Article

Time lag analysis of novel arithmetic modeling in breast cancer

Kalyan Das[1], **Ranjith Kumar**[2], **Pankaj Taneja**[3], **M Lutfor Rahman**[4]

[1]National Institute of Food Technology Entrepreneurship and Management, HSIIDC Industrial Estate, Kundli, Haryana, India

[2]Anurag University, Hyderabad, Telangana, India

[3]Sharda University, Knowledge Park III , Greater Noida, Uttar Pradesh, India

[4]Institute of Statistical Research and Training, University of Dhaka, Dhaka 1000, Bangladesh

E-mail: daskalyan27@gmail.com, ranjithreddy1982@gmail.com, pankaj12750@rediffmail.com, lutfor@isrt.ac.bd

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Abstract

In this paper, a mathematical model which considers population dynamics among infected and uninfected cancer tumor cells has been proposed. Delay differential equations have been utilized to demonstrate the framework to consider the periods of the cell cycle. We examine the steadiness of the framework and demonstrate a hypothesis dependent on the contention standard to decide the dependability of a fixed point and show that the solidness may rely upon the delay. We show hypothetically as well as through numerical results that periodic oscillations may arise through Hopf bifurcations. In this paper we study a stochastic model for the conduct of malignancy tumors, depicted by a stochastic differential condition with multiplicative noise term. We study the existence of the solution process, as well as its behavior in the framework of stochastic inclusion problems and long time behavior.

Keywords breast cancer; time lag; stability; hopf-bifurcation; stochasticity.

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1 Introduction

Breast cancer is a frequently occurred cancer among women, which is affecting 2.1 million women each year. It caused approximately 15% of all cancer deaths among women in 2018 (WHO, 2020). Symptoms of breast cancer include breast lump, bloody discharge from the nipple, and changes in the nipple or breast shape or texture. The treatment of cancer depends on type of cancer. For instance, chemotherapy, radiation, hormone therapy, and surgery some options to treat breast cancer (Diaby et al., 2015). The target organ breast cancer is a collection of malignancies in breast epithelial cells. Histological subtypes include ductal (70-80% of diagnosed cases), lobular (10-15% of diagnosed cases) or medular (3-5% of diagnosed cases). About 65% of

To establish the Hopf bifurcation at $\tau = \overline{\tau}$, we have to show that $\frac{d\mu}{d\tau} \neq 0$. For this differentiating (14) and (15) with respect to τ and $\tau = \overline{\tau}, \mu = 0 \text{ and } v = \overline{v}$ we get

$$\begin{aligned} L\frac{d\mu}{d\tau} + M\frac{dv}{d\tau} &= U \\ -M\frac{d\mu}{d\tau} + L\frac{dv}{d\tau} &= V \end{aligned} \tag{17}$$

where

$L = P_1 - Q_2\tau\cos v\tau + Q_1\cos v\tau - Q_1 v\tau\sin v\tau$; $M = -2v - Q_2\tau\sin v\tau + Q_1\sin v\tau + Q_1 v\tau\cos v\tau$;
$U = Q_2 v\sin v\tau - Q_1 v^2\cos v\tau$; $V = Q_2 v\cos v\tau + Q_1 v^2\sin v\tau$;

Soling (17) we get

$\frac{d\mu}{d\tau}(\overline{\tau}) = \frac{LU - MV}{L^2 + M^2}$ and the sign of $\frac{d\mu}{d\tau}(\overline{\tau})$ is same as $LU - MV$.

$$LU - MV = \left(P_1Q_2v + 2Q_1v^3\right)\sin v\tau + \left(2Q_2v^2 - P_1Q_1v^2\right)\cos v\tau - Q_1^2v^2$$

$$= v\left[2v\left(Q_1v\sin v\tau + Q_2\cos v\tau\right) - P_1\left(Q_1v\cos v\tau - Q_2\sin v\tau\right) - Q_1^2v^2\right]$$

Let $\phi(h) = h^2 + A_1h + A_2$

Clearly $\phi(h)$ is the L.H.S. of (16) with $v^2 = h$ and $\phi(v^2) = 0$ then v is the first positive root of (16) and $\frac{d\mu}{d\tau} > 0$ at $\tau = \overline{\tau}$ and stability cannot take the place these values of τ and

$\frac{d\mu}{d\tau}(\overline{\tau}) = \frac{v^2}{L^2 + M^2}\frac{d\varphi}{dh}(v^2)$ where $\overline{\tau} = \frac{1}{\overline{v}}\arctan\frac{Q_1\overline{v}^3 - (P_2Q_1 + P_1Q_2)\overline{v}}{\overline{v}^2(Q_2 - P_1Q_1) - P_2Q_2} + \frac{n\pi}{\overline{v}}, n = 0,1,2,3...$

By Hopf bifurcation theorem we can state that if $M_1 > 0, \text{and } M_2 < 0$, then there exist $\overline{\tau} > 0$ such that E^* possesses a supercritical type Hopf-bifucrcation as τ increases and passes through $\overline{\tau}$.

4 Stochastic Stability Analysis

In this section we presented the environmental disturbances on the system (1) by white noise theory. These results are discussed at positive equilibrium point. To discuss the stability of the stochastic system, we consider the linearized model with the perturbations $x_1 \text{ and } x_2$. By using mean-square fluctuations we characterized the stochastic stability of the system. The stochastic perturbed system with delay is given by

$$\frac{dx(t)}{dt} = \left[rx\left(1 - \frac{x+y}{k}\right) - dx - \beta xy\right]dt + q_1\xi_1(t) \tag{18}$$

$$\frac{dy(t)}{dt}=\left[\beta x(t-\tau)y(t-\tau)+sy\left(1-\frac{x+y}{k}\right)-\alpha y\right]dt+q_2\xi_2(t)$$

Linearising above system with the perturbations $x_1(t)\,and\,x_2(t)$ i.e., $x=x_1+x^*, y=x_2+y^*$ then

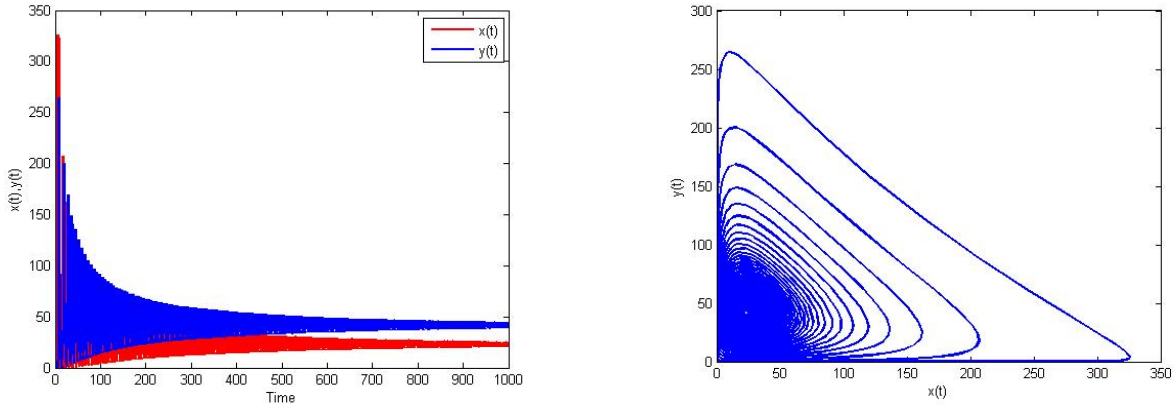

Fig. 2 The trajectories and phase graphs of system (1) with the parameter values as $r=2; k=400; d=0.211; \beta=0.035; s=0.01; \alpha=0.814;$ and $\tau=0.09$ is stable.

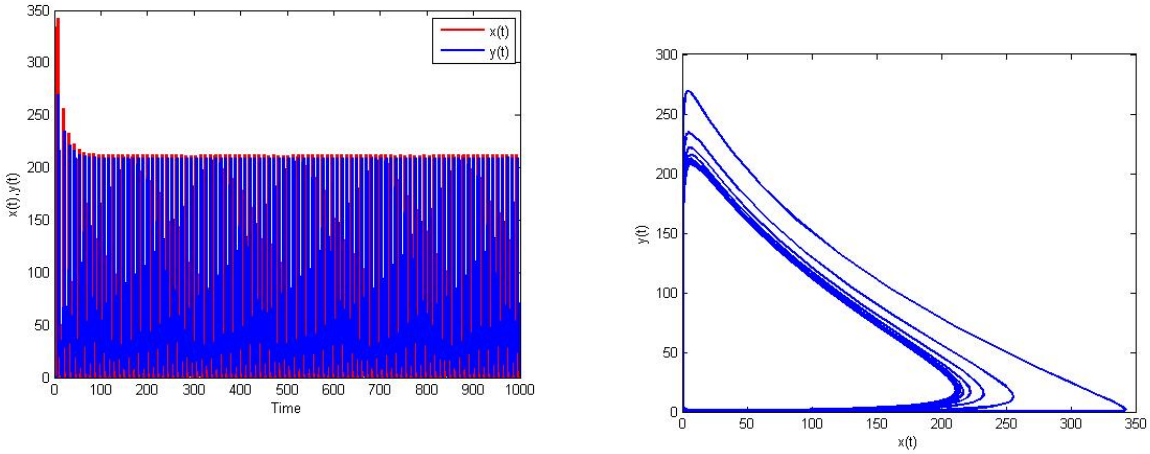

Fig. 3 The trajectories and phase graphs of system (1) with the parameter values as $r=2; k=400; d=0.211; \beta=0.035; s=0.01; \alpha=0.814;$ and $\tau=0.2$ is unstable and a periodic orbit bifurcate from E^*

$$\frac{dx_1(t)}{dt}=\frac{-2rx^*x_1}{k}-\frac{-rx^*x_2}{k}-\beta x^*x_2+q_1\xi_1(t) \tag{19}$$

$$\frac{dx_2(t)}{dt}=\beta x_1(t-\tau)-\frac{sy^*x_1}{k}-\frac{2sy^*x_2}{k}+q_2\xi_2(t)$$

Applying Fourier transforms both sides and we obtain,

$$q_1\xi_1(t)=\left(i\omega+\frac{2rx^*}{k}\right)x_1(\omega)+\left(\frac{rx^*}{k}+\beta x^*\right)x_2(\omega)$$

$$q_2\xi_2(t)=\left(\frac{sy^*}{k}-\beta e^{-i\omega\tau}\right)x_1(\omega)+\left(i\omega+\frac{2sy^*}{k}\right)x_2(\omega)$$

The matrix form of above system is given by

$$\overline{\xi}(\omega)=M(\omega)X(\omega) \tag{20}$$

and denoting the elements of $M(\omega)$ are $m_{11},m_{12},m_{21}\,\&\,m_{22}$ (row wise) then

$M(\omega)=\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}$ where

$$\overline{\xi}(\omega)=\begin{bmatrix} q_1\xi_1(t) \\ q_2\xi_2(t) \end{bmatrix};\quad X(\omega)=\begin{bmatrix} x_1(\omega) \\ x_2(\omega) \end{bmatrix};\quad m_{11}=i\omega+\frac{2rx^*}{k};m_{12}=\frac{rx^*}{k}+\beta x^*;$$

$$m_{21}=\frac{sy^*}{k}-\beta e^{-i\omega\tau};m_{22}=i\omega+\frac{2sy^*}{k}.$$

Here $M(\omega)$ is non-singular matrix then inverse of this matrix exist, therefore from (20)

$$X(\omega)=M^{-1}(\omega)\overline{\xi}(\omega)=N(\omega)\overline{\xi}(\omega) \tag{21}$$

Where $N(\omega)=\begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}=M^{-1}(\omega)=\begin{bmatrix} \dfrac{m_{22}(\omega)}{|M(\omega)|} & -\dfrac{m_{12}(\omega)}{|M(\omega)|} \\ -\dfrac{m_{21}(\omega)}{|M(\omega)|} & \dfrac{m_{11}(\omega)}{|M(\omega)|} \end{bmatrix}$

Now from the spectral density, we define

$$S_g(\omega)d\omega=\lim_{T\to\infty}\frac{\overline{|\overline{g}(\omega)|^2}}{T}$$

Where $g(t)$ a random function with is mean zero and $S_g(\omega)$ represents the variance of the elements of $g(t)$ within the interval $[\omega,\omega+d\omega]$.

The inverse transform of $S_g(\omega)$ is the auto covariance function is given by

$$C_g(\tau')=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_g(\omega)e^{i\omega\tau'}d\omega$$

and the variance function $g(t)$ is given by

$$\sigma^2{}_g = C_g(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_g d\omega$$

From (21), the mean value of the population is $\overline{x}_i = \sum_{j=1}^{2} n_{ij}\xi_j(\omega)$ where $b_{ij}, i, j = 1,2$

Therefore, $S_{x_i} = \sum_{j=1}^{2} q_j \left|n_{ij}(\omega)\right|^2$, $(i = 1,2)$

The fluctuations of $x_i (i = 1,2)$ are given by

$$\sigma^2{}_{x_i} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{x_i} d\omega = \frac{1}{2\pi}\sum_{j=1}^{2}\int_{-\infty}^{\infty} q_j \left|n_{ij}(\omega)\right|^2 d\omega$$

Therefore, from above variances and from system (18), we can find

$$\sigma_{x_1}^2 = \frac{1}{2\pi}\left[q_1 \int_{-\infty}^{\infty} \left|\frac{m_{22}(\omega)}{|M(\omega)|}\right|^2 d\omega + q_2 \int_{-\infty}^{\infty} \left|\frac{-m_{12}(\omega)}{|M(\omega)|}\right|^2 d\omega \right]$$

$$\sigma_{x_2}^2 = \frac{1}{2\pi}\left[q_1 \int_{-\infty}^{\infty} \left|\frac{-m_{21}(\omega)}{|M(\omega)|}\right|^2 d\omega + q_2 \int_{-\infty}^{\infty} \left|\frac{m_{11}(\omega)}{|M(\omega)|}\right|^2 d\omega \right]$$

Where $|M(\omega)| = M_1(\omega) + iM_2(\omega)$ and

$$M_1(\omega) = -\left(\omega^2 - \frac{3rsx^* y^*}{k^2} + \frac{s\beta x^* y^*}{k} - \frac{r\beta x^*}{k}\cos\omega\tau - \beta^2 x^* \cos\omega\tau \right)$$

$$M_2(\omega) = -\left(-\frac{2sy^*\omega}{k} - \frac{2rx^*\omega}{k} + \frac{r\beta x^*}{k}\sin\omega\tau + \beta^2 x^* \sin\omega\tau \right)$$

The variances of $x_i (i = 1,2)$ are given by

$$\sigma_{x_1}^2 = \frac{1}{2\pi}\left[q_1 \int_{-\infty}^{\infty} \frac{\left|m_{22}(\omega)\right|^2}{M_1^2(\omega) + M_2^2(\omega)} d\omega + q_2 \int_{-\infty}^{\infty} \frac{\left|m_{12}(\omega)\right|^2}{M_1^2(\omega) + M_2^2(\omega)} d\omega \right]$$

$$\sigma_{x_2}^2 = \frac{1}{2\pi}\left[q_1 \int_{-\infty}^{\infty} \frac{\left|m_{21}(\omega)\right|^2}{M_1^2(\omega) + M_2^2(\omega)} d\omega + q_2 \int_{-\infty}^{\infty} \frac{\left|m_{11}(\omega)\right|^2}{M_1^2(\omega) + M_2^2(\omega)} d\omega \right] \quad (22)$$

Where $|m_{11}|^2 = \omega^2 + \left(\frac{2rx^*}{k}\right)^2$; $|m_{12}|^2 = \left(\frac{rx^*}{k} + \beta x^*\right)^2$

$$|m_{21}|^2 = \left(\frac{sy^*}{k} - \beta\cos\omega\tau\right)^2 + (\beta\sin\omega\tau)^2; |m_{22}|^2 = \omega^2 + \left(\frac{2sy^*}{k}\right)^2$$

The results in (22) gives the variances of the system (18) populations x and y. Generally, to find these integrals it is very difficult. Therefore, by using numerical simulations we can explain these results easily. Taking different parameter values and for the some time lag we can calculate variance and this is small then the corresponding population is stable, otherwise unstable.

5 Conclusion and Discussions

In this paper we have formulated a delayed cancer model with infected cells and antiseptic tumor cells. Breast cancer risk assessments using statistical data based on epidemiologic data are valid; however, no single model includes family history, estrogen-dependent mechanisms, and abnormal breast disease. For this reason it is helpful to use different models in a specialized risk assessment clinic, but this requires a full understanding of the strengths and powers of a particular relationship.

From Fig. 2, we have a tendency to look at how quickly the positive balance of program (1) is stabilized as at home. Modeling within this state of affairs produces sustainable energy. With the same set of parameter values, we find that when the system is stable, it can be seen in Fig. 2 and when it is unstable Fig3 with constant stability. That means the system changes its behavior from stable to unstable when time delay crosses its critical value, which gives Hopf-bifurcation. From the biological point of view delay has influence in spread of virus in tumor cells. Conjointly one should bear in mind the pause to allow for the management of the germ cell growth. The existence of time-resolved solutions is consistent in cancer models. It means that the tumor level can get around a difficult and immediate purpose despite no other treatment. We have accomplished numerical simulation of the random model quickly so as to add the increasing white noise of the same amount of fluctuations present in the system and that we often look at the fluctuations in the amplitude of the sounds (Fig. 4).

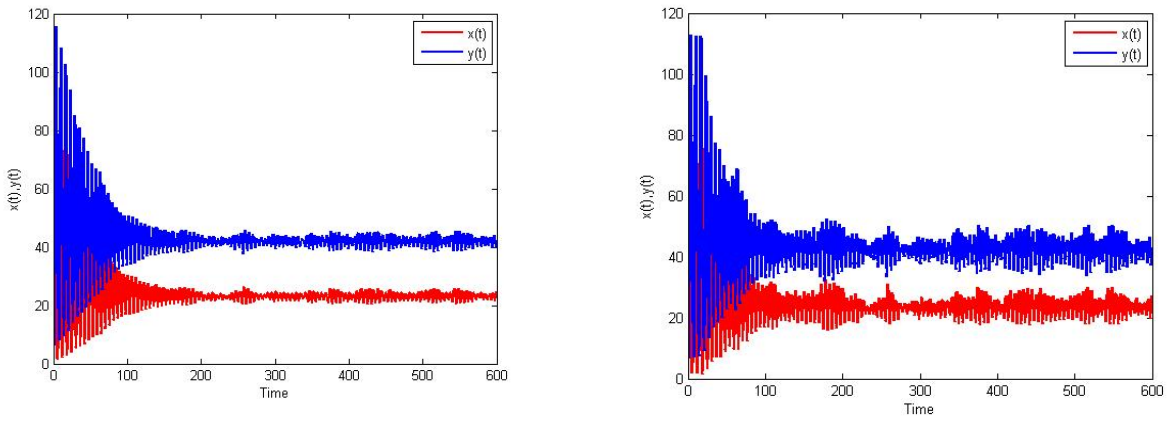

Fig. 4 The trajectories of system (18) with the parameter values as $r = 2; k = 400; d = 0.211; \beta = 0.035; s = 0.01; \alpha = 0.814;$ and $q_1 = 0.1, q_2 = 0.4$ (left figure) & $q_1 = 0.7, q_2 = 0.9$ (right figure)

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