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Article

Stability analysis of a mathematical model for the abatement of methane: Effect of mitigation options

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Abstract

Atmospheric methane is one of the main greenhouse gases which contribute significantly to increase the burden of global warming. The production of rice paddies and livestock farming are the major sources of methane emissions in the atmosphere. The control of methane emissions using some efficient mitigation options is crucial to lower the concentration of methane in the atmosphere so that the adverse effects of global warming can be reduced to some extent. In this paper, a nonlinear mathematical model is proposed to study the effects of mitigation options on abatement of methane discharged by rice paddies and livestock populations in the atmosphere. In the modeling process, four nonlinearly interacting variables namely, the cumulative density of rice paddies, the cumulative density of livestock populations, the atmospheric concentration of methane and the cumulative density of mitigation options are considered. The cumulative density of mitigation options is assumed to be proportional to the increased level of atmospheric methane concentration from its equilibrium.

The proposed nonlinear model is analyzed using the stability theory of differential equations and computer simulations. The study shows that without implementation of mitigation options, the concentration of methane in the atmosphere increases continuously with increase in the rates of its emissions by rice paddies and livestock populations. This increase in the atmospheric methane can be reduced considerably by efficient management of mitigation options. The increase in the implementation rate coefficient of various mitigation options and depletion rate coefficient due to net effectiveness of mitigation options further reduces the atmospheric methane concentration. The numerical simulation of the model confirms the analytical findings.

Keywords mathematical model; methane; rice paddies; livestock populations; mitigation options; stability analysis; numerical simulation.

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1 Introduction

The average earth temperature has been increasing continuously in past several decades due to emission of global warming gases such as carbon dioxide (CO_2), methane (CH_4) etc. in the atmosphere and is expected to rise further in future. The effect of global warming can be seen in the form of melting of glaciers, sea level rise, loss of biodiversity, drought, flood, etc. (Ramanathan and Feng, 2009). Methane, after carbon dioxide, is the most affecting greenhouse gas having global warming potential 25-fold than that of carbon dioxide. It is produced in abundance mainly due to agricultural practices (such as farming of rice paddies) (Seiler et al., 1983; Sander et al., 2014) and farming of cattle populations (such as buffaloes, goats, cows, sheep, etc.) (Steinfeld et al., 2006; Ramanathan and Feng, 2009; Mogensen et al., 2015). Worldwide rice production is responsible for nearly 20% of global anthropogenic methane emission (Cao et al., 1996). Also, farming of livestock population is responsible for about 33% of global anthropogenic methane emission (Eckard et al., 2010).

The importance of rice as a source of food for the populations residing in America, India and certain parts of Africa is well-known and it is predicted that the rice production must be increased by 50% till 2030 in order to fulfill their needs for food (Osman et al., 2012). Further, it has been estimated that China has to increase its rice production up to 20% by 2030 to ensure the food security to its increasing population (Peng et al., 2009). Thus, a sustainable increase in rice production is essential to feed the populations whereas significant reduction in methane emission from rice fields is also needed. Methane emission from rice paddies can be reduced significantly by taking into account the appropriate water management system like mid season drainage, intermediate irrigation, etc. (Tyagi et al., 2010; Khosa et al., 2011; Setyanto et al., 2018). Further, methane emission can also be controlled by effective use of fertilizers, proper selection of low methane emitting cultivars, etc. (Singh et al., 2003; Linquist et al., 2012).

The livestock populations (ruminants) like buffalo, cattle, goat and sheep etc., also produce significant amount of methane directly with enteric fermentation and indirectly with manure management. The methane emission from manure production depends on the size, density and quantity of feces, type of fodder, temperature, humidity etc. (Johnson and Johnson, 1995; Gonzalez-Avalos and Ruiz-Suarez, 2001; Boadi et al., 2004; Chagunda et al., 2009; Shibata and Terada, 2010; Patra, 2012). The methane emission can be reduced by proper management of this manure using chemical compounds, increasing dry matter intake, using legume instead of grass forage, upgradation of forages time to time, etc (Patra, 2012).

Very few investigations have been conducted to study the effect of global warming and its control using nonlinear mathematical models (Misra and Verma, 2014, 2017; Shukla et al., 2015; Sundar et al., 2020). For example, Shukla et al. (2015) have proposed a nonlinear mathematical model to study the removal of carbon dioxide from the atmosphere to reduce global warming considering two cases; (i) removal of carbon dioxide from the near earth atmosphere by greenbelt plantation and by introducing liquid droplets in the atmosphere, and (ii) removal of carbon dioxide from the upper earth atmosphere by spraying external liquid species and particulate matters. Their study reveals that the concentration of carbon dioxide can be reduced in the atmosphere using above processes. Misra and Verma (2017) have proposed and analyzed a nonlinear model to study the effect of mitigation options on the abatement of methane emission from livestock populations and observed considerable decrease in methane concentration on implementation of efficient mitigation options. But in these investigations, the simultaneous effect of methane discharged by rice paddies and livestock populations on global warming has not been studied.

Therefore, in this study, we tried to model and analyze the effects of mitigation options on the abatement of methane discharged simultaneously by rice paddies and livestock populations in the atmosphere.

2 Mathematical Model

To formulate a mathematical model for the problem under consideration, let $B(t)$ and $C_a(t)$ be the cumulative densities of rice paddies and livestock populations respectively. Let $C(t)$ be the atmospheric concentration of methane caused by rice paddies, livestock populations and natural sources such as water swamp. Further, let $M(t)$ be the cumulative density of mitigation options applied to mitigate methane emissions from rice paddies and livestock populations.

It is reasonable to assume that the cumulative density of rice paddies decreases due to presence of livestock population and the cumulative density of livestock population increases due to consumption of rice paddies. Thus, the decrease in cumulative density of rice paddies is taken to be proportional to the cumulative density of livestock population and that of rice paddies (i.e., s_1BC_a) where the constant s_1 denotes the consumption rate coefficient of rice paddies due to livestock population. The growth rate of cumulative density of rice paddies is, therefore, governed by the logistic model with its intrinsic growth rate s and carrying capacity L.

As pointed out above, the cumulative density of livestock population increases due to use of rice paddies and therefore the growth rate of cumulative density of livestock population is assumed to be proportional to the cumulative density of rice paddies as well as that of livestock population (i.e., $r_{a1}BC_a$) where r_{a1} is its rate of increase due to consumption of rice paddies. The cumulative density of livestock populations is also assumed to vary logistically with its intrinsic growth rate r_a and the carrying capacity K_a.

The growth of atmospheric concentration of methane is assumed to be increased by the cumulative density of livestock populations with emission rate coefficient λ_1 and by cumulative density of rice paddies with emission rate coefficient λ_2 with constant input of methane Q_o from various natural sources such as wetland, water swamp, etc. The constant λ_0 represents the natural depletion rate coefficient of atmospheric methane. The decrease in the atmospheric concentration of methane is also assumed to be proportional to the increased level of mitigation options from its equilibrium level M_0 as well as the concentration of atmospheric methane (i.e. $\lambda C(M-M_0)$). The constant M_0 defines the basic level of mitigation options applied at all time in order to maintain the atmospheric methane concentration at the level C_o. This implies that $C=C_o$ when $M=M_0$. The constant λ is the depletion rate coefficient of atmospheric methane due to net effectiveness of mitigation options.

It is noted here that if the atmospheric concentration of methane goes beyond its equilibrium level C_0, it may contribute significantly to increase global warming necessitating its abatement using mitigation options. Therefore, the cumulative density of mitigation options M is assumed to be proportional to the increased level of atmospheric methane concentration from its equilibrium level C_0 (i.e., $\phi(C-C_0)$) where ϕ denotes implementation rate coefficient of mitigation options. It is further assumed that the base line mitigation options M_0 are always applied in order to maintain the equilibrium level of atmospheric methane concentration. Therefore, the natural depletion of mitigation options is assumed to be proportional to the increased level of mitigation options from its equilibrium level M_0 (i.e., $\phi_0(M-M_0)$) where ϕ_0 denotes its natural depletion rate coefficient. This depletion is caused due to inefficient working of some mitigation options.

In view of the above assumptions and considerations, a four dimensional nonlinear mathematical model is proposed to study the effects of mitigation options on the abatement of methane discharged by rice paddies and livestock populations in the atmosphere as follows,

$$\frac{dB}{dt} = sB\left(1-\frac{B}{L}\right) - s_1 BC_a \tag{1}$$

$$\frac{dC_a}{dt} = r_a C_a\left(1-\frac{C_a}{K_a}\right) + r_{a1} BC_a \tag{2}$$

$$\frac{dC}{dt} = Q_0 + \lambda_1 C_a + \lambda_2 B - \lambda_0 C - \lambda C(M - M_0) \tag{3}$$

$$\frac{dM}{dt} = \phi(C - C_0) - \phi_0 (M - M_0) \tag{4}$$

where $C_0 = \dfrac{Q_0}{\lambda_0}$ and $B(0) \geq 0,\ C_a(0) \geq 0,\ C(0) \geq C_0 > 0,\ M(0) \geq M_0 > 0$

Remark 1. It is noted from the model system that $C = C_0$ when $M = M_0$.

Remark 2. From equation (1) of the model system, it is noted that $s - s_1 C_a > 0$ for all $C_a > 0$.

In order to analyze the model system (1) - (4), the following lemma is stated without proof.

Lemma The region of attraction for all solutions of model system (1) - (4) initiating in the positive octant is given by the set,

$$\Omega = \left\{ (B, C_a, C, M) : 0 \leq B \leq L,\ 0 \leq C_a \leq C_{am},\ C_0 \leq C \leq C_m,\ M_0 \leq M \leq M_0 + \frac{\phi}{\phi_0}(C_m - C_0) \right\}$$

where $C_{am} = \dfrac{K_a}{r_a}(r_a + r_{a1} L),\ C_m = \dfrac{Q_0 + \lambda_1 L + \lambda_2 C_{am}}{\lambda_0}$.

3 Equilibrium Analysis

The model system (1) – (4) has four equilibria namely,

(i) $E_0(0, 0, C_0, M_0)$, where $C_0 = \dfrac{Q_0}{\lambda_0}$

This equilibrium E_0 always exists. It implies that in the absence of rice paddies and livestock populations, the atmospheric concentration of methane is always maintained at its equilibrium level C_0 with equilibrium mitigation options M_0.

(ii) $E_1(L, 0, C_1, M_1)$, where

$$C_1 = \frac{\phi_0}{2\lambda\phi}\left\{ -\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right) + \sqrt{\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right)^2 + \frac{4\lambda\phi(Q_0 + \lambda_1 L)}{\phi_0}} \right\}$$

and $M_1 = M_0 + \dfrac{\phi}{\phi_0}(C_1 - C_0)$

The existence of equilibrium E_1 is obvious and it implies the presence of rice paddies at its carrying capacity without livestock populations in the system and therefore increase in the atmospheric concentration of methane depends only on methane emission from rice paddies. In such a case, the atmospheric concentration of methane exceeds its equilibrium value requiring increased level of mitigation options above its equilibrium value.

(iii) $E_2(0, K_a, C_2, M_2)$,

where $C_2 = \frac{\phi_0}{2\lambda\phi}\left\{-\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right) + \sqrt{\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right)^2 + \frac{4\lambda\phi(Q_0 + \lambda_2 K_a)}{\phi_0}}\right\}$

and $M_2 = M_0 + \frac{\phi}{\phi_0}(C_2 - C_0)$

The existence of equilibrium E_2 is also trivial. It implies the presence of livestock populations at its carrying capacity with no availability of rice paddies in the system. Thus, the atmospheric concentration of methane is only dependent on the methane emission from livestock populations. In this case also, the atmospheric concentration of methane is higher than its equilibrium level and hence the mitigation options above its equilibrium value are needed to be applied for abatement of atmospheric methane.

(iv) $E^*(B^*, C_a{}^*, C^*, M^*)$

The non-trivial equilibrium E^* implies the presence of both rice paddies and livestock populations to enhance the level of atmospheric methane concentration. In this case, the atmospheric concentration of methane is higher than its natural level due to emission from both rice paddies and livestock populations requiring increase in the level of mitigation options. The existence of E^* is proved below.

3.1 Existence of $E^*(B^*, C_a{}^*, C^*, M^*)$

The values of different variables in the equilibrium $E^*(B^*, C_a{}^*, C^*, M^*)$ can be obtained by solving the following set of equations obtained by equating the right hand side of equations in model system (1) - (4) to zero,

$$s\left(1 - \frac{B}{L}\right) - s_1 C_a = 0 \tag{5}$$

$$r_a\left(1 - \frac{C_a}{K_a}\right) + r_{a1}B = 0 \tag{6}$$

$$Q_0 + \lambda_1 C_a + \lambda_2 B - \lambda_0 C - \lambda C(M - M_0) = 0 \tag{7}$$

$$\phi(C - C_0) - \phi_0(M - M_0) = 0 \tag{8}$$

Solving equations (5) and (6), we get

$$B = \frac{s - s_1 K_a}{\frac{s}{L} + s_1 r_{a1}\frac{K_a}{r_a}} = B^* \text{(say)} \tag{9}$$

$$C_a = \frac{r_a + r_{a1}L}{\left(\frac{r_a}{K_a} + r_{a1}L\frac{s_1}{s}\right)} = C_a{}^* \text{(say)} \tag{10}$$

Using equations (8), (9) and (10) in equation (7), we get

$$\frac{\lambda\phi}{\phi_0}C^2 + \left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right)C - (Q_0 + \lambda_1 B^* + \lambda_2 C_a{}^*) = 0 \tag{11}$$

From which, we obtain

$$C = \frac{\phi_0}{2\lambda\phi}\left\{-\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right) + \sqrt{\left(\lambda_0 - \frac{\lambda\phi C_0}{\phi_0}\right)^2 + \frac{4\lambda\phi(Q_0 + \lambda_1 B^* + \lambda_2 C_a{}^*)}{\phi_0}}\right\} = C^* \text{(say)} \tag{12}$$

Now from equation (8), we obtain the equilibrium value M^* as,

$$M^* = M_0 + \frac{\phi}{\phi_0}(C^* - C_0) \tag{13}$$

Thus, the equilibrium $E^*(B^*, C_a{}^*, C^*, M^*)$ exists uniquely without any condition.

3.2 Variations of C with ϕ

Differentiating equation (11) with respect to ϕ, we have

$$\left(\frac{\lambda\phi C^2}{\phi_0} + Q_0 + \lambda_1 B^* + \lambda_2 C_a{}^*\right)\frac{dC}{d\phi} = -\frac{\lambda C^2(C - C_0)}{\phi_0} \tag{14}$$

From which we note that $\frac{dC}{d\phi} < 0$.

This implies that the atmospheric concentration of methane decreases with increase in the implementation rate of mitigation options required for abatement of methane discharged by rice paddies and livestock populations.

4 Stability Analysis

4.1 Local stability of the equilibria

By computing Jacobian matrix of the model system (1) – (4) about each equilibrium, it can be easily checked that,

(i) $E_0(0,0,C_0,M_0)$ is a saddle point unstable manifold in $B-C_a$ plane and stable manifold in $C-M$ plane.

(ii) $E_1(L,0,C_1,M_1)$ is a saddle point unstable manifold in C_a- direction and stable manifold in $B-C-M$ space.

(iii) $E_2(0,K_a,C_2,M_2)$ is a saddle point unstable manifold in $B-$ direction and stable manifold in C_a-C-M space.

The Jacobian matrix of the model system (1) – (4) about E^* is given as,

$$J(E^*)=\begin{bmatrix} -\frac{s}{L}B^* & -s_1B^* & 0 & 0 \\ r_{a1}C_a{}^* & -\frac{r_a}{K_a}C_a{}^* & 0 & 0 \\ \lambda_2 & \lambda_1 & -(\lambda_0+\lambda(M^*-M_0)) & -\lambda C^* \\ 0 & 0 & \phi & -\phi_0 \end{bmatrix}$$

Consider a positive definite function as,

$$V=\frac{1}{2}k_1B_1{}^2+\frac{1}{2}k_2C_{a1}{}^2+\frac{1}{2}k_3C_1{}^2+\frac{1}{2}k_4M_1{}^2$$

where B_1, C_{a1}, C_1 and M_1 are small perturbations about E^* such that $B=B^*+B_1$, $C_a=C_a{}^*+C_{a1}$, $C=C^*+C_1$ and $M=M^*+M_1$. The positive constants k_i $(i=1...4)$ are to be chosen appropriately.

Differentiating V with respect to $'t'$ we get,

$$\frac{dV}{dt}=k_1B_1\frac{dB_1}{dt}+k_2C_{a1}\frac{dC_{a1}}{dt}+k_3C_1\frac{dC_1}{dt}+k_4M_1\frac{dM_1}{dt}$$

Putting the values of derivatives and simplifying, we get,

$$\frac{dV}{dt}=-k_1\frac{sB^*}{L}B_1{}^2-k_2\frac{r_aC_a{}^*}{K_a}C_{a1}{}^2-k_3(\lambda_0+\lambda(M^*-M_0))C_1{}^2-k_4\phi_0M_1{}^2$$

$$-(k_1s_1B^*-k_2r_{a1}C_a{}^*)B_1C_{a1}+k_3\lambda_1B_1C_1+k_3\lambda_2C_1C_{a1}+(k_4\phi-k_3\lambda C^*)C_1M_1$$

Now after choosing,

$$k_1=1, k_2=\frac{s_1B^*}{r_{a1}C_a{}^*}\ ,\ k_3<\frac{2}{3}B^*(\lambda_0+\lambda(M^*-M_0))\min\left(\frac{s}{\lambda_1{}^2L},\frac{s_1r_a}{\lambda_2{}^2r_{a1}K_a}\right)\text{and } k_4=\frac{\lambda C^*}{\phi}k_3$$

we note that $\frac{dV}{dt}$ is negative definite showing that V is a Liapunov function and hence E^* is locally asymptotically stable. Thus, the following result is obtained.

Theorem 1 The equilibrium E^* is locally asymptotically stable without any condition.

In the following, we study the global stability behavior of E^*.

4.2 Global stability

Consider a positive definite function as,

$$U=m_1\left(B-B^*-B^*\log\frac{B}{B^*}\right)+m_2\left(C_a-C_a{}^*-C_a{}^*\log\frac{C_a}{C_a{}^*}\right)+\frac{1}{2}m_3(C-C^*)^2+\frac{1}{2}m_4(M-M^*)^2$$

where the positive constants m_i $(i=1...4)$ are to be chosen appropriately.

Differentiating U with respect to $'t'$ we get,

$$\frac{dU}{dt}=\frac{m_1}{B}(B-B^*)\frac{dB}{dt}+\frac{m_2}{C_a}(C_a-C_a{}^*)\frac{dC_a}{dt}+m_3(C-C^*)\frac{dC}{dt}+m_4(M-M^*)\frac{dM}{dt}$$

Putting the values of derivatives and simplifying, we get,

$$\frac{dU}{dt} = -m_1 \frac{s}{L}(B-B^*)^2 - m_2 \frac{r_a}{K_a}(C_a - C_a{}^*)^2 - (\lambda_0 + \lambda(M-M_0))m_3(C-C^*)^2$$

$$-m_4\theta_0(M-M^*)^2 - (m_1 s_1 - m_2 r_{a1})(B-B^*)(C_a - C_a{}^*) + m_3\lambda_1(B-B^*)(C-C^*)$$

$$+m_3\lambda_2(C_a - C_a{}^*)(C-C^*) + (m_4\phi - m_3\lambda C^*)(C-C^*)(M-M^*)$$

After some algebraic manipulations and by choosing,

$$m_1 = 1, m_2 = \frac{s_1}{r_{a1}} \;,\; m_3 < \frac{2}{3}\lambda_0 \min\left(\frac{s}{\lambda_1{}^2 L}, \frac{s_1 r_a}{\lambda_2{}^2 r_{a1} K_a}\right) \text{and } m_4 = \frac{\lambda C^*}{\phi} m_3$$

we note that $\frac{dU}{dt}$ is negative definite showing that U is a Liapunov function and hence E^* is globally asymptotically stable. The result obtained is stated in the form of the following theorem.

Theorem 2 The equilibrium E^* is globally asymptotically stable within the region of attraction Ω without any condition.

5 Numerical Simulation

To validate the analytical findings, we perform numerical simulation of the model system (1) – (4) for different values of parameters. For that the system (1) – (4) is integrated numerically with the help of MAPLE 18 using the set of parameters values given in Table 1.

The equilibrium values of different variables in E^* are obtained as follows.

$$B^* = 989.8020396 \text{ ton}, C_a{}^* = 2039.592082 \text{ million}$$

$$C^* = 2398.218886 \text{ ppb}, M^* = 1148.515738 \text{ dollar}$$

Table 1 Parameter values for the model system (1) – (4).

Parameter	Value	Parameter	Value
Q_0	120 ppb (year)$^{-1}$	λ	0.00003 (dollar year)$^{-1}$
λ_0	0.1 year^{-1}	ϕ	0.025 dollar (ppb year)$^{-1}$
r_a	0.5 year^{-1}	ϕ_0	0.03 year^{-1}
K_a	2000 million	s	0.6 year^{-1}
L	1000 ton	s_1	0.000003 (million year)$^{-1}$
r_{a1}	0.00001 (ton year)$^{-1}$	C_0	1200 ppb
λ_1	0.06 ppb (million year)$^{-1}$	M_0	150 dollar
λ_2	0.07 ppb (ton year)$^{-1}$		

All the eigenvalues of the Jacobian matrix J corresponding to E^* are obtained as -0.593153 , -0.510625 , -0.106418 and -0.0535370 which are all negative showing that the non-trivial equilibrium E^* is locally asymptotically stable. The global stability behavior of equilibrium E^* for the model system (1) – (4) in $B-C_a-C$ space is shown in Fig. 1 where all trajectories with different initial starts approach to E^* showing that the equilibrium E^* is globally asymptotically stable.

The variation of model variables is plotted with time for different values of relevant parameters in Figs. 2–7. In Figs. 2 and 3, the variation of the atmospheric concentration (C) of methane with time $'t'$ is shown for different values of the emission rate coefficient of methane due to livestock population (i.e., $\lambda_1 = 0.06, 0.07, 0.08$) and the emission rate coefficient of methane due to rice paddies (i.e., $\lambda_2 = 0.07, 0.08, 0.09$) respectively. From these figures, it is found that the atmospheric concentration of methane increases as the emission rate coefficient of methane due to livestock populations (λ_1) and the emission rate coefficient of methane due to rice paddies (λ_2) increases. This indicates the crucial role of rice paddies and livestock populations in elevating the atmospheric concentration of methane which is significantly responsible for global warming. Thus, the resulting increase in atmospheric concentration of methane requires the implementation of mitigation options for the abatement of methane discharged by rice paddies and livestock populations in the atmosphere. The effect of mitigation options to curtail the methane emission from rice paddies and livestock farming is shown in Figs. 4 and 5 where the variation of the atmospheric concentration (C) of methane with time $'t'$ is shown for different values of implementation rate of mitigation options (i.e., $\phi = 0.025, 0.030, 0.035$) and the depletion rate coefficient of atmospheric methane due to net effectiveness of mitigation options (i.e., $\lambda = 0,\ 0.00003, 0.00004$) respectively. From these figures, it is seen that the atmospheric concentration of methane decreases with increase in the implementation rate of mitigation options (ϕ) and the depletion rate coefficient of atmospheric methane due to net effectiveness of mitigation options (λ). It is also noted from Fig. 5 that if the net effectiveness of mitigation options is zero then the atmospheric concentration of methane will increase continuously due to emission of methane from rice paddies, livestock populations and other natural sources and attains its maximum equilibrium. In Fig. 6 and 7, the cumulative density of mitigation options (M) with time $'t'$ is shown for different values of the emission rate coefficient of methane due to livestock populations (i.e., $\lambda_1 = 0.06, 0.07, 0.08$) and the emission rate coefficient of methane due to rice paddies (i.e., $\lambda_2 = 0.07, 0.08, 0.09$) respectively.

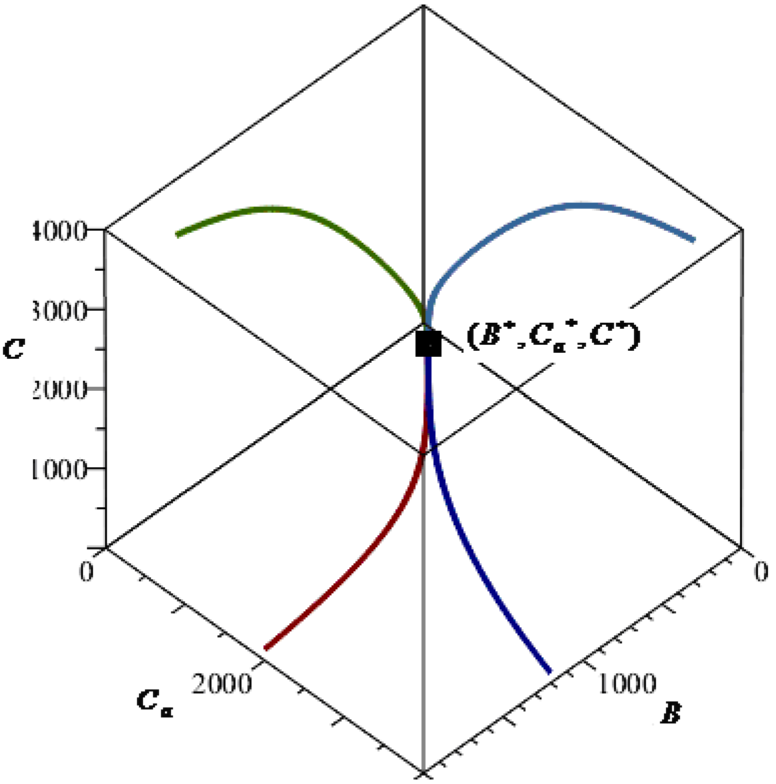

Fig. 1 Global stability in $B-C_a-C$ plane.

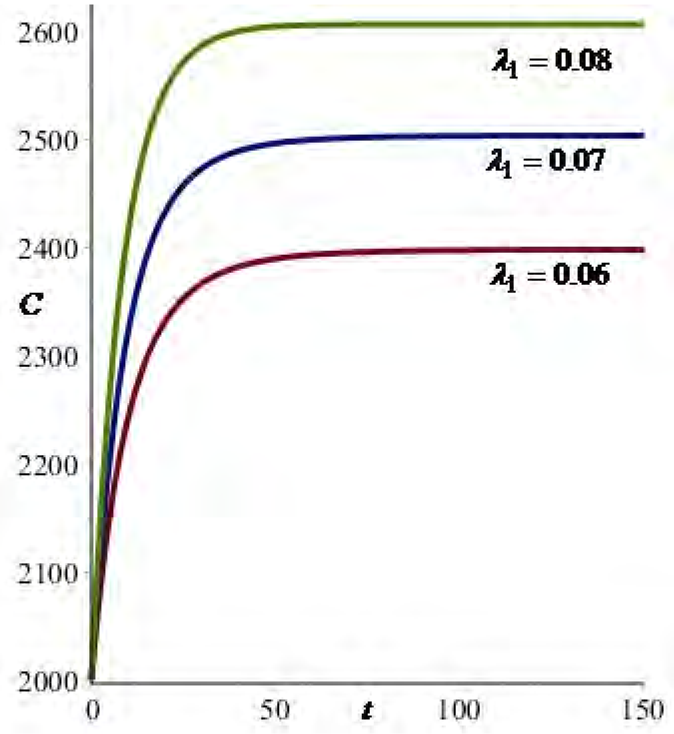

Fig. 2 Variation of atmospheric concentration C of methane with time *t* for different values of λ_1.

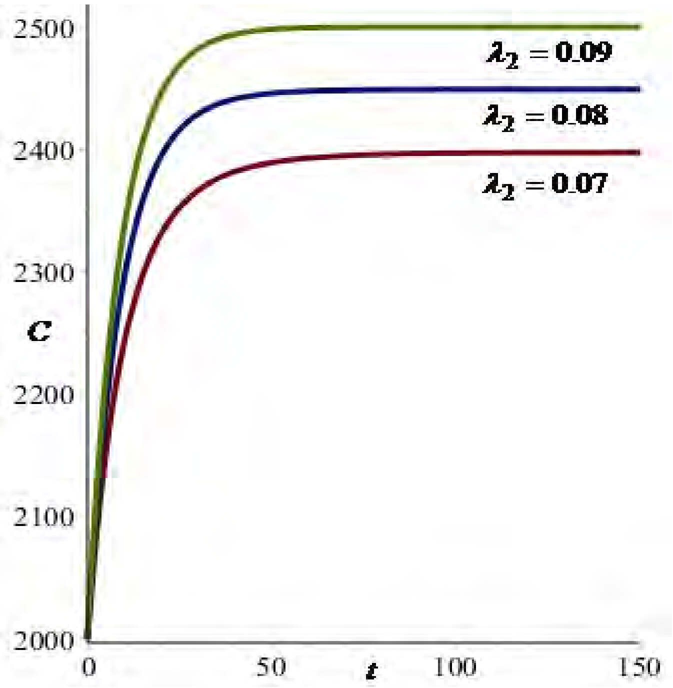

Fig. 3 Variation of atmospheric concentration C of methane with time *t* for different values of λ_2.

From these figures, it is found that as the emission rate coefficients of methane due to livestock populations (λ_1) and that due to rice paddies (λ_2) increase, the cumulative density of mitigation options increases. This is due to the fact that increasing these parameters elevates the atmospheric concentration of methane and subsequently the requirement of mitigation options increases to curtail the level of methane in the atmosphere.

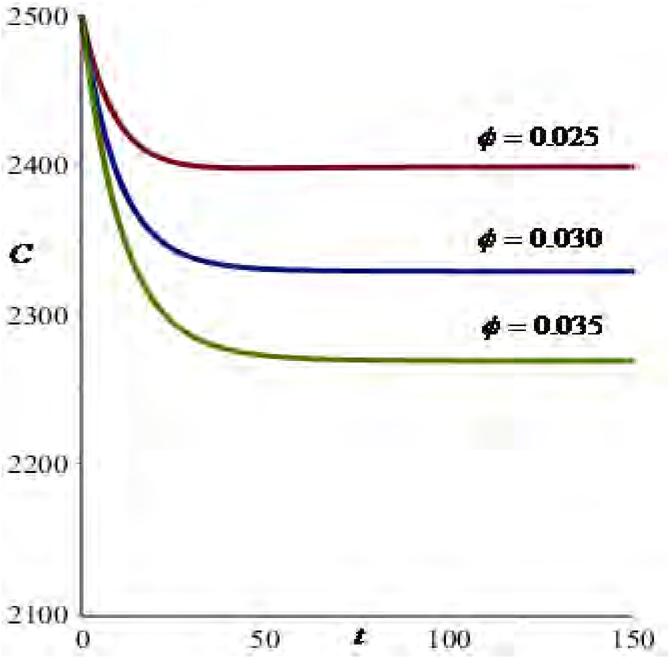

Fig. 4 Variation of atmospheric concentration C of methane with time *t* for different values of ϕ.

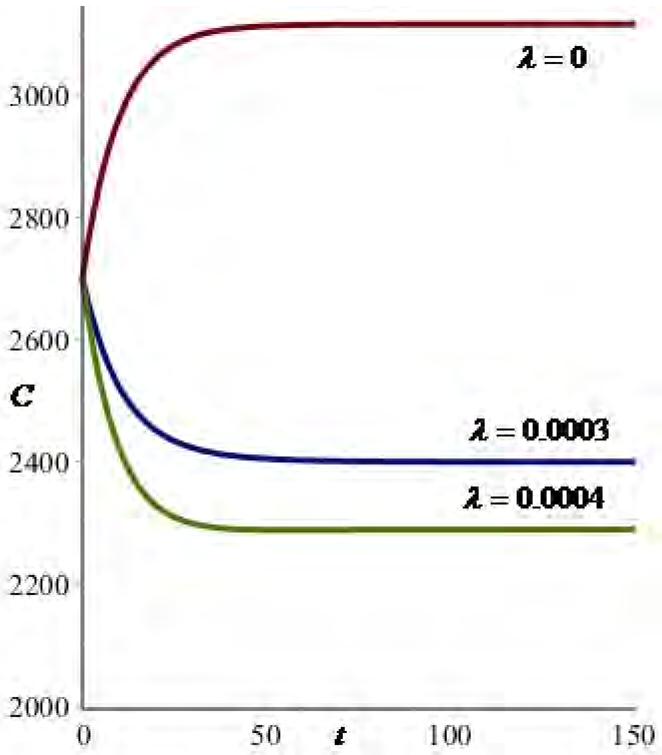

Fig. 5 Variation of atmospheric concentration C of methane with time t for different values of λ.

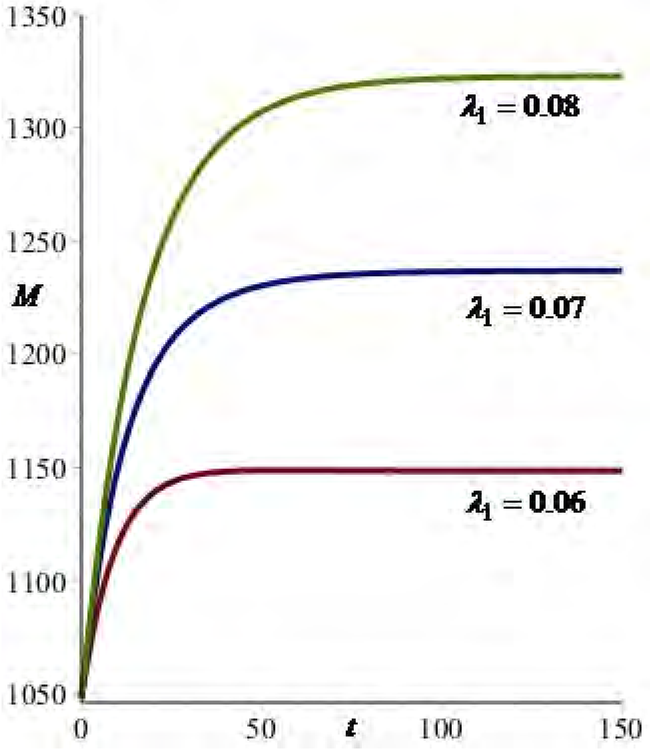

Fig. 6 Variation of cumulative density of mitigation options M with time t for different values of λ_1.

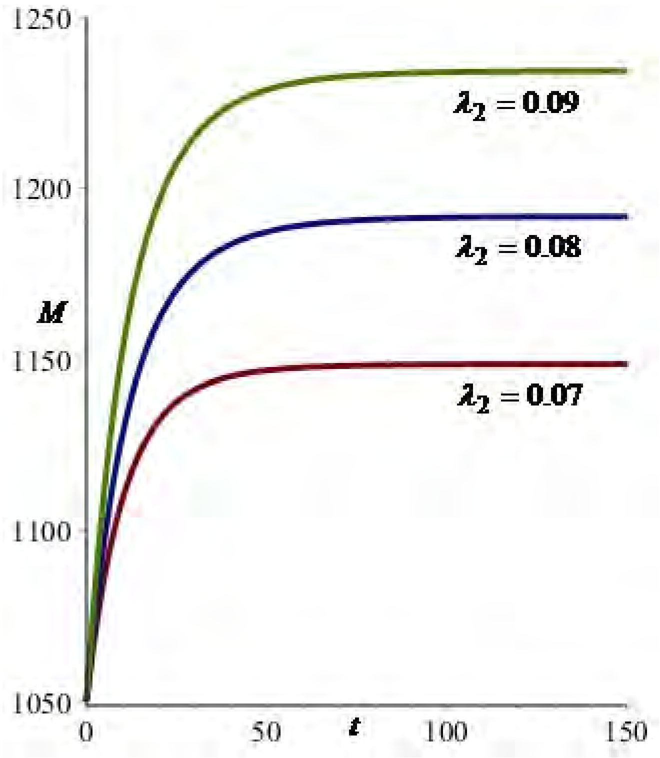

Fig. 7 Variation of cumulative density of mitigation options M with time t for different values of λ_2.

6 Conclusions

In this paper, a nonlinear mathematical model has been proposed to study the effect of mitigation options on abatement of methane emissions caused by rice paddies and livestock populations in the atmosphere. In the modeling process, four nonlinearly interacting variables, namely, the cumulative density of rice paddies, the cumulative density of livestock populations, the atmospheric concentration of methane and the cumulative density of mitigation options have been considered. The cumulative densities of rice paddies and the livestock populations have been assumed to grow logistically with their respective intrinsic growth rates and carrying capacities. The increase in the atmospheric concentration of methane is taken to be directly proportional to the cumulative densities of rice paddies and livestock populations. The cumulative density of mitigation options, assumed to be proportional to the increased level of atmospheric methane concentration from its equilibrium, is however to decrease the atmospheric methane concentration. The analysis of the proposed model has been carried out using stability theory of ordinary differential equations and numerical simulations. It has been shown analytically and numerically that the atmospheric concentration of methane increases with increase in the emission rate coefficients of methane due to rice paddies and livestock populations. This increase in the atmospheric concentration of methane above its equilibrium level requires increased level of cumulative density of mitigation options. On increasing the implementation rate coefficient of various mitigation options the atmospheric concentration of methane decreases. It reduces further with increase in the depletion rate coefficient due to net effectiveness of mitigation options. Thus, implementation and efficiency of mitigation options for abatement of methane emission caused by rice paddies and livestock populations can be of immense use to curb the elevated level of methane concentration in the atmosphere.

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