Article

Multi-objective mathematical programming subject to box boundary constraints on system parameters

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Abstract
Multi-objective mathematical programming subject to box constraints on system parameters is an important problem in the fields of computer-aided design and decision-making theory, when there is an apparent conflict between multiple preference criteria. In this work, mathematical programming methodology has been proposed using a modern algorithm to solve multi-objective mathematical problems. The proposed algorithm to perform the ranking and classification procedures for the set of alternative solutions with boundary constraints is based on the concepts of parametric multi-aspect modeling of flexible complex systems. Interactive software has been implemented in the .Net environment to aid the decision-maker to identify the preference criteria and constraints on the system parameters, and then to choose the best solution from the morphological set depending on the developed algorithm. In addition, an illustrative numerical example solved by the proposed algorithm is presented at the end of this work.

Keywords flexible complex systems; parametric modeling; CAD; multi-objective programming; multi-criteria optimization; multi-criteria decision-making; ranking and classification methods; fuzzy logic.

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1 Introduction

Currently, the modular order for building and designing systems has become more prevalent due to its high flexibility and the ability to quickly aggregate and combine systems from the standard units (modules) (Levin, 2015; Vasawade et al., 2015; Miraglia, 2014). One of the areas, in which the modular approach is used intensively, is the design and manufacture of radio-electronic equipment and systems (Zakarian and Rushton, 2001). Modular order is a global standard arrangement for the development of equipment and systems according to functional and structural complexity. However, modular systems and complexes (flexible complex systems) represent a multi-component hierarchical structure with complex links between its constituent units (Khoder et al., 2017; Efatmaneshnik and Ryan, 2015). There are many aspects that must be taken into account in the process of automating the aggregation of flexible complex systems, including different types of module compatibility. The process of the aggregation of flexible complex systems can be reduced to the process of structural-parametric synthesis, which is performed using multi-criteria search algorithms. This type of synthesis requires a mathematical model, an objective function and a numerical algorithm.

Mathematical optimization problemshave received great interest from researchers (Taherdoost and Madanchian, 2023; IAEA, 2019; Kumar et al., 2017; Tlas, 2013; Tlas and Abdul Ghani, 2020). Thus, algorithms have been developed to quickly solve single-objective optimization problems. However, the interest in multi-objective optimization problems has also increased. The algorithms for solving these problems need special techniques because they deal with more than one objective function, which often conflict with each other. Moreover, multi-objective mathematical programming with box boundary constraints on system parameters is an important problem in computer-aided design and automating the aggregation of flexible complex systems. In order to automate the design/aggregation, there is a need to develop a numerical algorithm for classifying and ranking a set of alternative solutions based on a new method of parametric multi-aspect modeling of flexible complex systems (Khoder et al., 2017; Verkhova and Akimov, 2017). This method provides automatic formation of the objective function based on the information stored in qualimetric models.

The objectives of this article are to present a modern mathematical algorithm to perform ranking and classification procedures for the set of alternative solutions, and also to implement an interactive computer software for the proposed algorithm that aids the decision-maker to choose the most appropriate preference criteria on the system parameters. Rest of this article is organized as follows: Section 2 is designed to describe the mathematical model of the problem. Section 3 is consisted of definitions and formulations of box constraints on system parameters. Numerical algorithm for ranking and classifying the set of alternative solutions with boundary constraints is presented in Section 4. Illustrative example and interactive computer code algorithm are presented in Section 5 and 6, respectively. Finally, in Section 7, conclusions about this work are presented.

2 Description of the Problem

A description of the structure of the modular system and the parameters of the modular units, that make it up, will be called modular system's specifications and denoted by a letter S. The system's specifications include a description of the design or development requirements that a system or artifact should fulfill. The synthesized system S can be represented as a triple (Khoder et al., 2017):

$$S = < M, R, P >, \qquad (1)$$

where M – the set of units (modules) making up the system; R – the set of links between the modules; P – the set of modules parameters.

The set of technical-economic characteristics (TeX) completely determining all the essential properties of the synthesized system S can be represented by the following formula (2):

$$\text{TeX} = F_{Eval}(S) = F_{Eval}(< M, R, P >), \qquad (2)$$

where $Eval$ – the rules for calculating technical-economic characteristics by $S = < M, R, P >$.

Expression)2(shows the possibility of creating mathematical functions that allow calculating the individual technical-economic characteristics of the system, which reflect various aspects (functional, structural, constructive, economic, etc.) (3):

$$\mathrm{TeX}_1 = F_{Eval\,1}(S);\mathrm{TeX}_2 = F_{Eval\,2}(S);...;\mathrm{TeX}_n = F_{Eval\ \mathrm{n}}(S). \quad (3)$$

However, the set of technical and economic characteristics can be represented on the basis of multi-aspect parametric modeling, taking into account the various aspects of the units, as follows (4):

$$\mathrm{TeX}=\bigcup_{i=1}^{n} P_{A_i}; \quad P_{A_i} \in \boldsymbol{P}, \quad (4)$$

where P_{A_i} –the set of parameters of *i*-th aspect; ***P*** – the set of parameters used in the complex model that represents the modular system.

Technical-economic requirements (TeR) put constraints on the set of technical-economic characteristics (TeX) (Verkhova and Akimov, 2017). The technical-economic requirements are obtained by placing qualitative measures on the set of technical-economic characteristics, as follows (5):

$$\begin{gathered} \mathrm{TeR}^u = Q(\mathrm{TeX}^u) = \{Q_i(P_i)\},\ i \in \overline{1,n}; \\ P_i \in \boldsymbol{P}. \end{gathered} \quad (5)$$

where TeR^u – the set of technical and economic requirements of modules belonging to the considered class; TeX^u – the set of technical-economic characteristics of modules belonging to the considered class; ***P***– the set of parameters used in the complex model; Q –Qualimetric Model (Kirillov, 2014).

It should be noted that one of the important features of the proposed qualimetric models is the ability to automatically form an objective function (*OF*) according to the specific technical and economic requirements. The following expression shows the relationship between TeR and the objective function (6)(Podinovskiy and Nogin, 1982):

$$\begin{gathered} OF \overset{def}{=} F(\mathrm{TeR}) = F\left(\alpha_1 K_1,\ \alpha_2 K_2, ...,\ \alpha_n K_n\right); \\ \alpha_i = \varphi(a_i^{\mathrm{TeR}}),\ i \in \overline{1,n}; \qquad K_i = F_i(\mathrm{TeR_i}),\ i \in \overline{1,n}; \\ \mathrm{TeR_i} = < P_i^{\mathrm{TeR}}, a_i^{\mathrm{TeR}}, P_i >,\ \mathrm{i} \in \overline{1,n}; \\ P_i \in \boldsymbol{P}. \end{gathered} \quad (6)$$

where K_i – the normalized value of the *i*-th criterion; α_i – the weighting coefficient of the *i*-th criterion; P_i^{TeR} –the constraints on the parameter P_i; a_i^{TeR} – the significance factor of the parameter P_i, representing the value of a linguistic variable a^{TeR}.

In the processes of aggregation automation and design automation of modular systems, it is very important to have a mechanism for filtering out units (modules) that are incompatible with already defined units, as well as comparing units within a whole class of units among themselves according to several criteria (multi-criteria filtering). The concept of filtering on a set of alternatives can be formulated with the following formula (7):

$$Filter \overset{def}{=} < X, Y, Cond, Mech >, \qquad (7)$$

where *X*– the original set; *Y*– the set obtained as a result of filtrating; *Cond*– filtering conditions; *Mech*– a filtering mechanism (numerical algorithm).

In the following sections, the realization of this mechanism through a numerical algorithm will be presented.

3 Definitions and Formulations of Box Constraints on System Parameters

The parameters space of complex models is *n*-dimensional space consisting of points with cartesian coordinates ($v|P_1,\ldots,v|P_k,\ldots,v|P_n$),), where $v|P_k$ is the nominal value of the parameter P_K. The set of possible solutions (alternatives) can be continuous or discrete, and the boundaries of parameters change (v_{max}, v_{min}) is determined by experts (expert judgment) as shown in Fig. 1.

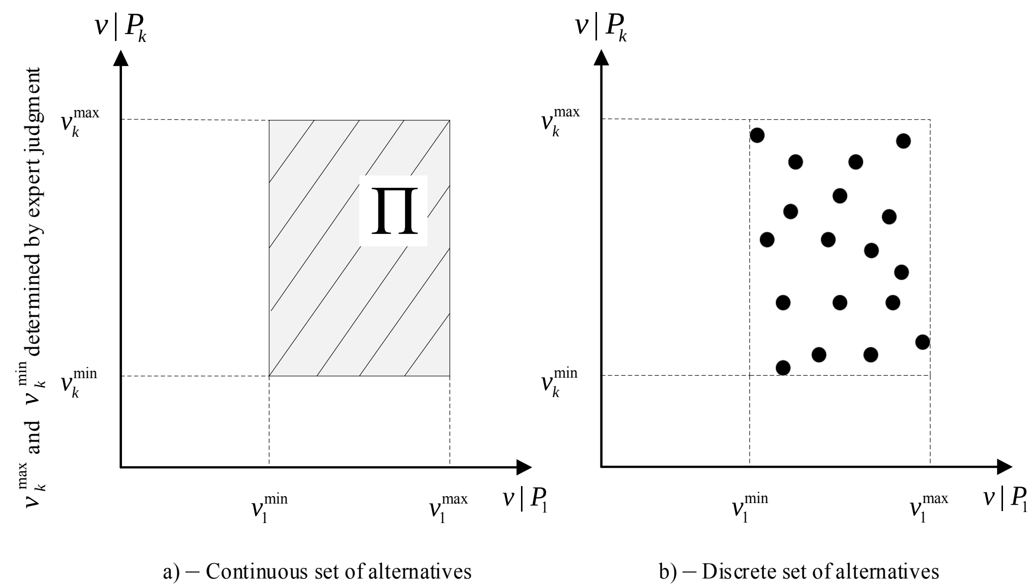

Fig. 1 Field of parameters change.

However, in order to calculate the value of the criteria of the objective function given in the expression)6(, it is necessary to analyze the specific requirements on the modules. Table 1 shows the types of functional requirements for module characteristics.

In the classical solution of optimization problems, strict constraints are introduced on the area of parameters change. However, according to Sobol I.M. "floating" constraints can be used as criteria (Sobol and Statnikov, 1981). This allows solving the problem in an extended range of changes in operating parameters. Moving away from the hard constraints dictated by functional requirements, the desired direction of change in the functional characteristics can be obtained. Table 2 shows the functional requirements for characteristics (hard constraints) and their desired direction of change.

Table 1 Types of functional requirements.

Requirement type	Meaning
A	$a \le x \le b$: $a, b \in$ {Real, Int}
B	$x \ge b$: $b \in$ {Real, Int}
C	$x \le b$: $b \in$ {Real, Int}
D	$x = b$: $b \in$ {Real, Int}
F	Does not matter

Table 2 Hard constraints and the desired direction of changing characteristics.

Functional requirements (hard constraints)	Desired direction of change in functional characteristics
$A : a \le x \le b$	←→–provide maximum distance from the borders (outside): ↑– maximizing value (not less than a) ↓–minimizing value (not more than b)
$B : x \ge b$	↑– maximizing value (not less than b)
$C : x \le b$	↓–minimizing value (not more than b)
$D : x = b$	→←–provide minimum deviation from the set value: ↓–minimizing value (not more than b_{max}) ↑– maximizing value (not less than b_{min})

There are four possible cases of box boundary constraints on system parameters can be distinguished as follows:

$$v \mid P_i \le b_{\max}, \;\; type = lt. \tag{8}$$

$$v \mid P_i \ge b_{\min}, \;\; type = gt. \tag{9}$$

$$b_{\min} \le v \mid P_i \le b_{\max}, \;\; type = gap. \tag{10}$$

$$v \mid P_i = b, \;\; type = eq. \tag{11}$$

It is logical to assume that in the case of expression)8(, the parameter value $v|P_i$ is minimized, in the case of expression)9(– maximization is performed, for expression)10(– the maximum distance from the boundaries given by b_{min} and b_{max} is achieved, and in expression)11(, It is necessary to achieve the minimum possible deviation from b. The last option for real systems should be transformed as follows: $|v \mid P_i - b| < \delta$.

Thus, expression)11(can be reformulated intoexpression)10(as follows (12):

$$b_{\min} = b - \delta,\ \ b_{\max} = b + \delta. \tag{12}$$

The use of expressions (8−12) makes it possible to move from the strict boundary constraints on the field of parameters change, and turn them into "floating" constraints (criteria). Then these criteria (or some of them) are convoluted into one integrated criterion.

4 NumericalAlgorithmfor Ranking and Classifying the Set of Alternative Solutions with Boundary Constraints

In expression)6(, F(TeR) implements one of the well-known types of convolution of individual criteria based on the method of combining them into a generalized criterion (global criterion), the most popular of which are additive, multiplicative, maximin (minimax), etc. In order to make a decision based on the generalized objective function method, a linear convolution of a set of criteria with weighting coefficients determining their significance is performed. Then the alternative, whose value of the generalized objective function is the maximum or minimum, is selected. In the case of applying the additive method for combining particular criteria into a generalized criterion, we obtain (13):

$$F_{j\ global} = \sum_{i=1}^{n} \alpha_i K_{ji},\ j \in \overline{1,m},\ i \in \overline{1,n}, \tag{13}$$

where K_{ji}– the normalized value of the i-th criterion for the j-th alternative; α_i – the weighting coefficient of the i-th criterion; m– the number of alternatives; n– the number of evaluation criteria.

Fig. 2 shows the hierarchy of decision-making levels in a multi-criteria problem.

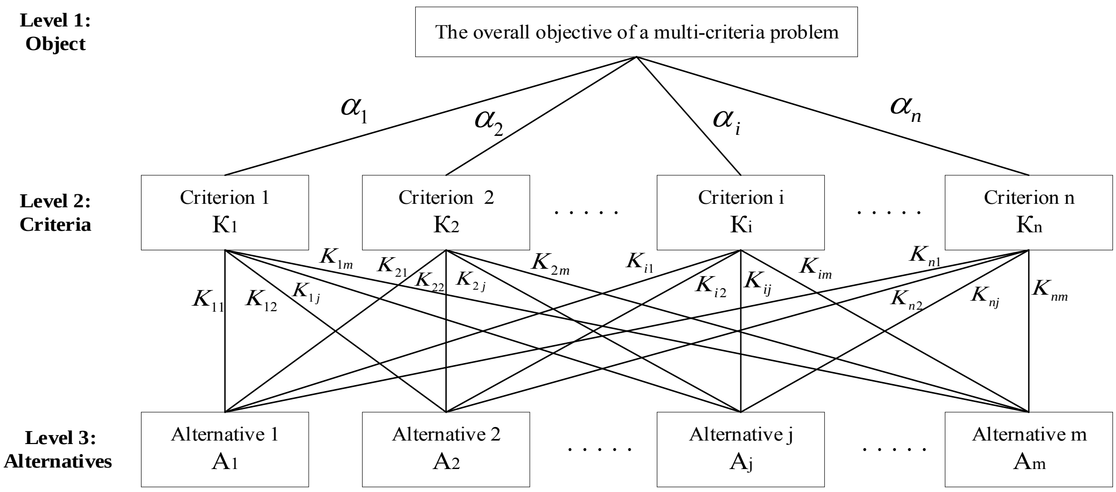

Fig. 2 Hierarchy of decision-making levels in a multi-criteria problem.

Expression)*13*(can be represented in the matrix form as follows (14):

$$\begin{pmatrix} F_{1\,global} \\ F_{2\,global} \\ \dots \\ F_{m\,global} \end{pmatrix} = \sum_{i=1}^{n} \alpha_i \begin{pmatrix} K_{1i} \\ K_{2i} \\ \dots \\ K_{mi} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \dots & \dots & \dots & \dots \\ K_{m1} & K_{m2} & \dots & K_{mn} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \dots \\ \alpha_n \end{pmatrix}. \quad (14)$$

In case of applying the additive method for combining the individual criteria into a generalized criterion while maximizing the objective function, we obtain (15):

$$OF \overset{def}{=} F(\text{TeR}) = \sum_i \alpha_i K_i \to \max, i \in \overline{1,n}; \\ \sum_i \alpha_i = 1, \quad \alpha_i \geq 0. \quad (15)$$

To maximize the generalized objective function (*OF*), the function of weighting coefficients can be represented as follows (16):

$$\alpha_i = \varphi(a_i^{\text{TeR}}) = \frac{a_i}{\sum_j a_j}, \; i, j \in \overline{1,n}; 0 \leq a \leq 1. \quad (16)$$

where a_i^{TeR} – the significance factor of the parameter P_i, representing the value of a linguistic variable a^{TeR}, defined as follows: $a^{\text{TeR}} \in T$ where *T* is the set of fuzzy variables *T = {null, very low, low, medium, high, very high}* (Zadeh, 1975).

The significance coefficient a_i (value of the linguistic variable) in expression)*16*(can be calculated from the compatibility function $C{:}T \longrightarrow [0,1]$ as follows (17):

$$a_i = \underset{a_i^{\text{TeR}} \in T}{C}(a_i^{\text{TeR}}) = \frac{i-1}{n-1}, \quad i \in \overline{1,n}. \quad (17)$$

thus, when $n=6$ then $a_i \in \{0_{null}, 0.2_{very\ low}, 0.4_{low}, 0.6_{medium}, 0.8_{high}, 1_{very\ high}\}$.

However, it is required that for the values of individual criteria K_i, the condition $K_i \in [-1, 1]$ is satisfied. Such a constraint will give a visual representation of the fulfillment of the TeR requirements for each parameter. When the global criterion (generalized criterion) is subjected to maximization $F_{global} \longrightarrow max$, then two cases for computing the individual criterion can be presented (shown in Fig. 3).

• If the individual criterion is maximized (the larger the indicator, the better), then the criteria K_i can be calculated as follows (18):

$$K_i(v) = \begin{cases} -1, & v \le v_{\min} \\ -\dfrac{b-v}{b-v_{\min}}, & v_{\min} < v \le b \\ \dfrac{v-b}{v_{\max}-b}, & b < v \le v_{\max} \\ 1, & v \ge v_{\max} \end{cases} \quad (18)$$

• Else if the individual criterion is minimized (the smaller the indicator, the better), then the criteria K_i can be calculated as follows (19):

$$K_i(v) = \begin{cases} 1, & v \le v_{\min} \\ \dfrac{b-v}{b-v_{\min}}, & v_{\min} < v \le b \\ -\dfrac{v-b}{v_{\max}-b}, & b < v \le v_{\max} \\ -1, & v \ge v_{\max} \end{cases} \quad (19)$$

where $v_{\max}$ and $v_{\min}$ are determined by experts (expert judgment).

Thus, the best solution A^* from the set of possible (acceptable) alternatives $\boldsymbol{A} = \left\{A_j,\ j \in \overline{1,m}\right\}$ is determined in the formula (20):

$$A^* = \arg\left(\max_{A \in \boldsymbol{A}} \left\{\sum_i \alpha_i K_i(A)\right\}\right), i \in \overline{1,n}; \\ \sum_i \alpha_i = 1, \quad \alpha_i \ge 0. \quad (20)$$

The number of morphological set alternatives can be very large and this makes the selection of the alternatives that best meet the technical and economic requirements is a very difficult task. Obviously, there is a need for an efficient algorithm performing morphological set classification and ranking. As a result of the classification and ranking procedures, ineffective options will be eliminated, and the rest of solutions will be ranked according to their level of compliance with the TeR. The block diagram of the ranking algorithm is shown in Fig. 4, and the steps of the algorithm are shown below:

Step 1. Definition of the set of alternatives (modules).

Step 2. Determining the number of alternatives (m).

Step 3. Calculating the real value of the weighting coefficient.

Step 4. Calculating the value of criterion complying with the conditions of TeR.

Step 5. Calculating the value of objective function for each alternative.

Step 6. Ranking/classification by criteria complying with conditions of TeR.

As a result of the ranking of the morphological set according to the TeR compliance criterion, the decision-maker will be able to choose the most appropriate solution.

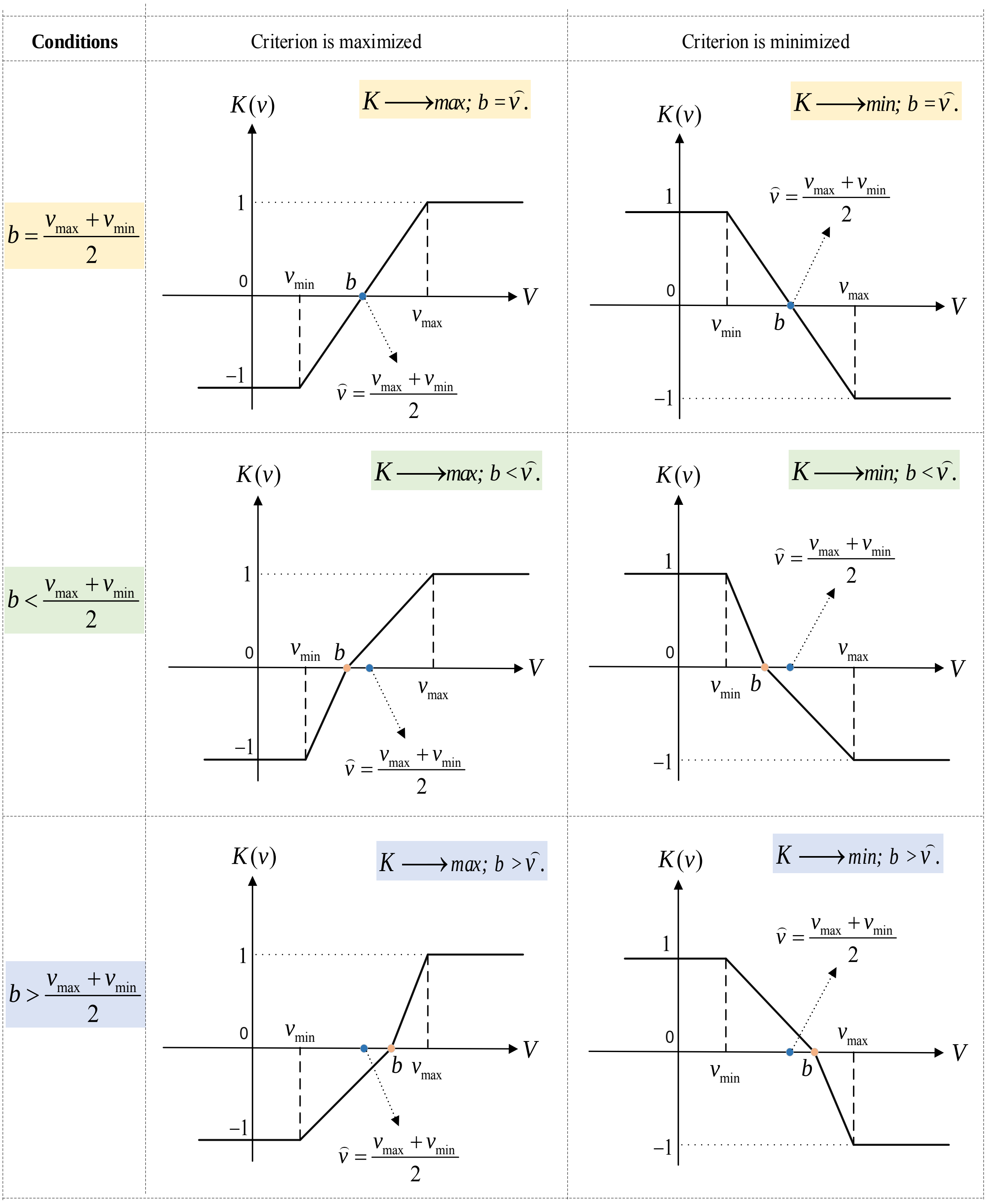

Fig. 3 Calculation of the criterion valuecomplied with constraints.

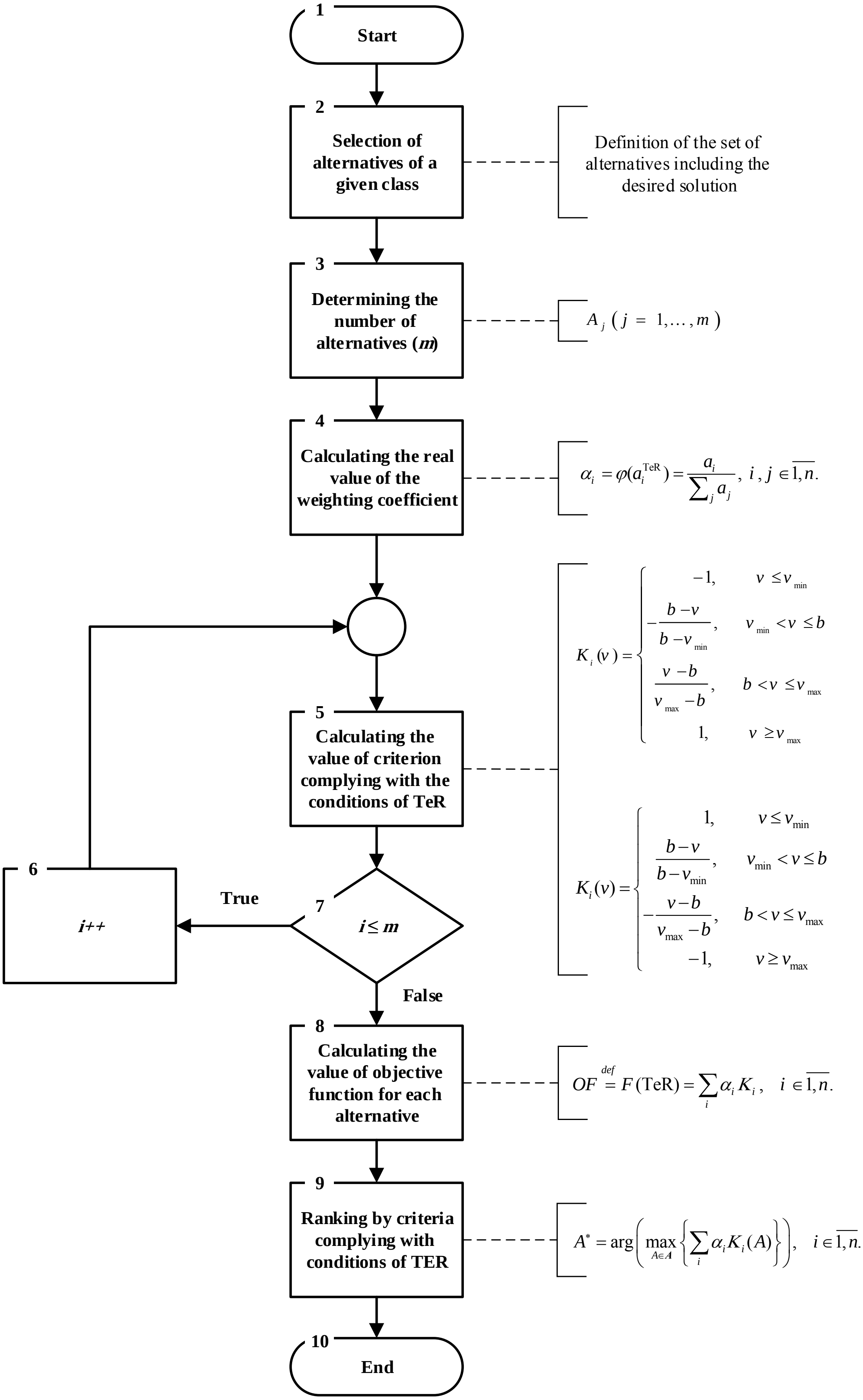

Fig. 4 Block diagram of ranking algorithm of alternatives according to the criterion complied with constraints.

5 Illustrative Example

In order to classify and rank a set of alternatives belonging to a given class using the numerical algorithm that has been developed, it is necessary to form the objective function including calculating the value of the criteria complied to the technical requirements and calculating the real value of the weighting coefficient of these criteria. Let's take a low-noise microwave amplifier as an example (Table 3 shows the requirements on technical characteristics). Table 4 shows the constraints and their type according to Table 1, as well as the degree of importance of individual characteristics according to expressions)*16*(and)*17*(.

Table 3 Requirements on the technical and economic characteristics of the low-noise amplifier.

Technical and economic characteristics	Boundary constraints	Units of measurement
Operating frequency range	*50-500*	*MHz*
Noise coefficient	*4.5*	*dB*
Number of inactive items	*10*	*Unit*
Nominal power	*5*	*W*
Weight	*1800*	*g*
Overall width (one of the dimensions)	*20*	*mm*

Table 4 Analysis of low-noise amplifier requirements.

Technical and Economic Characteristics TeX	Units of measurement	Technical and Economic Requirements TeR			
		Boundary constraints			Significance coefficient
		Requirement type	*min*	*max*	
Operating frequency range	*MHz*	*A*	50	500	*very high*
Noise coefficient	*dB*	*C*	–	4.5	*high*
Number of inactive items	*Unit*	*C*	–	10	*medium*
Nominal power	*W*	*B*	5	–	*high*
Weight	*g*	*C*	–	1800	*very low*
Width (one of the dimensions)	*mm*	*C*	–	20	*low*

To apply the algorithm developed in this paper to the presented example, there is a need to formulate the technical and economic requirements for the amplifier using the algorithm symbols previously defined. Table 5 shows the derivation of amplifier parameters (criteria) from the technical and economic characteristics, as well as the desired direction of changing in these characteristics according to Table 2. The value of symbol b in addition to the values of (v_{max}, v_{min}) for calculating the values of the criteria according to expressions)*18*(and)*19*(are shown in Table 5 as well.

Table 6 shows the description of four amplifiers (four alternatives), which are: $\{A_1, A_2, A_3, A_4\}$ respectively. The set of criteria and requirements for the alternatives (low-noise amplifier) are shown in Table 5. Moreover, Table 7 shows the calculation of the values of criteria complying to the requirements and conditions according to expressions)*18*(and)*19*(. The values of the weighting coefficientfor each criterion were calculated according to expression)*16*(.

Table 5 Formulation of low-noise amplifier requirements.

Parameters	Desired direction	Value *b*	Expert judgment		Units of measurement	Significance coefficient
			v_{min}	v_{max}		
Minimum operating frequency	↓	*50*	*10*	*100*	*MHz*	*very high*
Maximum operating frequency	↑	*500*	*300*	*1000*	*MHz*	*very high*
Noise coefficient	↓	*4.5*	*0*	*8*	*dB*	*high*
Number of inactive items	↓	*10*	*1*	*20*	*Unit*	*medium*
Nominal power	↑	*5*	*3*	*8*	*W*	*high*
Weight	↓	*1800*	*1000*	*5000*	*g*	*very low*
Width (one of the dimensions)	↓	*20*	*10*	*30*	*mm*	*low*

Table 6 The set of alternatives.

Technical and economic characteristics	Symbols of Alternatives				Units of measurement
	A_1	A_2	A_3	A_4	
Operating frequency range	*50-500*	*50-500*	*50-700*	*50-550*	*MHz*
Noise coefficient	*4.5*	*4.5*	*4*	*4.5*	*dB*
Number of inactive items	*12*	*9*	*5*	*11*	*Unit*
Nominal power	*4*	*5*	*7*	*5*	*W*
Weight	*2000*	*1700*	*1500*	*1800*	*g*
Width (one of the dimensions)	*24*	*19*	*15*	*27*	*mm*

Table 7 Calculate the value of the objective function for each alternative from the set.

Symbols of criteria	Alternatives				Weighting coefficient α_i
	A_1	A_2	A_3	A_4	
K_1	*0*	*0*	*0*	*0*	$\alpha_1 = \frac{1}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.208$
K_2	*0*	*0*	*0.4*	*0.1*	$\alpha_2 = \frac{1}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.208$
K_3	*0*	*0*	*0.111*	*0*	$\alpha_3 = \frac{0.8}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.167$
K_4	*-0.2*	*0.111*	*0.56*	*-0.1*	$\alpha_4 = \frac{0.6}{1+1+0.8+0.6+0.8+0.4+0.2} = 0.125$
K_5	*-0.5*	*0*	*0.67*	*0*	$\alpha_5 = \frac{0.8}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.167$
K_6	*-0.062*	*0.125*	*0.375*	*0*	$\alpha_6 = \frac{0.2}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.042$
K_7	*-0.4*	*0.1*	*0.5*	*-0.7*	$\alpha_7 = \frac{0.4}{1+1+0.8+0.6+0.8+0.4+0.2} \approx 0.083$
$OF = \sum_i \alpha_i K_i$	*-0.144*	*0.027*	*0.340*	*-0.05*	

However, the values of the objective function (*OF*) for each option of the four alternatives were calculated at the bottom of Table 7 according to expression)*15*(. As a result, Table 7 shows that the low-noise amplifier (A_3) is the best option according to the technical and economic requirements (determined in the Table 3), since the value of the corresponding objective function is the highest among the alternatives. Module (A_2) comes second, followed by (A_4) and the worst option is (A_1).

6 Interactive Computer Code Algorithm

Interactive software program has been developed to help the decision-maker to choose the best solution from among the morphological set solutions stored in the database. This interactive computer code is implemented in the .NET environment by C-Sharp programming language and using Windows Forms Applications technology to develop desktop programs in the integrated development platform "Microsoft Visual Studio 2019". The interactive software helps in making the right decision for the best solution among the alternatives, through the implementation of the developed numerical algorithm to rank and classify the set of alternative solutions with boundary constraints. Then the decision maker makes the appropriate decision based on the ranking result generated by the algorithm according to the proposed preference criteria. The code algorithm is composed from the next steps:

Step 1: Define and create the set of alternatives

The code provides the ability to manually create and add alternatives to the database through the interface *"Create Module"*. This interface contains a template for creating alternatives of a low-noise amplifier, for example, and other templates can be added to create modules belonging to different classes such as electronic components, radio electronic devices, etc. In order to create a new alternative belonging to the class of low-noise amplifier, the code requests to enter the name in addition to a set of technical properties, as shown in Fig. 5.

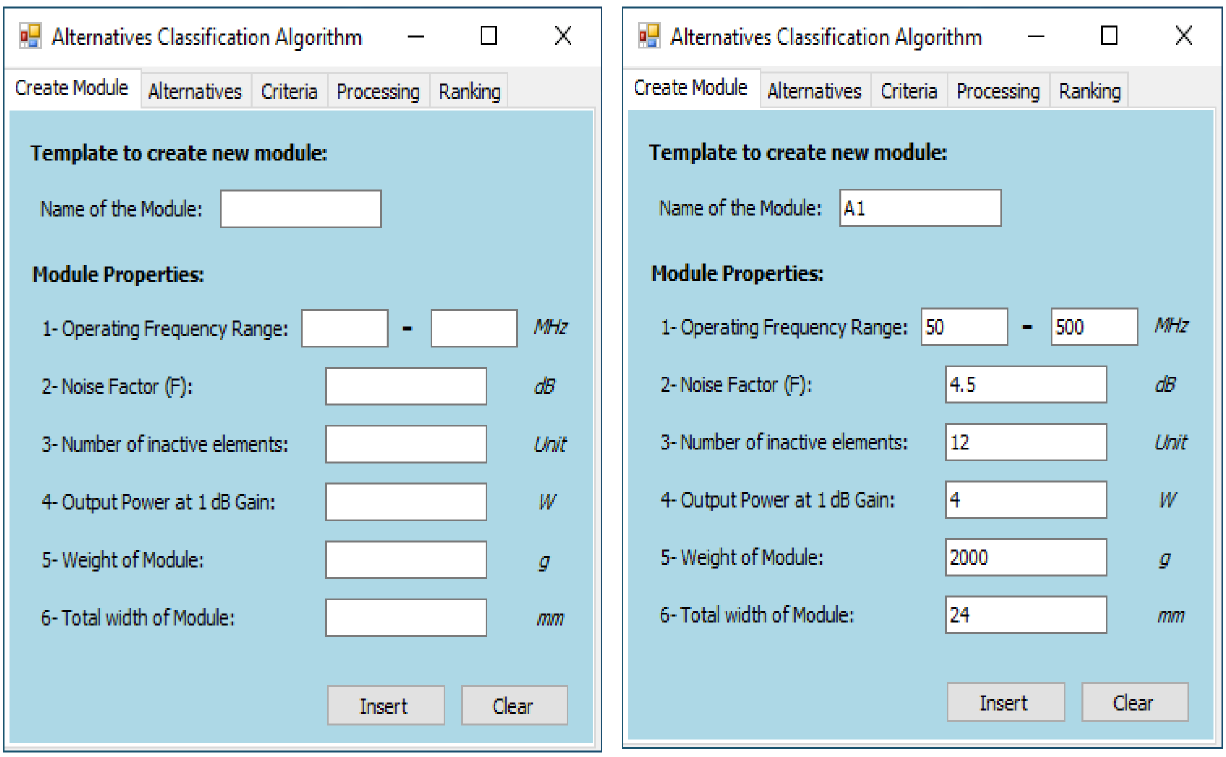

Fig. 5 Template for creating and adding alternatives belonging to the class of low-noise amplifier.

The created modules are stored in the *"Modules"* database by pressing the *"Insert"* button in the interface. Fig. 6 shows the database that has been designed. This database includes a table *"dbo.Table_1"* containing all the properties of the amplifier that will be stored. Moreover, Fig. 7 shows four amplifiers (four alternatives) created and stored in the database, which are: $\{A_1, A_2, A_3, A_4\}$.

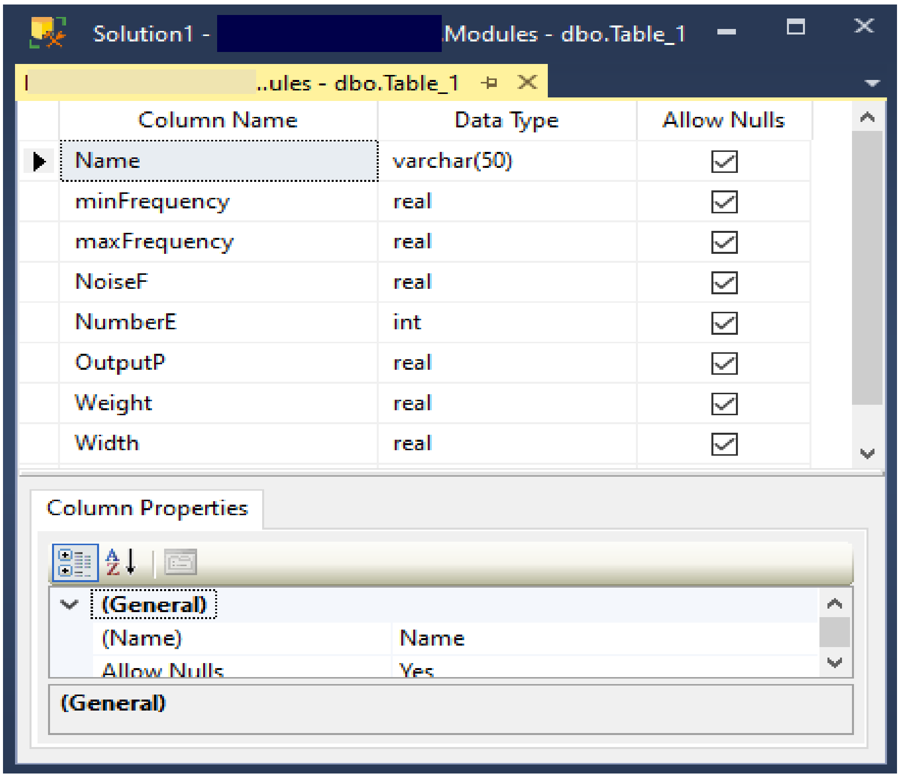

Fig. 6 Database in SQL to store alternatives.

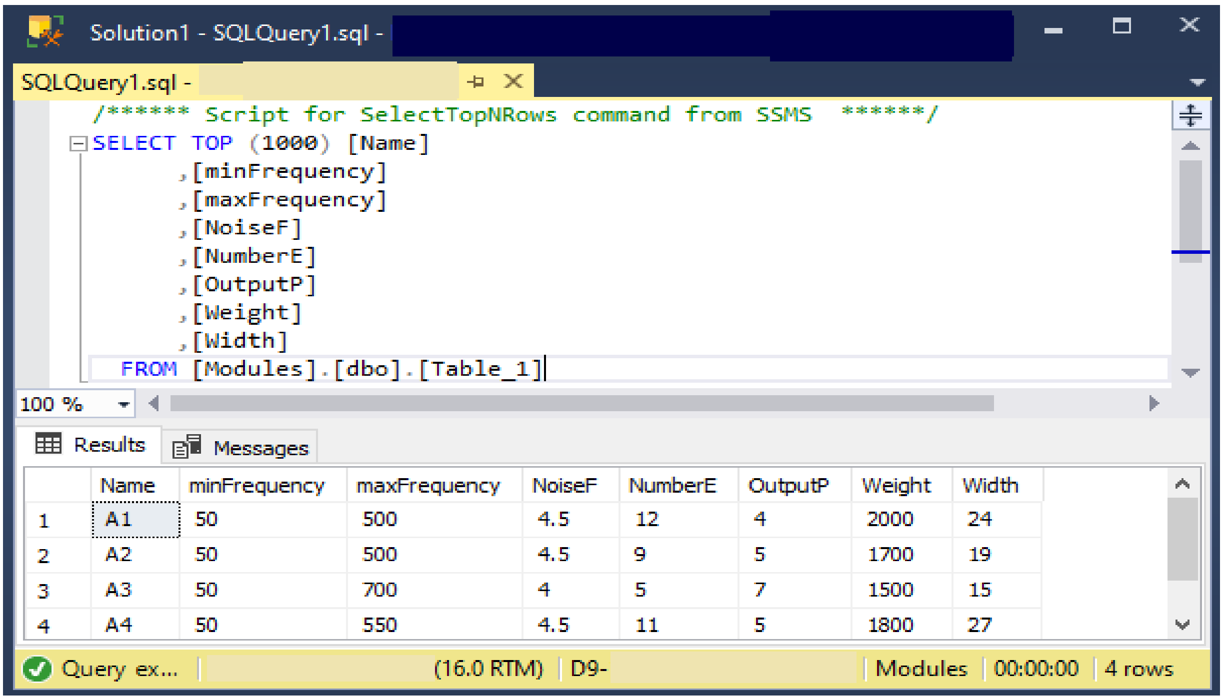

Fig. 7 Alternatives stored in the database.

Step 2: Determine the alternatives $A_j(j = 1,\ldots,m)$

The code allows showing all the alternatives belonging to a specific class and stored in the database through the *"Alternatives"* interface. Figure 8 shows the number of stored alternatives (*m*) as well as a list of all the alternatives. Through the *"Alternatives"* interface, it is also possible to access to create a new alternative or to determine the criteria for rating alternatives by clicking on the *"Create New Alternatives"* and *"Determine Criteria"* buttons, respectively.

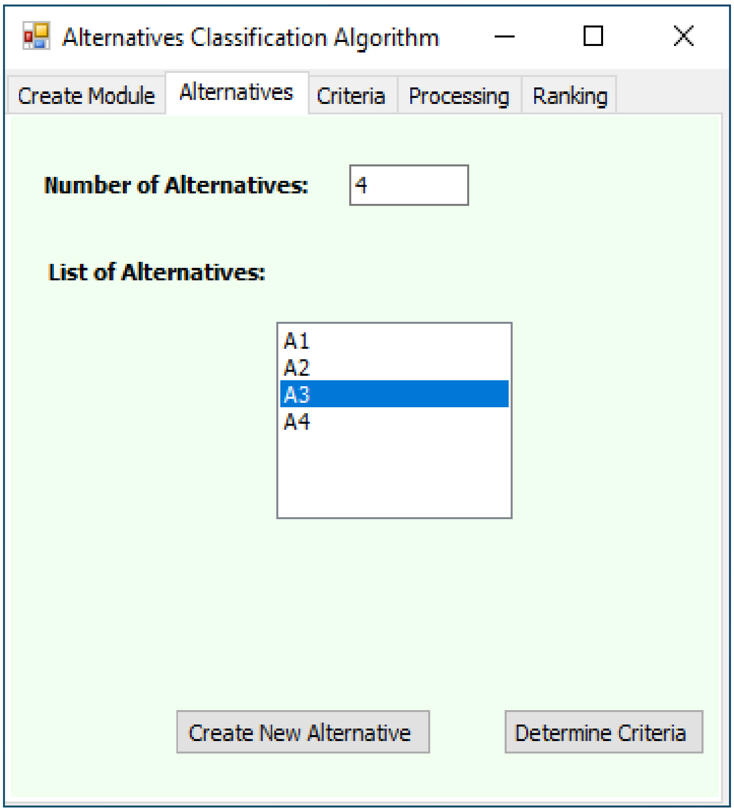

Fig. 8 List of alternatives stored in the database belonging to the amplifier class.

Step 3: Construct the criteria K_i (i= 1,…,n)

The code requests to enter the detail description of each criterion K_i (i= 1,…,n) through the *"Criteria"* interface. This interface (Fig. 9) contains 7 parameters related to low-noise amplifier {K_1, K_2, K_3, K_4, K_5, K_6, K_7}. For each criterion requires: selecting the desired direction for the criterion (maximizing or minimizing) from the *"Desired direction"* drop-down list containing the two values: {*Min, Max*}; specifying the value of the minimum or maximum boundary (*b*) in the *"Constraint"* text box; and determining the importance of the criterion by selecting from the *"Significance factor"* drop-down list that contains the values: {*null, very low, low, medium, high, very high*}.

Step 4: Processing and Calculating

After constructing the criteria, data processing comes which includes (Fig. 10): calculating the real value of the weighting coefficients for each of the ranking criteria; calculating the values of the criteria for all the alternatives; and then calculating the value of the objective function for each alternative.

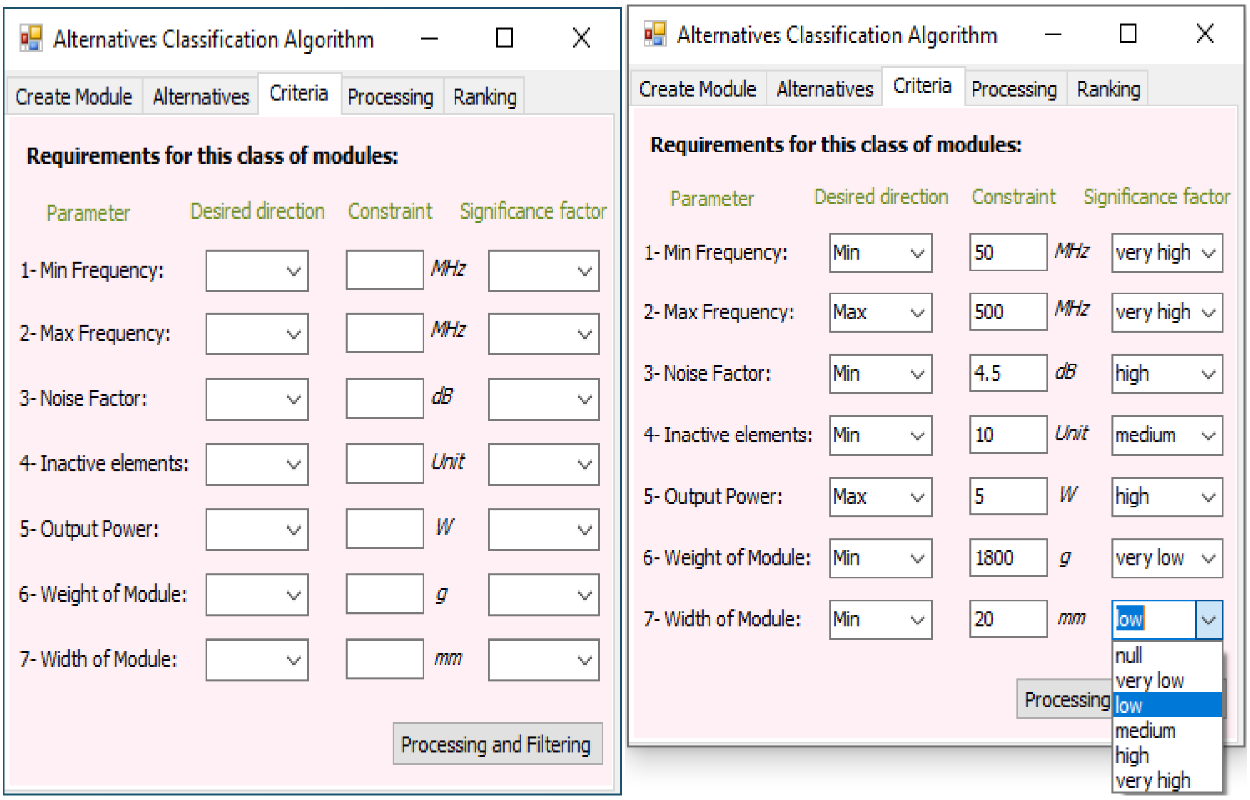

Fig. 9 Construct the criteria for classifying and rankingalternatives.

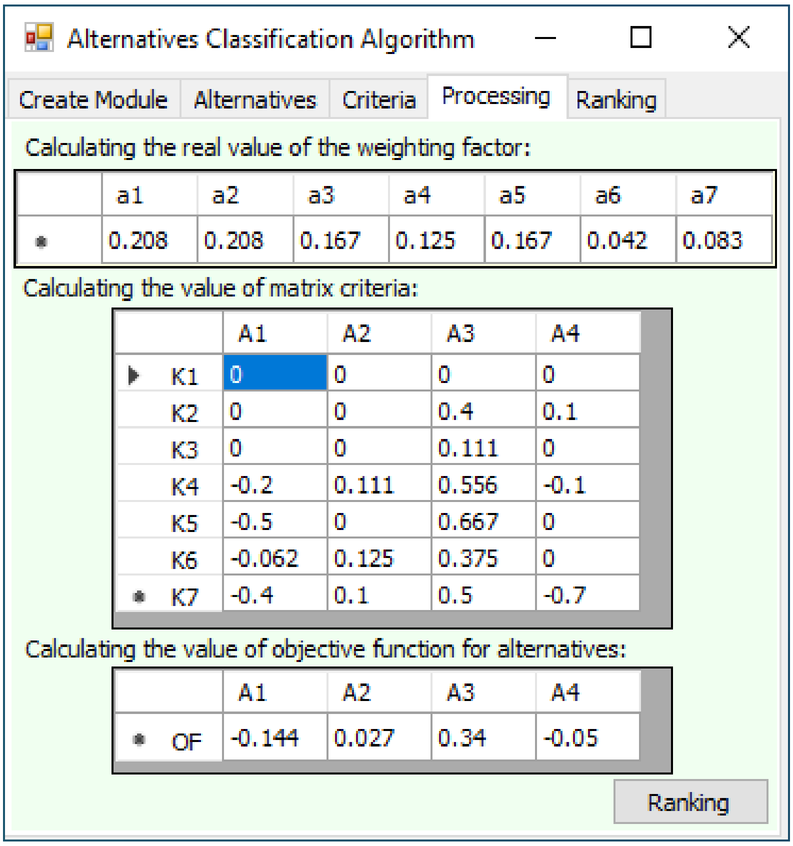

Fig. 10 Data processing.

Step 5: Ranking and Classification

The *"Processing"* interface (Fig. 10) contains the *"Ranking"* button, and by clicking on it, the algorithm developed in this research is called. Thealternatives are classified and ranked according to the criteria complying with the required conditions. The *"Ranking"* interface (Fig. 11) contains a table including the rank of alternatives from the best to the worst. Under each alternative, the value of corresponding objective function has been shown.

Step 6: Displaying and Drawing chart of results

The *"Ranking"* interface provides the ability to display the results in a graphical form by clicking on the *"Display"* button (as shown in Figure 11). According to the results shown in Figure 11, alternative A_3 is the best, since the value of its objective function is the highest compared to other alternatives. The results of ranking the alternatives according to Fig. 11 are: $\{A_3, A_2, A_4, A_1\}$. These results are identical to the results of the calculations shown in Table 7.

Step 7: Filtering of results

The *"Ranking"* interface also allows the decision maker to determine a threshold value on the objective function (threshold value: a numerical value between -1 and +1). Then the decision maker clicks the *"Show"* button to filter and display the alternatives meeting the specified threshold. Figure 12 shows that when the threshold value is set to *"0.02"*, the filter results are appeared in the *"Results"* list and on the graph within the shaded area.

Step 8: Exportation of Ranking and Classification results

The code provides the ability to save the ranking results in a word file or a text file.

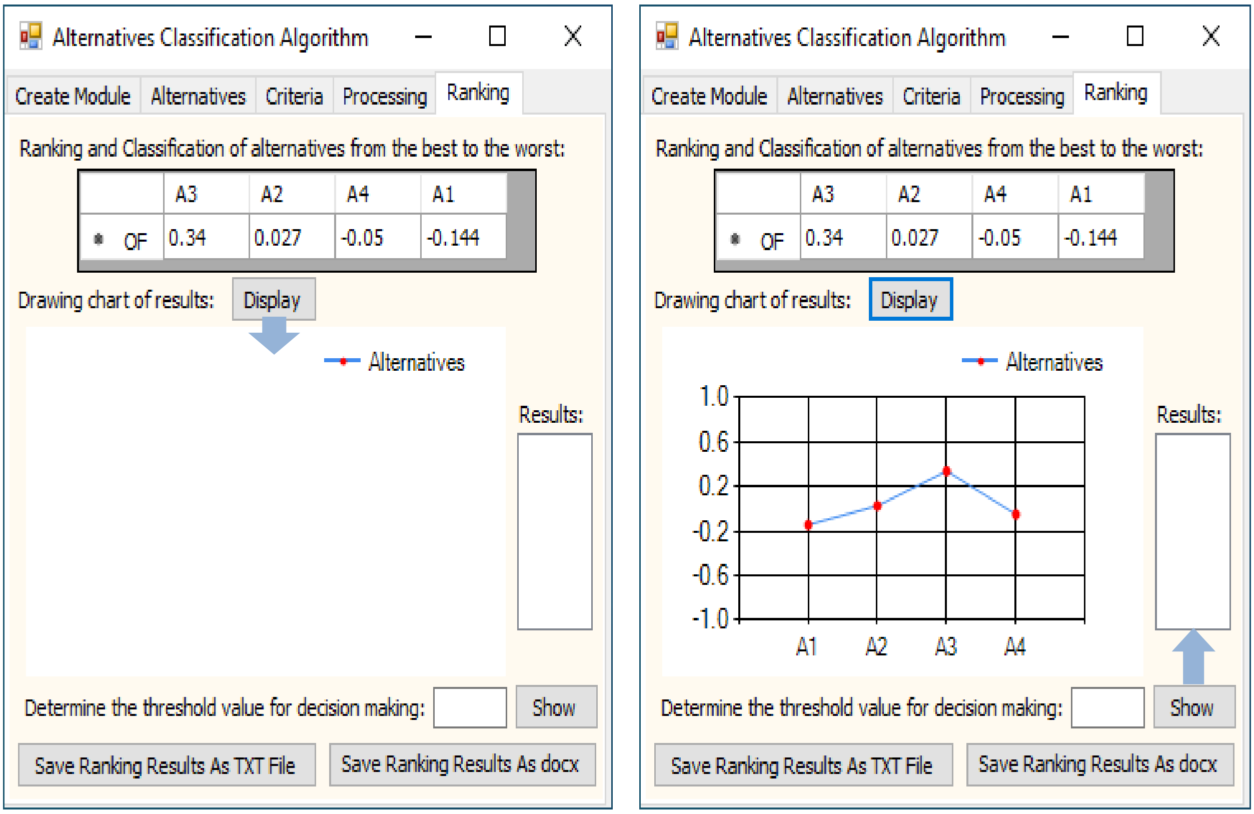

Fig. 11 Ranking alternatives and Displaying the results.

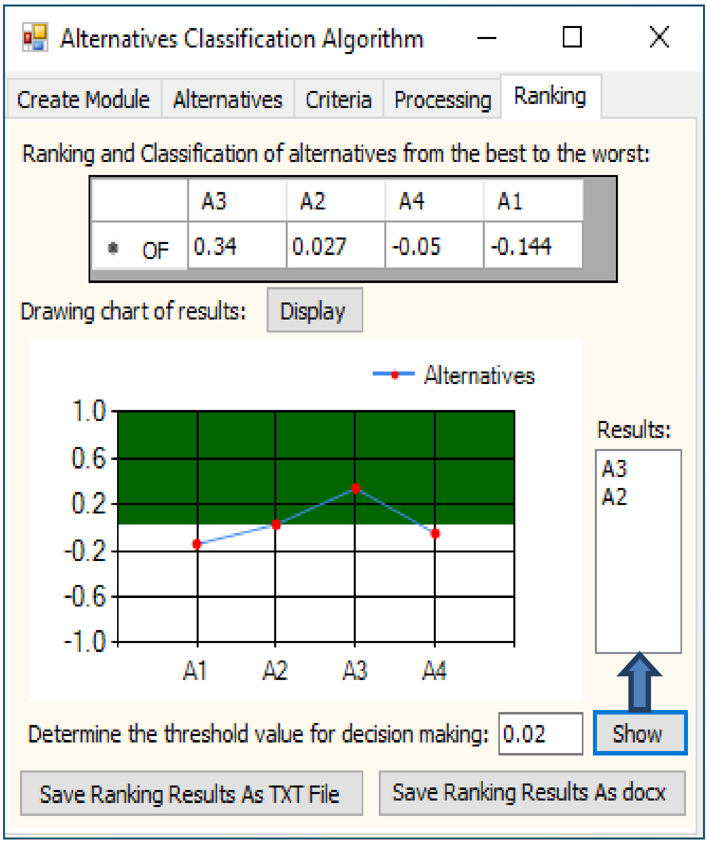

Fig. 12 Filtering of results according to the threshold defined by the decision maker.

7 Conclusions

Numerical algorithm for ranking and classifying the set of alternative solutions has been presented. This algorithm has a significant and useful role in the fields of design, manufacturing, innovation, control, artificial intelligence, and decision-making theory applied in production and design processes. The algorithmbasically aims to solve a multi-objective mathematical programming problem with box constraints by moving away from the hard constraints to "floating" constraints which can be used as criteria (practical method for obtaining an approximation of a Pareto set). However, a fast and low-complexity method for calculating the values of criteriacomplied with the constraints without using penalty functions (penalty and barrier functions methods) has been shown. In addition, fuzzy logic was used to calculate the weighting coefficient function of the criteria. The demonstration of the proposed algorithm for solving the multi-objective mathematical programming with mathematical constraints has been done though a real numerical example. Moreover, easy interactive computer software has been proposed. The code algorithm implemented and programmed by the C# is mainly based on the numerical algorithm developed in this article.

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References

Efatmaneshnik M, Ryan M. 2015. System construction cost of hierarchical-modular structures. Annual IEEE Systems Conference (SysCon) Proceedings, 153-157

IAEA (International Atomic Energy Agency). 2019. Application of Multi-criteria Decision Analysis Methods to Comparative Evaluation of Nuclear Energy System Options: Final Report of the INPRO Collaborative Project KIND. NO. NG-T- 3.20

Khoder H, Verkhova GV, Akimov SV. 2017. Modular technology in design of flexible complex systems. T-Comm, 11(9): 86-90

Khoder HM, Verkhova GV, Akimov SV. 2017. Parametric multi-aspect modeling of distributed bus-modular control systems. Proceedings of the IInd International Conference on Control in Technical Systems (CTS, St. Petersburg, Russia). IEEE, 259-262. DOI:10.1109/CTSYS.2017.8109540

Kirillov VI. 2014. Kvalimetriiaisistemnyianaliz [Qualimetry and System Analysis]. Minsk: Novoeznanie Publ., Moscow: INFRA-M Publ.; 2014. 439 p. (in Russ.)

Kumar A, Sah B, Singh AR, Deng Y, He X, Kumar P, Bansal RC. 2017. A review of multi-criteria decision making (MCDM) towards sustainable renewable energy development. Renewable and Sustainable Energy Reviews, 69: 596-609, 2017.

Levin M. 2015. Modular System Design and Evaluation. Springer International Publishing, Switzerland. DOI: 10.1007/978-3-319-09876-0

Miraglia S. 2014. System Architectures and Innovation:theModularity-Integrality Framework. United Kingdom: Cambridge Service Alliance.

Podinovskiy VV, Nogin VD. 1982. Pareto-optimalnyeresheniiamnogokriterialnykhzadach [Pareto-Optimal Solutions of the Multi-objective Problems]. Moscow: Nauka; 1982. 254 p. (in Russ.)

Sobol IM, Statnikov RB. 1981. Vyboroptimalnykhparametrov v zadachakh so mnogimikriteriiami [The Choice of Optimal Parameters in Multicriteria Problems]. Science Publ, Moscow, Russia

Taherdoost H, Madanchian M. 2023. Multi-Criteria Decision Making (MCDM) methods and concepts. Encyclopedia. 3(1), 77-87.

Tlas M, Abdul Ghani B. 2020. Interactive software for classification and ranking procedures based on multi-criteria decision-making algorithms. Computational Ecology and Software, 10(3): 133-150

Tlas M. 2013. An Interactive Interior Point Method for Multi-objective Nonlinear Programming Problems, Journal of Mathematics. Statistics and Operations Research (JMSOR), 2(1): 34-44

Vasawade R, Deshmukh B, Kulkarni V. 2015. Modularity in design: A review. IEEE International Conference on Technologies for Sustainable Development (ICTSD).

Verkhova GV, Akimov SV. 2017. Multi-aspect modeling system objects in CALS. Proceedings of 2017 20th IEEE International Conference on Soft Computing and Measurements (SCM). 449-451, St Petersburg, Russia, Russia. DOI: 10.1109/SCM.2017.7970614

Zadeh LA. 1975. The concept of a linguistic variable and its application to approximate reasoning-I.Information Sciences, 8(3): 199-249

Zakarian A., Rushton G. 2001. Development of modular electrical systems. IEEE/ASME Transactions on Mechatronics, 6(4): 507-520. DOI: 10.1109/3516.974865