*Article*

# Susceptible-Infected-Treatment model with disease infection in prey population recovered by treatment

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**Abstract**
This paper describes a mathematical model with Susceptible-Infected-Treatment under disease infection in prey population. The controls methods are one is separating control that is to separate the prey from infected prey by means of self quarantine and the another one as a treatment control that is to reduce the rate of death caused by the disease. A local study of the model is performed around the disease-free equilibrium and the endemic equilibrium to estimate the effect of parameters that control disease spread and species coexistence. Some numerical solutions are given to explain most of the analytical results.

**Keywords** susceptible-infected; prey-predator system; global stability; treatment; Hopf bifurcation; harvesting.

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## 1 Introduction

Ecology and epidemiology are two major and distinct fields of study (Su and Wang, 2015; Vijaya et al., 2017; Bezabih et al., 2021). Populations of each level are under constant threat of diseases (Banerjee, 2017; Saikh and Gazi, 2018; Zhang et al., 2020). This type prey-predator model is susceptible infected-treatment model assumption of four nonlinear differential equations. One of the important components of the predator-prey relationships is the rate of predator's feeding depends upon the prey (Upadhyay and Iyengar, 2013).

Below we consider a mathematical model for covid-19 epidemic around a period of about 15 days. For this period it is valuable to assume that infection will spread all parts of the body will destroy one by one. At finally it will affect the lungs it may cause breathing problems to reduce the oxygen level and leads to death. Many diseases are spread by infected individuals in the population coming into close contact with susceptible individuals. These include influenza, measles, chicken pox and AIDS (Vijaya et al., 2017). Covid-19 also spread by the infected individuals through sneezing, cold and cough, touch each other and it is easily transmitted through all aged people (Alshammari and Talay, 2021; Musa et al., 2021; Yadav et al., 2021).

We introduce harvesting factor in the predator to balance the predator-prey population (Amalia and Arif, 2018). Harvesting policies is one of the major and familiar problems from ecologically and economic point of view (Kar, 2006). Two control methods to avoid this disease as treatment given for the lower immunity level population and boost their immunity power for infected individuals through vaccination. Another method to reduce Covid-19 is by self quarantine because the incubation period is around 15 days. We take some home remedies like natural medicine, Ayurveda, Yoga and meditation.

In this prey-predator epidemiological model in section 2, we modify the model with four species and explanation. In section 3, the recent researchers are proved that the population is always positive and bounded. In section 4, we find that the system has stability with equilibrium points. In section 5, the researchers have been examined the stability of equilibrium points. In section 6, the researchers obtain conditions for Hopf bifurcation. In final section presents the conclusion about the key findings of the problem.

## 2 Method

$$\frac{dS}{dt} = aS(t)\left(1 - \frac{S(t)}{k}\right) - \frac{cN(t)S(t)}{1+N(t)} - \beta P(t)S(t)$$

$$\frac{dN}{dt} = \frac{cN(t)S(t)}{1+N(t)} - jN(t)P(t) - \delta N(t) - eN(t)$$

$$\frac{dT}{dt} = vT(t) - fT(t) + \delta N(t)$$

$$\frac{dP}{dt} = \psi N(t)P(t) + \phi S(t)P(t) - hP(t) \tag{1}$$

with initial densities

$$S(0)>0,\ N(0)> 0,\ T(0) > 0,\ P(0) > 0 \tag{2}$$

Here, $S(t)$, $N(t)$ ,$T(t)$ and $P(t)$ denote the susceptible prey, infected prey, treatment and predator respectively and all the parameters are positive.

The methods to select parameter values for is based on the dynamical presentation of a given model

We described the following model parameters below:

a is the growth rate of susceptible prey.

$k$ is the carrying capacities of susceptible prey.

c is the interference coefficients of two prey species with Holling type II functional response .

$\beta$, $j$ are the force of infection of preys from predator.

e is the natural death rate of infected prey.

$\delta$ is the recovered rate of infected prey.

f relates the death of infected prey under treatment.

$v$ denotes the growth rate of prey under treatment.

$\varphi$, $\psi$, are the conversion coefficient of predator from susceptible and infected preys population.

h is the harvest effort of predator.

This model has some assumptions which consist of the followings:

1. The susceptible and infected prey interaction take place with Holling type II functional response.
2. We identify the number of infected prey and given treatment for them.
3. The treatment will be given for infected individuals as some home remedies like natural medicine, ayurveda, yoga and meditation.
4. We consider the growth rate by the raise of immunity power and death rate for the species under treatment.
5. For healthy prey will be given vaccine before they affected by the predator.

### 3 Positiveness and Boundedness of Theorem

In this section, we establish the conditions to get positive as well as bounded solutions of the system.

#### 3.1 Positivity

**Theorem 1.** Every solution of system (1) with initial conditions (2) always exists in the interval $[0,\infty)$ and $S(t) > 0, N(t) > 0, T(t) > 0, P(t) > 0$, for all $t \geq 0$

**Proof.**

In the right hand side of equation (1) is completely continuous and relates to locally Lipschitzian on C, the conditions (2) exists and is unique on $[0,\xi)$, where $0 < \xi \leq +\infty$ (Sharma and Samanta, 2015). From equation (1) and (2), we have

$$S(t) = S(0)\ exp\ [\int_0^t \{a\left(1-\frac{S}{k}\right) - \frac{cN}{1+N} - \beta P\}\ dt\ ] > 0$$

$$N(t) = N(0)\ exp\ [\int_0^t \{\frac{cS}{1+N} - jP - \delta - e\}\ dt\ ] > 0$$

$$T(t) = T(0)\ exp\ [\int_0^t \{v - f + \frac{\delta N}{T}\} dt] > 0$$

$$P(t) = P(0)\ exp\ [\int_0^t \{\psi N + \phi S - h\}\ dt] > 0$$

hence we proved.

#### 3.2 Boundedness

**Theorem 2.** Susceptible prey is always bounded above for $a > 0$, $k > 0$.

**Proof.**

If $S(0) = 0$ then the result is obvious, if $S(0) > 0$, Then $S(t) > 0$ for all t on adding equation (1) we obtain

$$\frac{dS}{dt} \leq a\left(1-\frac{S}{k}\right) \qquad \lim_{t\to\infty}(\sup S(t)) \leq k$$

**Theorem 3.** Infected prey, Treatment and Predator are bounded above.

**Proof.**

If $N(0) = 0$ the result is obvious. We obtain the equation (1)

If $N(0) > 0$, then $\frac{dN}{dt} < 0$ if $d_1N > 1$, $\frac{dT}{dt} < 0$ if $d_2T > 1$, $\frac{dP}{dt} < 0$ if $d_3T > 1$

where $d_1 = \delta + e$, $d_2 = v + f$, & $d_3 = h$

$$\Rightarrow \lim_{t\to\infty}(\sup N(t)) \leq \frac{1}{d_1}\ ,\ \lim_{t\to\infty}(\sup T(t)) \leq \frac{1}{d_2}\ ,\ \lim_{t\to\infty}(\sup P(t)) \leq \frac{1}{d_3}$$

**Theorem 4.** The path of the equation (1) are bounded.

**Proof.**

Define the function $\Omega = S + N + T + P$ and take its time derivative along the solution of (1)

$\frac{d\Omega}{dt} = \frac{dS}{dt} + \frac{dN}{dt} + \frac{dT}{dt} + \frac{dP}{dt}$ since the parameters are non negative and solutions initiating in $R_+^4$ remain in the non negative quadrant, then $\frac{d\Omega}{dt} + \rho\Omega \leq aS\left(1 - \frac{S}{k}\right) - eN + vT - fT - hP + \rho S + \rho N + \rho T + \rho P$

$$=(\rho+a)S+(\rho-e)N+\left(\frac{\rho+v}{f}\right)T + (\rho - h)P - \frac{aS^2}{k}$$

where $\rho$ is a nonnegative constant for $\rho > e$ or $\rho > h$ given ε>0 there exists to such that t on t ≥ $t_0$

$\frac{d\Omega}{dt} + \rho\Omega \leq m + \varepsilon$, $m = \min\{(\rho+a), \left(\frac{\rho+v}{f}\right), (\rho-e), (\rho - h)\}$

**Lemma .** Let φ be an absolutely-continuous function satisfying the differential inequality

$$\frac{d\phi(t)}{dt} + \alpha_1\phi(t) \leq \alpha_2, t \geq 0 \text{ where}(\alpha_1, \alpha_2) \in R_+^3, \alpha_1 \neq 0 \text{then } \forall t \geq T \geq 0, \phi(t) \leq \frac{\alpha_2}{\alpha_1} - \left(\frac{\alpha_2}{\alpha_1}\phi(\underline{T})\right)e^{-\alpha_1(t-T)}.$$

From the above lemma (Aziz-Alaoui and Daher Okiye. 2003),

$$\Rightarrow \Omega(t) \leq \Omega(t_0)e^{-\rho(t-t_0)} + \left(\frac{m+\varepsilon}{\rho}\right)\left(1 - e^{-\rho(t-t_0)}\right)$$

Letting $t \to \infty$ then letting $\varepsilon \to 0$ $\lim_{t\to\infty}(\sup \Omega(t)) \leq \frac{m}{\rho}$

On the initial conditions, the system (1) are bounded.

## 4 Nonlinear Systems and Stability Analysis

### 4 .1 Equilibrium points

The equilibrium points of the parametric model (1) is given by steady state equations $\frac{dS}{dt} = \frac{dN}{dt} = \frac{dT}{dt} = \frac{dP}{dt}$. The system has four equilibrium points and after some algebraic calculations we get the trivial, axial and non-trivial equilibrium points (Table 1).

**Table 1** Stability analysis of equilibrium points.

| Equilibrium point | Explanation | Stable/Unstable |
|---|---|---|
| $E1\{S = 0, N = 0, T = 0, P = 0\}$ | Trivial | saddle point |
| $E2\{S = k, N = 0, T = 0, P = 0\}$ | infected prey, treatment &predator-free | stable |
| $E3\{S = \frac{h}{\phi}, N = 0, T = 0, P = \frac{-a(-k\phi+h)}{k\beta\phi}\}$ ) | infected prey& treatment free | stable |
| $E4\{S = S^*, N = N^*, T = T^*, P = P^*\}$ | Coexistence | stable |

### 4.2 Analysis stability and existence of equilibrium points

The jacobian matrix of the system (1) at equilibrium point E=(S(t),N(t),T(t),P(t)) is given by

$$J = \begin{bmatrix} m_{11} & m_{12} & 0 & -S\beta \\ m_{21} & m_{22} & 0 & -Nj \\ 0 & \delta & m_{33} & 0 \\ P\phi & P\psi & 0 & m_{44} \end{bmatrix}$$

where $m_{11} = a - 2\frac{aS}{k} - \frac{cN}{1+N} - \beta S$,

$m_{12} = S\left(-\frac{c}{1+N} + \frac{cN}{(1+N)^2}\right)$,

$m_{21} = \frac{cN}{1+N}$,

$m_{22} = \frac{cS}{1+N} - jP - \delta - e - \frac{cNS}{(1+N)^2}$

$m_{33} = v - f$, $m_{44} = \psi N + \phi S - h$

based on the condition of eigen values, the dynamical system (1) gets stable if all the eigen values are non positive in case of real roots or non positive real parts in case of complex roots of the characteristic equation for the above jacobian matrix. Otherwise the system is unstable.

**Theorem 5.** Given the linearized system of equations (1) is infected prey, treatment and predator-free equilibrium point. Then the equilibrium point $\{S = k, N = 0, T = 0, P = 0\}$ is locally asymptotically stable provided that $\lambda_1 < 0$, $\lambda_2 < 0$, $\lambda_3 < 0$ & $\lambda_4 < 0$ with the condition
$\delta + e > ck, f > \upsilon$ & $h > k\varphi$.

**proof.**

The jacobian matrix is

$$E2 = \begin{bmatrix} -a & -kc & 0 & -k\beta \\ 0 & kc - \delta - e & 0 & 0 \\ 0 & \delta & v - f & 0 \\ 0 & 0 & 0 & k\phi - h \end{bmatrix}$$

Now the eigen values are

$\lambda_1 = -a$, $\lambda_2 = ck - \delta - e$, $\lambda_3 = \upsilon - f$, $\lambda_4 = k\alpha - h$

Hence the equilibrium point (1) is locally stable if $\lambda_1 < 0$, $\lambda_2 < 0$, $\lambda_3 < 0$ & $\lambda_4 < 0$ with the condition $\delta + e > ck$, $f > v$ & $h > k\varphi$.

**Theorem 6.** Given the linearized system of equations (1) is infected prey & treatment-free equilibrium point. Then the equilibrium point $\{S = \frac{h}{\phi}, N = 0, T = 0, P = \frac{-a(-k\phi+h)}{k\beta\phi}\}$ is locally asymptotically stable provided that $\lambda_1 < 0$, $\lambda_2 < 0$ with the condition $ajk\varphi + k\beta\delta\varphi + ek\beta\varphi > chk\beta + ahj$, $f > v$ and $\lambda_3$ & $\lambda_4$ have negative real parts.

**proof.**

The jacobian matrix is

$$E3 = \begin{bmatrix} a - 2\frac{ah}{k\phi} + \frac{a(-k\phi + h)}{k\phi} & \frac{-ch}{\phi} & 0 & -\frac{\beta h}{\phi} \\ 0 & \frac{ch}{\phi} + \frac{aj(-k\phi + h)}{k\beta\phi} - \delta - e & 0 & 0 \\ 0 & \delta & v - f & 0 \\ -\frac{a(-k\phi + h)}{k\beta} & -\frac{a\psi(-k\phi + h)}{k\beta\phi} & 0 & 0 \end{bmatrix}$$

Now the eigenvalues are

$\lambda_1 = \frac{-ajk\phi + ck\beta h - \beta\delta\phi k - \beta\phi ek + ahj}{k\beta\phi}$, $\lambda_2 = v - f$ ,

$\lambda_3 = \frac{1}{2}\frac{-ha + \sqrt{-4ahk^2\phi^2 + 4ah^2k\phi + a^2h^2}}{\phi k}$, $\lambda_4 = -\frac{1}{2}\frac{ha + \sqrt{-4ahk^2\phi^2 + 4ah^2k\phi + a^2h^2}}{\phi k}$

hence the equilibrium point (1) is locally stable if $\lambda_1 < 0$, $\lambda_2 < 0$, with the condition $ajk\varphi + k\beta\delta\varphi + ek\beta\varphi > chk\beta + ahj$, $f > \upsilon$ and $\lambda_3$ & $\lambda_4$ have negative real parts.

**Theorem 7.** The interior equilibrium point {S = S*,N =N*,T =T*, P=P*} is locally asymptotically stable.

**proof.**

The Variation of the Jacobian matrix is

$$E4 = \begin{bmatrix} m_{11} & m_{12} & 0 & -S^*\beta \\ m_{21} & m_{22} & 0 & -N^*j \\ 0 & \delta & m_{33} & 0 \\ P^*\phi & P^*\psi & 0 & m_{44} \end{bmatrix}$$

Where $m_{11} = a - 2\frac{aS^*}{k} - \frac{cN^*}{1+N^*} - \beta P^*$, $m_{12} = S^*\left(-\frac{c}{1+N^*} + \frac{cN^*}{(1+N^*)^2}\right)$, $m_{21} = \frac{cN^*}{1+N^*}$,

$m_{22} = \frac{cS^*}{1+N^*} - jP^* - \delta - e - \frac{cN^*S^*}{(1+N^*)^2}$

$m_{33} = v - f$, $m_{44} = \psi N^* + \phi S^* - h$

The characteristic equation is $\Lambda(\lambda) = \lambda^4 + B_3\lambda^3 + B_2\lambda^2 + B_1\lambda^1 + B_0$

Where $B_3 = 4I-(m_{11}+m_{22}+m_{33}+m_{44})$

$B_2 = 6I^2-(m_{11}+m_{22}+m_{33}+m_{44})3I+m_{22}m_{33}+m_{22}m_{44}+m_{33}m_{44}+m_{11}m_{22}+m_{11}m_{33}+m_{11}m_{44} - m_{12}m_{22} + S^*P^*\beta\phi$

$B_1 = 4I^3 -(m_{11} + m_{22} + m_{33} + m_{44})3I^2 + (2m_{22}m_{33} + 2m_{22}m_{44} + 2m_{33}m_{44} + 2m_{22}m_{11} + 2m_{33}m_{11} + 2m_{44}m_{11} - 2m_{12}m_{21} + 2S^*P^*\beta\phi + m_{11}N^*P^*j\psi)I - m_{22}m_{33}m_{44} - m_{22}m_{33}m_{11} - m_{22}m_{44}m_{11} - m_{33}m_{44}m_{11} + m_{12}m_{21}m_{33} + m_{12}m_{21}m_{44} + m_{11}N^*P^*j\psi + m_{12}N^*P^*j\phi + m_{21}S^*P^*\beta\psi - m_{22}S^*P^*\beta\phi - m_{33}S^*P^*\beta\phi$

$B_0 = I^4-(m_{11}+m_{22}+m_{33}+m_{44})I^3+(m_{11}m_{22}+m_{11}m_{33}+m_{11}m_{44}+m_{22}m_{33}+m_{33}m_{44}+ m_{22}m_{44} + N^*P^*j\psi - m_{12}m_{21})I^2 - (m_{11}m_{22}m_{33} + m_{11}m_{22}m_{44} + m_{11}m_{33}m_{44} + m_{22}m_{33}m_{44} + m_{11}N^*P^*j\psi + m_{33}N^*P^*j\psi - m_{33}m_{12}m_{21} - m_{44}m_{12}m_{21} - m_{12}N^*P^*j\phi)I + m_{11}m_{22}m_{33}m_{44} + m_{11}m_{33}N^*P^*j\psi - m_{33}m_{44}m_{12}m_{21} - m_{12}m_{33}N^*P^*j\phi$

By Routh Hurwitz criterion, all the eigenvalues of $j_4$ have negative real parts if

(i) $B_3 > 0$,

(ii) $B_3B_2 > B_1$ ,

(iii) $B_3B_2B_1 > B_1^2 + B_3^2 B_0$.

Therefore the given system of the nonlinear differential equation (1) is locally asymptotically stable around the non negative equilibrium point {S = S*,N = N*,T = T*,P = P*} if the conditions mentioned in the theorem holds.

### 4.3 Global stability analysis

We perform a global analysis of the system(1) around the non negative equilibrium point *G*(S*,N*,T*,P*) of the coexistence. The following theorem of Lyapunov function Δ is considered (Hugo et al., 2012; Ackleh et al., 2021).

**Theorem 8.** Let $\Delta = \frac{1}{2}(S - S^*)^2 + \frac{1}{2}\varsigma_1(N - N^*)^2 + \frac{1}{2}\varsigma_2(T - T^*)^2 + \frac{1}{2}\varsigma_3(P - P^*)^2$ where $\zeta_1,\zeta_2,\zeta_3>0$ are to be carefully chosen such that Δ' (G) = 0 then G(S*,N*,T*,P*) and Δ=(S,N,T,P) >0 S,N,T,P|G. If the time derivative of Δ is $\frac{d\Delta}{dt} \le 0, \forall\, S, N, T, P \in \Gamma^+$ then it follows that $\frac{d\Delta}{dt} = 0,\ \forall\, S, N, T, P \in \Gamma^+$s tends to $G^*$ of the system is Lyapunov stable and $\frac{d\Delta}{dt} < 0$ S,N,T,P∈ $\Gamma^+$ near implies that $G^*$ is globally stable.

**Proof.**

$$\frac{d\Delta}{dt} = (S - S^*)\frac{dS}{dt} + \varsigma_1(N - N^*)\frac{dN}{dt} + \varsigma_2(T - T^*)\frac{dT}{dt} + \varsigma_3(P - P^*)\frac{dP}{dt}$$

Now by substituting the equations (1) we get,

$$\frac{d\Delta}{dt} = (S - S^*)\{aS\left(1 - \frac{S}{k}\right) - \frac{cNS}{1+N} - \beta PS\} + \varsigma_1(N - N^*)\{\frac{cSN}{1+N} - jPN - \delta N - eN\} + \varsigma_2(T - T^*)\{vT - Tf + \delta N\} + \varsigma_3(P - P^*)\{\psi PN + \phi SP - hP\}$$

The above equation becomes

$$\frac{d\Delta}{dt} = (S - S^*)\left\{a\left(1 - \frac{S}{k}\right) - \frac{cN}{1+N} - \beta P\right\}\{(S - S^*)\} + \varsigma_1(N - N^*)\{\frac{cS}{1+N} - jP - \delta - e\}\{(N - N^*)\} + \varsigma_2(T - T^*)\{v - f + \frac{\delta N}{T}\}\{(T - T^*)\} + \varsigma_3(P - P^*)\{\psi N + \phi S - h\}\{(P - P^*)\}$$

By rearranging we obtain,

$$\frac{d\Delta}{dt} = (S - S^*)^2\left\{a\left(1 - \frac{S}{k}\right) - \frac{cN}{1+N} - \beta P\right\} + \varsigma_1(N - N^*)^2\{\frac{cS}{1+N} - jP - \delta - e\} + \varsigma_2(T - T^*)^2\{v - f + \frac{\delta N}{T}\} + \varsigma_3(P - P^*)^2\{\psi N + \phi S - h\}$$

Thus it is possible to set $\varsigma_1, \varsigma_2, \varsigma_3>0$ such that $\Delta' \le 0$ is an endemic positive equilibrium point is globally stable. Therefore it is pointed that the parameters play a key role in controlling the stability aspects of the system (Hugo and Simanjilo, 2019).

### 5 Hopf Bifurcation

In this section we examined the Hopf bifurcation around the interior equilibrium point E4. The parameter a is basic and represent the growth rate of prey S is identified as a bifurcation parameter. Hopf bifurcation occurs that the jacobian matrix E4 has a pair of merely imaginary eigenvalues and the other eigenvalues have non positive real parts and $Re[\frac{d\lambda}{da}]|_{a=a_0} \neq 0$ (Atena and Doust, 2021) . Presume that the characteristic equation at the interior equilibrium point E4 is as follows:

$$\lambda^4 + B_3\lambda^3 + B_2\lambda^2 + B_1\lambda^1 + B_0 = 0 \quad (3)$$

For merely imaginary eigenvalues, the coefficients of characteristic polynomial (3) satisfy the following condition:

$$B_3B_2B_1 - B_3^2 B_0 - B_1^2 = 0$$

Suppose ±iω is a pair of merely imaginary eigenvalues corresponding to $a_0$. We solve from the characteristic equation (3) tends to a (Hui Cao et al., 2021)

$$[4\lambda^3 + 3B_3\lambda^2 + 2B_2\lambda + B_1]\frac{d\lambda}{da} + (\lambda^3\frac{dB_3}{da} + \lambda^2\frac{dB_2}{da} + \lambda\frac{dB_1}{da} + \frac{dB_0}{da}) = 0$$

hence $$\frac{d\lambda}{da} = -\left(\frac{\lambda 3\frac{dB_3}{da} + \lambda 2\frac{dB_2}{da} + \lambda\frac{dB_1}{da} + \frac{dB_0}{da}}{4\lambda 3 + 3B_3\lambda 2 + 2B_2\lambda + B_1}\right) \quad (4)$$

We substitute *iω* in to equation (4), we have

$$\frac{d\lambda}{da}|_{i\omega} = -\left(\frac{-i\omega^3\frac{dB_3}{da} - \frac{\omega^2 dB_2}{da} + \frac{i\omega dB_1}{da} + \frac{dB_0}{da}}{-4i\omega^3 - 3B_3\omega^2 + 2B_2 i\omega + B_1}\right)$$

Hence

$$\mathrm{Re}\left(\frac{d\lambda}{da}|_{i\omega}\right) = -\left(\frac{[B_1-3B_3\omega^2]\left[\frac{dB_0}{da}-\omega^2\frac{dB_2}{da}\right]+[2B_2\omega-4\omega^3]\left[\omega^3\frac{dB_3}{da}-\omega\frac{dB_1}{da}\right]}{[B_1-3B_3\omega^2]^2+[2B_2\omega-4\omega^3]^2}\right)$$

**Theorem 9.** Consider the bifurcation parameter as ’a’ then System (1) sustains a Hopf bifurcation provided.

$$\left([B_1 - 3B_3\omega^2]\left[\frac{dB_0}{da} - \omega^2\frac{dB_2}{da}\right] + [2B_2\omega - 4\omega^3]\left[\omega^3\frac{dB_3}{da} - \omega\frac{dB_1}{da}\right]\right) \neq 0$$

## 6 Numerical Results

The system of the nonlinear differential equation (1) for the numerical solution. In this section we existing computer reflection of solutions under stability analysis.

1. First we take the parameter of the system as (β,k,ψ,j,υ,δ,c,f,φ,e,h,a) = (1.05,1.25,0.23,0.545,0.005,1.236,1.450,0.212,2.024,2.05,2.11,1.07) at the population (*S,N,T,P*) = (10.25,10.46,10.06,10.13) The given system is asymptotically stable (Fig. 1).

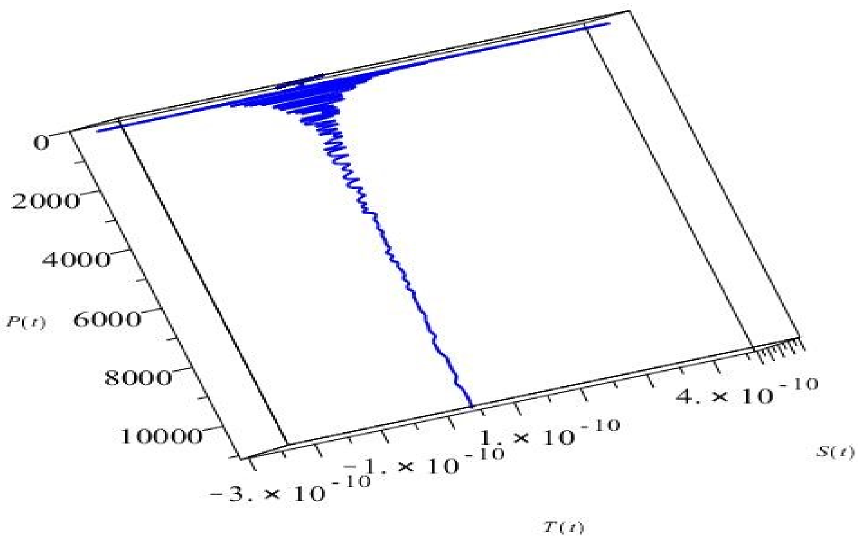


**Fig. 1** Rossler type of prey-predator is asymptotically stable.

2. If we take the parameter of the system as mentioned above in point (1). Then the initial condition satisfies $(S,N,T,P) = (10.2,0,0,0)$ is susceptible prey population (Fig. 2).

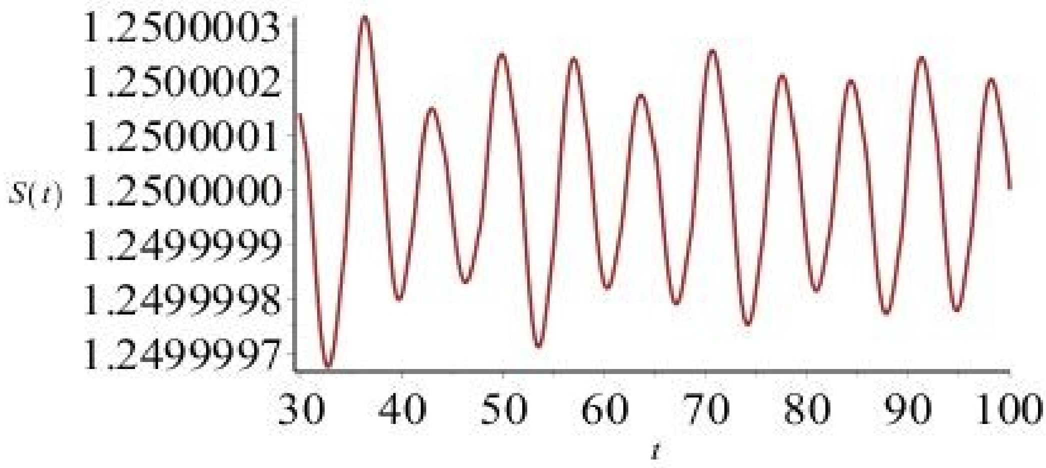


**Fig. 2** The stability analysis of susceptible prey.

3. If we take the parameter of the system as mentioned above in point (1). Then the initial condition satisfies $(S,N,T,P)=((1.2,0,0,1),(1.2,0,0,1.2))$ the contact rate of susceptible prey and predator interaction (Fig. 3).

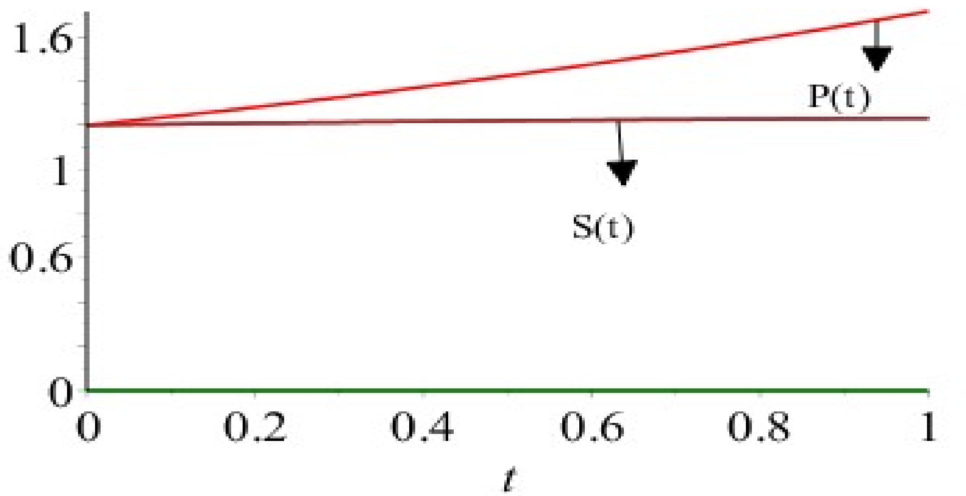


**Fig. 3** The stability analysis of susceptible prey and predator interaction.

4. If we take the parameter of the system as mentioned above in point (1). Then the initial condition satisfies $(S,N,T,P)$=((1.2,0.1,1,1),(1.3,0.6,0.7,0.2),(1.2,0.6,1.7,0.1),(1.2,1.46,1.7,1.2)) the contact rate of preys, treatment and predator interaction (Fig. 4).

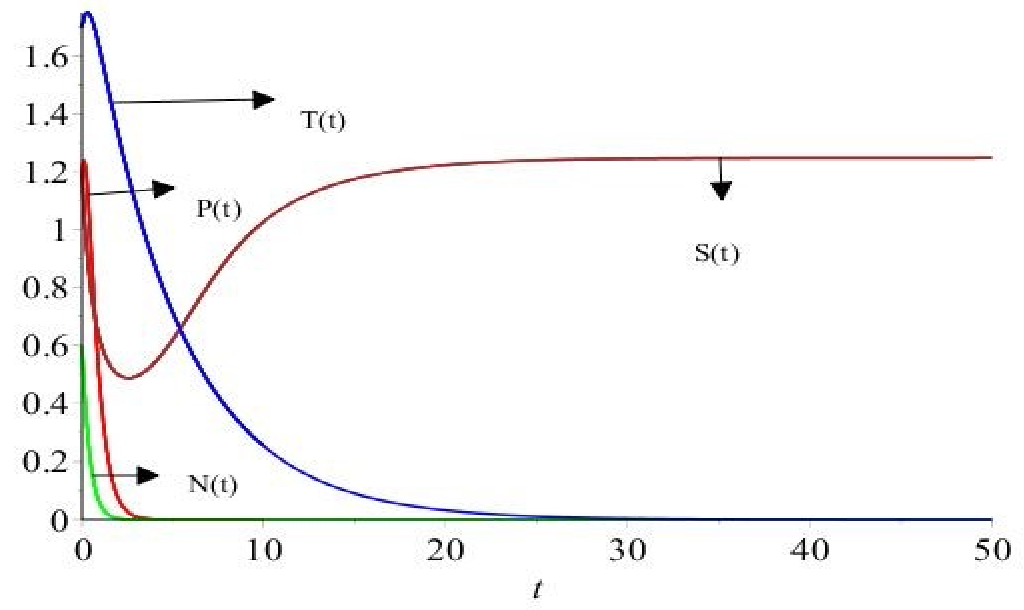


**Fig. 4** The stability analysis of susceptible-infected prey, treatment and predator interaction

## 7 Discussions and Conclusions

In this paper, we developed an epidemiological of S-I-T model. We conclude that our system of susceptible-infected prey, treatment and predator model the disease will spread in healthy prey. We take some control measure to reduce the spread of disease, the treatment will give for infected prey through vaccination and another is self quarantine. There must be some time lag called the raise of immunity power where steps to be taken by the control measure during this process. We assumed that the prey grows logistically. The system with time sustains a Hopf bifurcation around E* at a=a* taking time delay 'a' as bifurcation parameter.

This model analyzed the stability and the following conclusions were drawn (Table 1):

(i) If we analyze the second equilibrium point no interaction take place only the susceptible prey attains oscillatory dynamics of this model because of the absence of predator and infected prey (Fig. 2).

(ii) If the interaction occurs only in susceptible prey with predator, then the prey population size tends to normal growth but predator increases (Fig. 3).

(iii) In the interior equilibrium values, we conclude that the treatment given for infected prey then it will be decreasing and also affects the predator reduced completely. The susceptible prey decreases first and then increases towards its normal growth because of treatment given for infected prey recovered (Fig. 4).

The equations may reveal chaotic oscillation and attractor that may comes due to Rossler system. So all our significant diagnostic findings are numerically verified using Mapple. Hence we recommend that the self quarantine and natural or herbal medicines, yoga or meditation is the best way to avoid the spread of disease in prey population.

## References


Alfred H, Estomih MS, Oluwole DM. 2012. An eco-epidemiological mathematical model with treatment and disease infection in both prey and predator population. Journal of Ecology and the Natural Environment, 4(10): 266-279

Alshammari FS, Talay AF. 2021. Global stability for novel complicated SIR epidemic models with the nonlinear recovery rate and transfer from being infectious to being susceptible to analyze the transmission of covid-19. Journal of Function Spaces, 5207152

Amalia RUD, Arif DK, et al. 2018. Optimal control of predator-prey mathematical model with infection and harvesting on prey. Journal of Physics: Conference Series, 974: 012050

Atena A, Doust MHR. 2021. Investigating the dynamics of Lotka−Volterra model with disease in the prey and predator species. International Journal of Nonlinear Analysis and Applications, 12(1): 633-648

Aziz-Alaoui MA, Daher OM. 2003. Boundedness and global stability for a predator-prey model with modified Leslie-Gower and Holling-type II schemes. Applied Mathematics Letters, 16(7): 1069-1075

Banerjee S. 2017. A stage structured hybrid model for within-host emerging infectious disease modelling. Network Biology, 7(4): 94-97

Bezabih AF, Edessa GK, Rao PK. 2021. Eco-epidemiological modelling and analysis of prey-predator population. Science Journal of Applied Mathematics and Statistics, 9(1): 1-14

Cao H, Yan DX. 2021. Hopf bifurcation for an SIR model with age structure. Mathematical Modelling of Natural Phenomena, 16: 7

Hugo A, Simanjilo E. 2019. Analysis of an eco-epidemiological model under optimal control measures for infected prey. Applications and Applied Mathematics, 14(1): 117-138

Kar TK. 2006. Modelling and analysis of a harvested prey–predator system incorporating a prey refuge. Journal of Computational and Applied Mathematics, 185(1): 19-33

Musa SS, Qureshi S, Zhao S, Yusuf A, Mustapha UT, He DH. 2021. Mathematical modeling of covid-19 epidemic with effect of awareness programs. Infectious Disease Modelling, 6: 448-460

Saikh A, Gazi NH. 2018. Mathematical analysis of a predator–prey eco epidemiological system under the reproduction of infected prey. Journal of Applied Mathematics and Computing, 58(1): 621-646

Sharma S, Samanta GP. 2015. Analysis of a two prey one predator system with disease in the first prey population. International Journal of Dynamics and Control, 3(3): 210-224

Su M, Wang H. 2015. Modeling at the interface of ecology and epidemiology. Computational Ecology and Software, 5(4): 367-379

Upadhyay RK, Iyengar SRK. 2013. Introduction to Mathematical Modeling and Chaotic Dynamics. CRC Press, USA

Vijaya S, Singh JJ, Rekha E. 2017. Eco–epidemiological prey–predator model for susceptible–infected species. 2017, India

Yadav AS, Swami A, Ahlawat N, et al. 2021. A study of Covid-19 pandemic on fertilizer supply chain inventory management using travelling salesman problem for Cuckoo Search Algorithms. Selforganizology, 8(3-4): 21-35

Zhang WJ, Chen ZL, Lu Y, et al. 2020. A generalized discrete dynamic model for human epidemics. Computational Ecology and Software, 10(3): 94-104