < \frac{4}{9} a_{33} a_{44}. \tag{52}$$

(53)

Then from (48), (49), (50), (51), (52) and (53) the condition $\frac{dU}{dt}$ would be negative definite. This implies that equilibrium point E_7 to be locally asymptotically stable.

4.2 Global stability

For finding the condition of global stability at E_7 in region Ω we construct the Lyapunov function

$$H = \left(F - F^* - F^* \ln \frac{F}{F^*} \right) + p_0 \left(W - W^* - W^* \ln \frac{W}{W^*} \right) + p_1 \left(N - N^* - N^* \ln \frac{N}{N^*} \right) + \frac{p_2}{2} \left(H - H^* \right)^2. \tag{54}$$

Differentiating H with respect to time t along the solutions of the system (1), we get

$$\frac{dH}{dt} = \frac{\left(F - F^*\right)}{F} \frac{dF}{dt} + \frac{p_0 \left(W - W^*\right)}{W} \frac{dW}{dt} + \frac{p_1 \left(N - N^*\right)}{N} \frac{dN}{dt} + p_2 \left(H - H^*\right) \frac{dH}{dt}. \tag{55}$$

Using system of equations (1), we get after some algebraic manipulations as

$$\begin{aligned} \frac{dH}{dt} = &-b_{11} \left(F - F^*\right)^2 - b_{22} \left(W - W^*\right)^2 - b_{33} \left(N - N^*\right)^2 - b_{44} \left(H - H^*\right)^2 + \\ &b_{34} \left(N - N^*\right)\left(H - H^*\right) + b_{14} \left(F - F^*\right)\left(H - H^*\right) - b_{13} \left(N - N^*\right)\left(F - F^*\right) - \\ &b_{23} \left(W - W^*\right)\left(N - N^*\right) - b_{12} \left(F - F^*\right)\left(W - W^*\right) - b_{24} \left(H - H^*\right)\left(W - W^*\right). \end{aligned} \tag{56}$$

First we choose $p_0 = \frac{1}{\theta_1}, p_1 = \frac{1}{\theta_2}, p_2 = \frac{1}{\lambda}$ and simplify the above equation as follows

$$\begin{aligned} \frac{dH}{dt} = &-\left(\frac{s_0}{L} + \gamma N \right)\left(F - F^*\right)^2 - \delta_0 \left(W - W^*\right)^2 - \frac{r_0}{K} \left(N - N^*\right)^2 - \left(\mu_0 + \theta_3 \gamma F^2\right)\left(H - H^*\right)^2 \\ &+ \left(N - N^*\right)\left(H - H^*\right) + \frac{\theta_3 \gamma H^*}{\lambda} \left(F + F\right)\left(F - F^*\right)\left(H - H^*\right) - \gamma F^* \left(N - N^*\right)\left(F - F^*\right) \\ &- \left(\frac{\sigma}{\theta_2} + \frac{\nu_1}{\theta_1} \right)\left(W - W^*\right)\left(N - N^*\right) - \frac{\nu_2}{\theta_1} \left(H - H^*\right)\left(W - W^*\right). \end{aligned} \tag{57}$$

Then $\frac{dH}{dt}$ to be negative definite, the following inequality must hold.

$$\left(\frac{\theta_3\gamma\, H^*}{\lambda}\right)^2 < \frac{4s_0\mu_0}{3L},$$

(58)

$$\left(\gamma\, F^*\right)^2 < \frac{2s_0 r_0}{3LK},$$

(59)

$$1 < \frac{4r_0\mu_0}{9K},$$

(60)

$$\left(\frac{\sigma}{\theta_2}+\frac{\nu_1}{\theta_1}\right)^2 < \frac{2\delta_0 r_0}{3K},$$

(61)

$$\frac{\nu_2^2}{\theta_1^2} < \frac{2\delta_0\mu_0}{3}.$$

(62)

Then $\frac{dH}{dt}$ is negative definite under conditions (58), (59), (60), (61) and (62). Then equilibrium point E_7 is globally stable in the region Ω .

5 Persistence

Assume that $s > \alpha\, W_m + \beta\, N_m$, $\theta_1\alpha\, F_{\min} > \delta_1 + \nu_1 N_m + \nu_2 H_m$, $r + \theta_2\beta\, F_{\min} > \sigma\, W_m$ and $\mu_0 > \theta_3\,\gamma\, F_{\min}^2$. Here W_m, N_m and H_m are upper bounds of the populations W, N, H respectively and always positive. Then system (1) persists. From first equation of the system (1), we have

$$\frac{dF}{dt} \geq \left(s - \alpha\, W_m - \beta\, N_m\right)F - \left(\frac{s_0}{L} + \gamma\, H_m\right)F^2,$$

(63)

According to lemma 3.1 and comparison principle, it follows that

$$F_{\min} = \frac{\left(s - \alpha\, W_m - \beta N_m\right)L}{s_0 + \gamma\, H_m L}.$$

(64)

With condition $s > \alpha\, W_m + \beta\, N_m$, $F_{\min}$ remains always positive. From second equation of the system (1) , we have

$$\frac{dW}{dt} \geq \left(\theta_1\,\alpha\, F_{\min} - \delta_1 - \nu_1 N_m - \nu_2 H_m\right)W - \delta_0 W^2.$$

(65)

According to lemma 3.1 and comparison principle, it follows that

$$W_{\min} = \frac{\theta_1\,\alpha\,F_{\min} - \delta_1 - \nu_1 N_m - \nu_2 H_m}{\delta_0}.$$

(66)

With condition $\theta_1\,\alpha\,F_{\min} > \delta_1 + \nu_1 N_m + \nu_2 H_m$, $W_{\min}$ remains always positive. From the second last equation of the system (1) , we have

$$\frac{dN}{dt} \geq \left(r + \theta_2 \beta\,F_{\min} - \sigma\,W_m\right)N - \frac{r_0}{K}N^2.$$

(67)

Using comparison principle, it follows that

$$N_{\min} = \frac{\left(r + \theta_2 \beta\,F_{\min} - \sigma\,W_m\right)K}{r_0}.$$

(68)

With condition, $r + \theta_2 \beta\,F_{\min} > \sigma\,W_m$, $N_{\min}$ remains always positive.

From the last equation of the system (1) , we have

$$\frac{dH}{dt} \geq \lambda\,N_{\min} - \left(\mu_0 - \theta_3 \gamma\,F_{\min}^2\right)H.$$

(69)

According to lemma 3.1 and comparison principle, it follows that

$$H_{\min} = \frac{\lambda N_{\min}}{\mu_0 - \theta_3 \gamma\,F_{\min}^2}.$$

(70)

With condition $\mu_0 > \theta_3\,\gamma\,F_{\min}^2$, $H_{\min}$ remains always positive.

This completes the proof of the theorem. Thus, system (1) persists if $s > \alpha\,W_m + \beta\,N_m$, $\theta_1 \alpha\,F_{\min} > \delta_1 + \nu_1 N_m + \nu_2 H_m$, $r + \theta_2 \beta\,F_{\min} > \sigma\,W_m$ and $\mu_0 > \theta_3\,\gamma\,F_{\min}^2$.

6 Numerical Results

The stability of the non-linear model system (1), in the positive octant, is investigated numerically by using the following set of parameters.

$$s = 0.8, s_0 = 0.8, L = 100, \alpha = 0.4, \beta = 0.003, \gamma = 0.000004, \theta_1 = 0.8, \delta_1 = 0.01, \delta_0 = 0.08, \nu_1 = 0.002,$$

$$\nu_2 = 0.0001, r = 0.5, r_0 = 0.5, K = 100, \theta = 0.01, \theta_2 = 0.05, \sigma = 0.001, \lambda = 0.001,$$
$$\theta_3 = 0.002, \mu_0 = 0.01.$$

(71)

The interior equilibrium point of the model system (2.1) corresponding to the above parameters values is:

$F^* = 0.9658$, $W^* = 1.2313$, $N^* = 99.7827$, $H^* = 9.9783$.

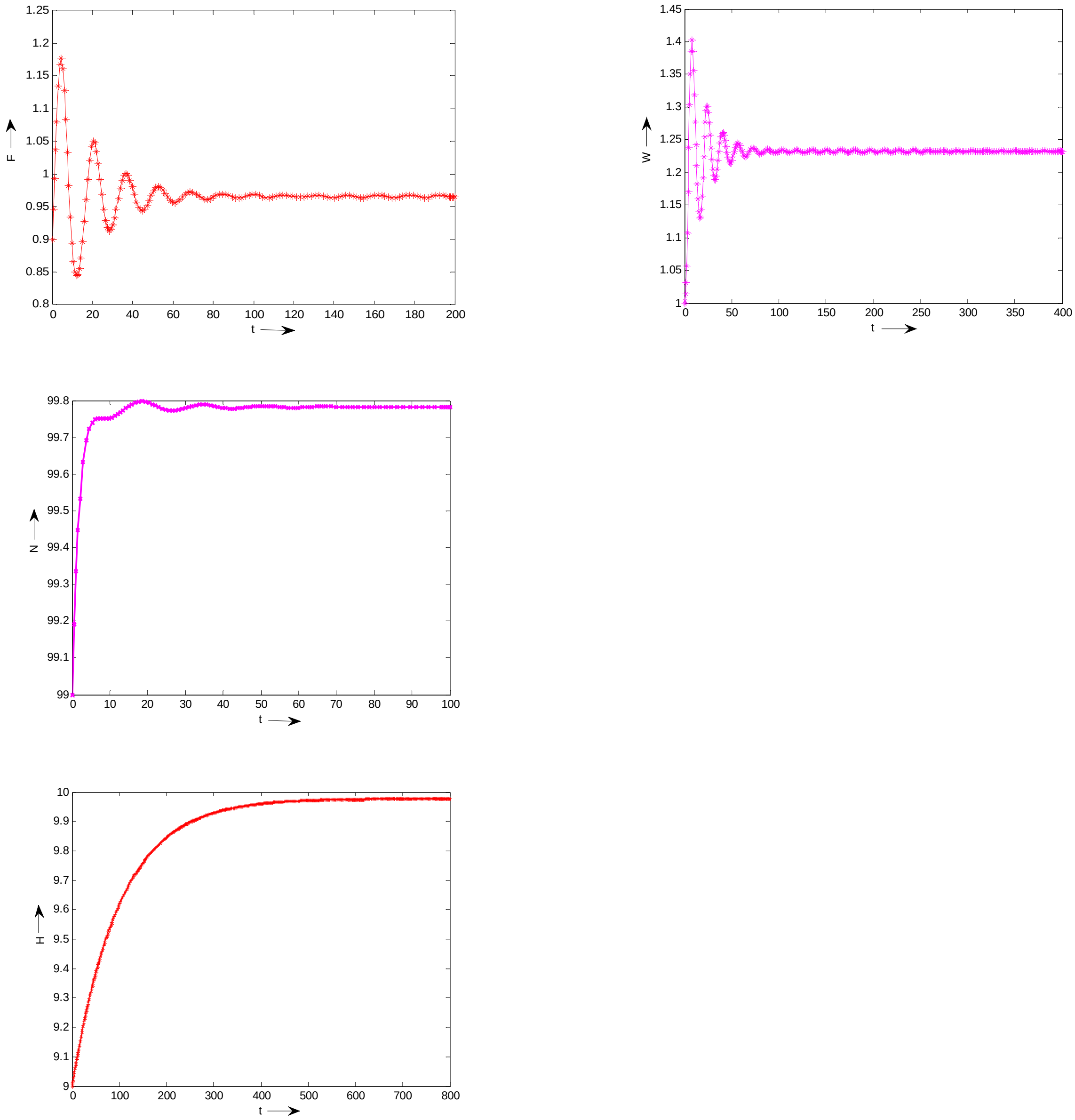

Fig. 1 Stable behavior of F, W, N and H with time and other parameter values are same as (71).

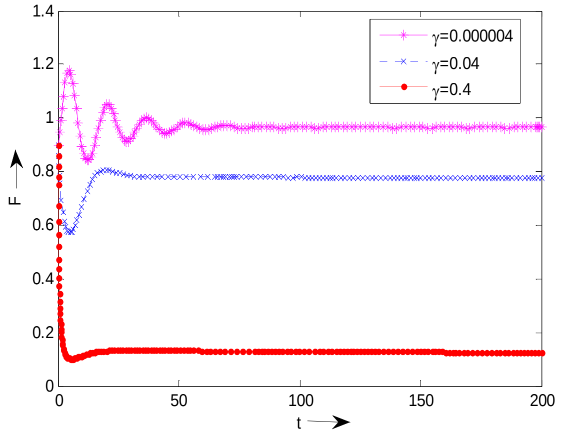

Fig. 2 Variation of F with time for different values of γ and other parameter values are same as (71).

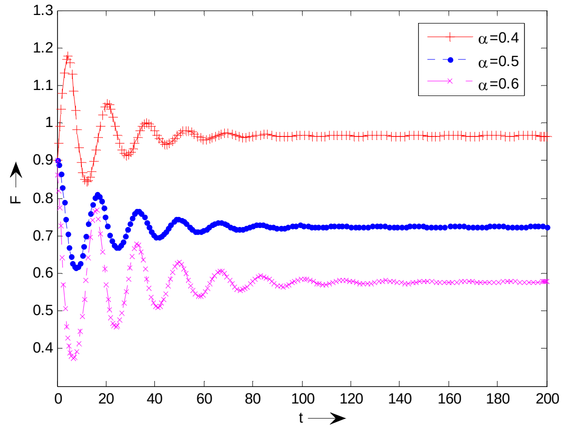

Fig. 3 Variation of F with time for different values of α and other parameter values are same as (71).

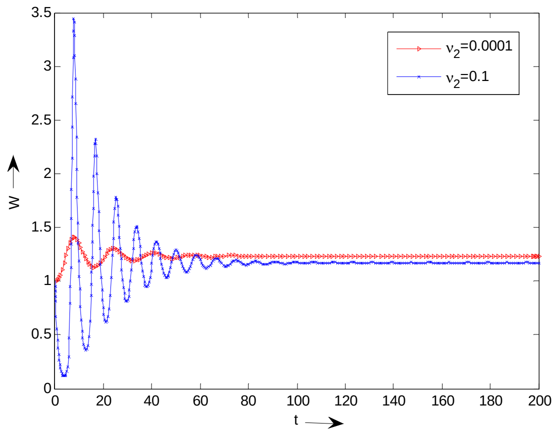

Fig. 4 Variation of W with time for different values of ν_2 and other parameter values are same as (71).

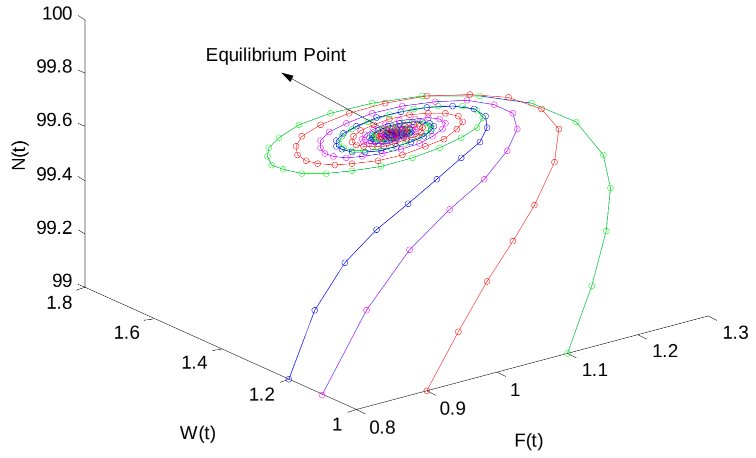

Fig. 5 Global behaviour of the system.

Using the above set of parameters, the model (1) has been solved numerically and various results are displayed graphically in Fig. 1-5. The variation of the populations with time is shown in Fig. 1 which shows the local stable behaviour of the system. Fig. 2 shows the effects of γ on the forestry biomass density F . This figure shows that as the value of γ increases the growth rate of forestry biomass decreases and it may become extinct after a long time. The effects of α on forestry biomass density have been shown in Fig. 3, which show that forestry biomass density decreases due to increase the value of α . The effect of human activities on wildlife population have been shown in Fig. 4. We see clearly from figure, increasing the value of ν_2 the density of wildlife population decrease. Fig. 5 shows the global stability of the system (1).

7 Conclusions

In this paper, we have proposed a nonlinear mathematical model to study the affect of human activities on the forest resources and its effect on wildlife population living in the forest. The proposed model has been analyzed by the stability theory of differential equations. The conditions of existence of equilibrium points and their stability in both local and global cases have been obtained. The condition, under which the system persists, by using differential inequality, has been found. By using numerical simulation, the effects of various parameters on the depletion of forestry biomass and wildlife population by human activities have been shown graphically. The time has come for decisive action to be taken to tackle depletion of forestry biomass head on. Mitigation strategies need to be taken to deal with the man-made causes of depletion of forestry biomass and wildlife population by controlling deforestation and encouraging afforestation of deforested areas there by protecting the CO_2 sink. Improved forest management and where possible, expansion of forest areas will help to reduce the concentration of CO_2 in the atmosphere since forest make use of CO_2 for photosynthesis and act as reservoirs of carbon. This study revealed the main activities in the region of the forestry biomass and demonstrates the impact of those activities on forest resources by looking at the percentage of forest cover change and meteorological changes. The findings suggest that the rates of forest cover loss have reduced in recent years. These findings imply that there is a relationship between human activities and forest resources. There is a need for the provision of more schools and adequate learning measures. This may help in increasing the education level, which will in the long run reduce unemployment rate and the use of forest resources. The study was able to give an insight into the possible impact on forest resources in the future. Although the situation is still manageable, so long there has not been any dramatic change in forest cover, the rates of forest cover loss seem to be under control, but could blow out of proportion if not controlled.

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