Article

Climate change impact of greenhouse gases: A mathematical modeling approach

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Abstract
A mathematical model is (newly proposed) to be defined the interconnection of the four state variables greenhouse gases (GHGs), atmospheric temperature, phytoplankton population and fishery population. Basically, it is a non-linear differential equation mathematical model. It contains environmental parameters related to climate change, caused by quick increment in amount of GHGs, which caused the disturbances in the marine environments that affects marine ecological system especially fisheries and phytoplankton population. The purpose of the study is to explain the current situation of the effect of climate change to marine ecosystem. In the mathematical modeling part, equilibrium analysis, local stability and numerical simulations have been done. A novel model for environment management using the time variant parameters is validated through analysis, and numerical simulations are used to test the study's analytical findings.

Keywords greenhouse gases; mathematical modeling; computational simulations; global warming.

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1 Introduction

Land and ocean are two distinct geophysical entities. Land offers transportation infrastructure as well as essential components for survival such as food, oxygen, and a diverse range of living organisms. Oceans, on the other hand, also provide transport infrastructure, food, oxygen, water, and a vibrant habitat for a wide array of living species including coral reefs, fish, Riftia, Shark, Orca, whale, and trees like mangroves and shrubs. The ocean is broken down into five oceanic divisions: Pacific, Atlantic, Arctic, Indian, and Antarctic. Marine ecosystem is the term used to describe a salty water ecosystem. This environment provides a special habitat to the variousliving organisms.Marine ecosystem provides oxygen to the ratio 70:30 with the terrestrial ecosystem and concurrently the largest CO_2absorber (Barange at el., 2014). Various anthropogenic activities like burning of fossil fuels, emissions of CFC’s, using pesticides, fertilizers, throughtoxic wastes (industrial, plastic)that have detrimental effects on aquatic ecosystem and simultaneously the natural increasing rate is taking place. Any movement that affects marine habitat is directly proportional to the earth habitat. Land is already damaged up to a great extent and the oceans also. But the biggest problem arises in the last century that troubles most is the climate change due to increasing global warming (Regnier at el., 2022).

In marine fish population and phytoplankton population are act like predator and preys. Bothpopulations are important as the fish population is one of the leading foods for the human population and the other living creatures in the water andPhytoplankton is the most important as it provides oxygen and one of the largest CO_2 absorbers (Mandal at el., 2022). But increase in global warming decreases the ability of phytoplankton to provide oxygen and absorb CO_2 as phytoplankton also requires a specific environmental condition to survive (Scheffer at el., 2000). Increase in atmospheric temperature also leads down to decrease in phytoplankton population. Excess GHGs emissions raise atmospheric temperature and have seriousirreversible effects on climate change (Sarker at el., 2020). Rapid greenhouse gas emissions and climate change are causing many unanticipated changes in the natural world, including the introduction of new and destructive natural disasters like flooding, droughts, hurricanes, cyclones, tornadoes, tsunamis, rising sea levels, and acidification.Scientists estimate that human activity accounts for around 83% of greenhouse gas emissions, and that human activity is the primary cause of climate change (National Research Council, 2010). However, the number of people on Earth is steadily reducing the amount of forest cover, which cleans the air through absorption of roughly 32.6 gigatonnes of CO_2 annually (Pan et al., 2013). As a result, as the amount of forest area decreases, the number of GHGs in the atmosphere and air temperature rises significantly. Because of this, an estimated 137 plants, insect species and plant are being distancing day by day and individualsare suffering from various skin ailments as a result of incoming UV light from former natural climates (Sahney et al., 2010). According to an estimate by CIEL (2019), if greenhouse gas emissions continue at their current rate, the environment is projected to emit 1.34 billion tons of GHGs annually until 2030 and 56 billion tons until 2050. Additionally, forecasting results shows the rise in atmospheric temperature may surpass the past records due to global warming in initially phases of year 2047, that would have a detrimental impact on a variety of ecosystems and earth dynamics (Mora, 2013).

While many works have been statistically examined, only a small number of studies use mathematical modelling to show excess emissions of greenhouse gases. In addition to introducing an optimal management strategy to control the influence of climate change, the authors highlighted a mathematical model to investigate the consequences of global warming in Bangladesh specifically the southwest area (Biswas et al., 2016). Devi and Mishra (2020) have presented and studied a new mathematical model to investigate the effects of rising environmental temperatures brought on by rising greenhouse gas emissions on the plant–pollinator system. In order to determine the correlation between the sources of CO_2 emissions and the expansion of the Indian population, a Bayesian model was also presented. As per population (Babbar and Babbar, 2018), the model incorporated the idea that the sources of CO_2 are growing in direct proportion to the size of the Indian. Other researchers provided a description of the practical implementation of climate adjustment strategies for climate change to enhance management for food safety and security (Islam et al., 2020). Additionally, the authors used crop modelling to identify the primary disasters that have been hastened by climate change.The authors utilized training and awareness campaigns for public as a control variable to reduce the CO_2 produced by vehicles, while reforestation was implemented by putting forth a non-linear mathematical model with time delay to manage the CO_2 present in atmosphere which is thought to be the primary greenhouse gas (Misra et al., 2015). In order to explain how climate change can affect the increment in angiosperm species especially in highland ecosystems of Himalayas, another model built using extensive collection of data (Manish et al. 2016). Montzka et al. (2011) highlighted anthropogenic non-CO_2 greenhouse gas emissions have been a major contributor to global warming; in contrast, GHGs, particularly CO_2 released from various fossil fuel combustion processes, have relatively less effects on global warming. Ledley et al. (1999) explored the potential root causes responsible for global warming and highlighted the relations among climate change and GHGs. Samimi et al. (2013) highlighted the controlling techniques and emphasized that root cause for climate change are GHGs.

Specifically, a range of alternate forecasts regarding Bangladesh's future carbon dioxide levels were quantitatively defined based on varying rates of increase in the global population and GDP (Gunter 2010). According to a recent brief description by (Avila et al., 2019), the airplane is one of the main causes of the rapid global warming because it creates high thin clouds that mix heated exhaust fumes and water vapour with lower thin clouds. On the other hand, (Pendrill et al., 2019) ranked deforestationwhich is fueled by the growth of forestry and agricultureas the second-largest source of greenhouse gas emissions. The researchers also mentioned that between 2010 and 2014, the growth of agricultural resulted in annual CO_2 emissions of about 2.6 gigatonnes.In addition, Rock et al. (2020) linked the climate issue to greenhouse gas emissions from buildings after examining greater than 650 life cycle assessments of these emissions. Several authors discussed detailed investigation on the activities and emissions of GHGs as well as latest advancements in mathematical modelling (Rosa et al., 2016; Pathak and Stoddard, 2018; Biswas et al., 2016; Planton, 2013; Dym, 2004; Kumi-Boateng and Stemn, 2020; Huntingford et al., 2013; Sanderson et al., 2011; Barros Santiago et al., 2019).

Environmental science, the threat posed by emitted GHGsfor the atmosphere, and the interactions between living things and the environment are all covered in this study. The research aims to delineate the effects of climate change, due torised greenhouse gas emissions, on the human population and the forest environment in the vicinity of coastal areas.A non-linear mathematical model that takes into account four dynamical species has been created in this instance. After determining if the species is bounded, stability analyses at eachequilibrium point have been calculated. This study also includes thenumerical simulations. So, the mechanism of providing oxygen and absorbing CO_2 is getting disturbed by increasing global warming. Mathematical modeling can explain the current scenario or explain the linkage between all variables and parameters that are involved.The schematic diagram of the model is shown in the following Fig. 1.

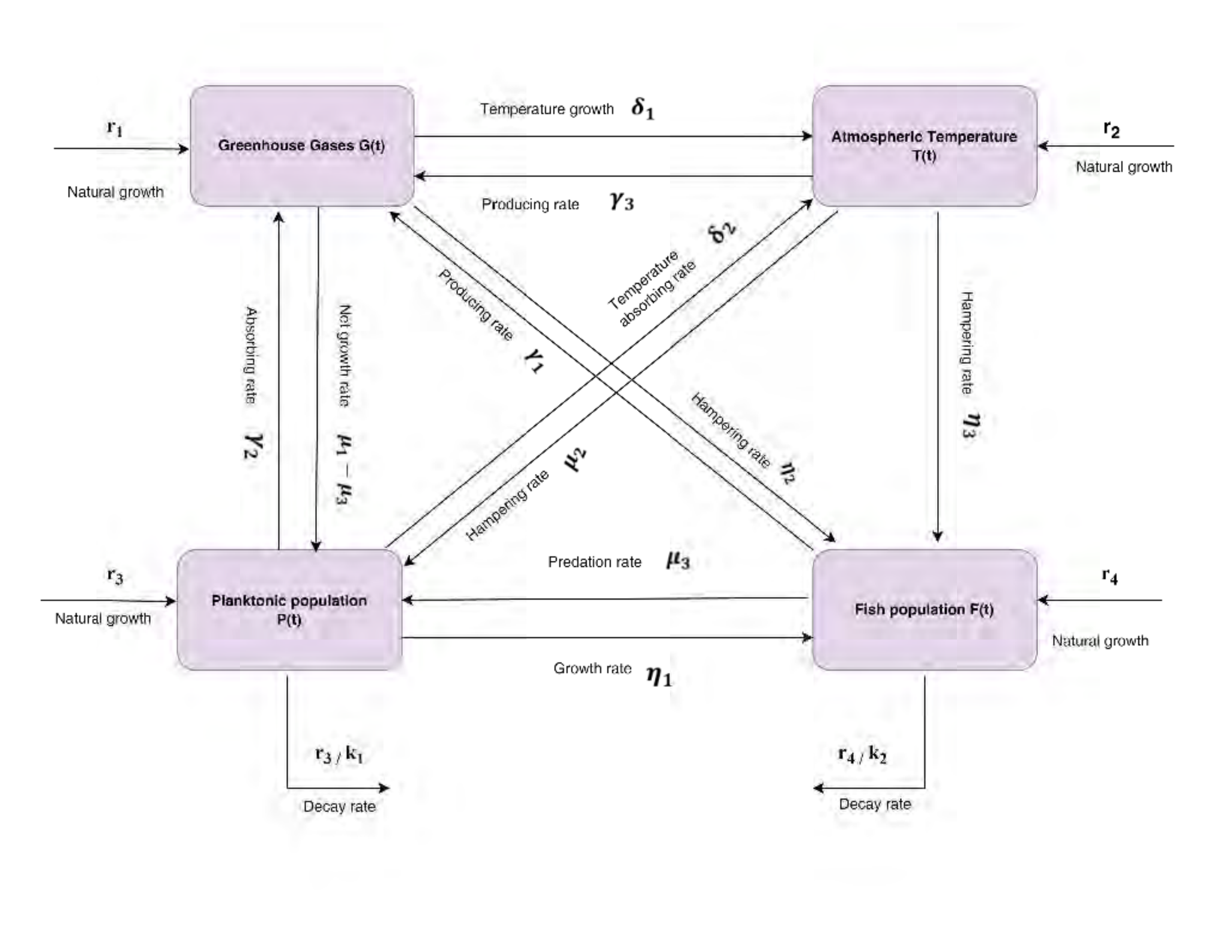

Fig. 1 Schematic diagram.

This is the schematic diagram that explain the interconnection of all four state variables with each other and the time varying parameters.

2 Formulation of Mathematical Model

In this section, following are the four mathematical differential equations involved in the differential equation mathematical model. These are the equations of the four state variables and the environmental parameters involved in the system. We assume that Greenhouse gases denoted by G(t) at time t with the atmospheric temperature denoted by T(t) and density of phytoplanktonic population and fish population is denoted by P(t) and F(t) respectively.

$$\frac{dG}{dt} = r_1G + \gamma_1FG - \gamma_2PG + \gamma_3T \quad (1)$$

$$\frac{dT}{dt} = r_2T + \delta_1GT - \delta_2PT \quad (2)$$

$$\frac{dP}{dt} = r_3P\left(1 - \frac{P}{k_1}\right) + \frac{M_1P}{\alpha + G} - r_2^4TP - \frac{\mu_3P^2F}{\beta^2 + P^2} - M_4P \quad (3)$$

$$\frac{dF}{dt} = r_4F\left(1 - \frac{F}{K_2}\right) + \frac{\eta_1\mu_3P^2F}{\beta^2 + P^2} - \frac{\eta_2F}{\alpha + G} - \eta_3TF \quad (4)$$

having conditions T(t)≥0, G(t)≥0, F(t)≥0 and P(t)≥0 initially.

The four differential equations that depict the behaviour of these state variables as they change over time. Each equation generally represents the rate at which a state variable changes over time, sometimes influenced by the interactions between variables and other environmental elements. These describes the temporal variation of greenhouse gases, potentially incorporating variables that consider natural emissions, environmental absorption, and human-induced influences; represents the relationship between atmospheric temperature and the concentration of greenhouse gases, potentially including radiative forcing factors; represents the population dynamics of phytoplankton, which may be affected by factors such as temperature and greenhouse gases; represents the dynamics of the fish population, which is presumably affected by the abundance of phytoplankton as a food source and the water temperature (Table 1).

Table 1 Description of variables and parameters involves in the model formulation.

Symbol	Descriptions
r_1	Rate of greenhouse gases in oceans increasing naturally
r_2	Atmospheric temperature natural growth rate
r_3	Plankton population natural growth rate
r_4	Fish population natural growth rate
Υ_1	Producing rate of greenhouse gasses by plankton populations in oceans
Υ_2	Absorbing rate of greenhouse gasses by plankton populations in oceans
Υ_3	Producing rate of Greenhouse gasses due to atmospheric temperature
δ_1	Rate of rising Atmospheric temperature due to greenhouse gasses
δ_2	Absorbing rate of temperature due to plankton population
μ_1	Growth rate plankton population due to carbon dioxide
μ_2	Hampering rate of plankton population because of global warming
μ_3	Rate of Predation of plankton by fish
μ_4	Decrement in plankton rate caused by acidity effect
$\frac{{}^2F}{P^2}$	The functional response of predator (Fish)-Holling type-3 functional response
η_1	The conversion factor denoting the number of newly born predators(fish) for each captured prey(plankton)
η_2	Decrement in fishes rate caused due to acidity effects by GHGs
η_3	Impeding the fish population’s rate due to climate change
A	Saturation constant

2.1 Boundedness of the system

The boundedness and non-negativity of the solution for system (1-4) are shown using Lemma 1 and Lemma 2.

Lemma 2.1: Set $\lambda=\left\{(G,T,P,F)\in R_4^+: p(t)=G(t)+T(t)+P(t)+F(t), 0\leq p(t)\leq\frac{\psi}{\Phi}\right\}$ is aarea of attractionsto every solution and all the variables are positive initially, where η is a constant and $\psi=\frac{K_1}{4r_3}(r_3+\Phi)^2+\frac{K_2}{4r_4}(r_4+\Phi)^2+\frac{1}{8\mu_4}(r_1+\Phi)^2+\frac{1}{4\eta_3}\left(\gamma_3+r_2+\Phi\right)^2$.

Proof: Let $h(t)=G(t)+T(t)+P(t)+F(t)$ be a constant. Then, we can write.

$$\frac{dh}{dt}+\Phi h=r_1G+\gamma_1FG-\gamma_2PG+\gamma_3T+\Phi G+r_2T+\delta_1GT-\delta_2PT+\Phi T+r_3P\left(1-\frac{P}{k_1}\right)$$
$$+\frac{\mu_1P}{\alpha+G}-\mu_2TP-\frac{\mu_3P^2F}{\beta^2+P^2}-\mu_4P+\Phi P+r_4F\left(1-\frac{F}{K_2}\right)+\frac{\eta_1\mu_3P^2F}{\beta^2+P^2}-\frac{\eta_2F}{\alpha+G}-\eta_3TF+\Phi F$$
$$\Rightarrow\frac{dh}{dt}+\Phi h=(r_1+\Phi)G+\left(\gamma_3+r_2+\Phi\right)T+(r_3+\Phi)P-\frac{r_3P^2}{K_1}+(r_4+\Phi)F-\frac{r_4F^2}{K_2}+\left(\gamma_1F-\gamma_2P\right)G$$
$$+(\delta_1G-\delta_2P)T+\left(\frac{\mu_1}{\alpha+G}-\mu_2T-\frac{\mu_3PF}{\beta^2+P^2}-\mu_4G\right)P+\left(\frac{\eta_1\mu_3P^2}{\beta^2+P^2}-\frac{\eta_2}{\alpha+G}-\eta_3T\right)F$$

Given that the rate of increase in temperature of the atmosphere caused by greenhouse gases (GHGs) is equal to or greater than the rate at which temperature is absorbed in the oceans by the planktonic population,hence $\delta_1\geq\delta_2$. While fish population has a relatively low rate of greenhouse gas (GHG) production, the planktonic population has a far higher rate of GHG absorption i.e. $\gamma_2P\geq\gamma_1F$.

However, assuming

$$\left(\frac{\mu_1}{\alpha+G}-\mu_2T-\frac{\mu_3PF}{\beta^2+P^2}-\mu_4G\right)P\approx2\mu_4G^2 \quad\text{and}\quad \left(\frac{\eta_1\mu_3P^2}{\beta^2+P^2}-\frac{\eta_2}{\alpha+G}-\eta_3T\right)F\approx\eta_3T^2$$

Then we have,

$$\frac{dh}{dt}+\Phi h=(r_3+\Phi)P-\frac{r_3P^2}{K_1}+(r_4+\Phi)F-\frac{r_4F^2}{K_2}+(r_1+\Phi)G-2\mu_4G^2+\left(\gamma_3+r_2+\Phi\right)T-\eta_3T^2$$

$$\frac{dh}{dt}+\Phi h=\frac{K_1}{4r_3}(r_3+\Phi)^2-\frac{r_3}{K_1}\left(P-\frac{K_1}{2r_3}(r_3+\Phi)\right)^2+\frac{K_2}{4r_4}(r_4+\Phi)^2$$
$$-\frac{r_4}{K_2}\left(F-\frac{K_2}{2r_4}(r_4+\Phi)\right)^2+\frac{1}{8\mu_4}(r_1+\Phi)^2-2\mu_4\left(G-\frac{r_1+\Phi}{4\mu_4}\right)^2$$
$$+\frac{1}{4\eta_3}\left(\gamma_3+r_2+\Phi\right)^2-\eta_3\left(T-\frac{1}{2\eta_3}\left(\gamma_3+r_2+\Phi\right)\right)^2$$
$$\frac{dh}{dt}+\Phi h\leq\frac{K_1}{4r_3}(r_3+\Phi)^2+\frac{K_2}{4r_4}(r_4+\Phi)^2+\frac{1}{8\mu_4}(r_1+\Phi)^2+\frac{1}{4\eta_3}\left(\gamma_3+r_2+\Phi\right)^2$$

Using the differential inequalities,

$$0\leq g\big(G(t),T(t),P(t),F(t)\big)\leq\frac{\Psi}{\Phi}(1-e^{-\Phi t})+\big(G(0)+T(0)+P(0)+F(0)\big)e^{-\Phi t}$$

Taking the limit as $t\rightarrow\infty$ then we have $0\leq g(t)\leq\frac{\Psi}{\Phi}$. Hence the system is bounded inλ.

2.2 Existence of positivity of the model

Lemma 2.2 For all $t\geq0$, system (1) has non-negative solutions.

Proof: The system hasequation (1) which can be written as:

$$\frac{dG}{dt}=r_1G+\gamma_1FG-\gamma_2PG+\gamma_3$$

In equation (S1), the positivity conditions were used and the modified equation is given below:

$$\frac{dG}{dt}\geq\big(r_1+\gamma_1F-\gamma_2P\big)G$$

$$\Rightarrow\frac{dG}{dt}\geq M_1G,\text{ where } M_1=r_1+\gamma_1F-\gamma_2P$$

$$\Rightarrow\ln G\geq M_1t+\ln c_1\text{ , where } c_1 \text{ is an integrating constant.}$$

So that we have, $G(t)\geq c_1e^{M_1t}$

Using the initial condition i.e. when $t=0$, equation (S2) becomes $G(0)=G_0>0$ that implies $G(0)=G_0\geq c_1$. Now, using the value of$c_1$in Eq. (S2), we have $G(t)\geq G_0e^{M_1t}$. Therefore, $G(t)>0$ when $t\rightarrow\infty$. $G(t)$ is positive for all$t\geq0$.

Uniformly, the authors got $T(t)>0,P(t)\geq0,F(t)\geq0\forall t\geq0$using equations (2-4). Therefore, the lemma is proved including $G(t)>0,T(t)>0,P(t)\geq0,F(t)\geq0\forall t\geq0$

3 Existence of Equilibrium Points

We obtain equilibrium points of the system by setting $G'(t)=T'(t)=P'(t)=F'(t)=0$. The system (1-4) produces three equilibrium points

i) $E_0(0,0,0,0)$, which is always exists.

ii) $E_1(\overline{G},\overline{T},0,0)$, where $\overline{G}=\frac{-r_2}{\delta_1}$, $\overline{T}=\frac{-r_1}{r_3}\left(\frac{-r_2}{\delta_1}\right)$.

iii) $E_2(G^{\phi},T^{\phi},P^{\phi},0)$, where

iv) $E_3(G^{\theta},0,P^{\theta},0)$, where $P^{\theta}=\frac{r}{\gamma_2}$, $G^{\theta}=\left[\left(\frac{\delta_2r_1}{\gamma_2}\right)-r_2\right]\frac{1}{\delta_1}$

For G^{θ} to be positive, we must have $r_2<\frac{\delta_2}{\gamma_2}r_1$.

v) $E_4(G^*,T^*,P^*,F^*)$ where $P^*=\frac{r_2+\delta_1G^*}{\delta_2}$, $T^*=\frac{1}{\gamma_3}\left[\frac{\gamma_2\delta_1}{\delta_2}(G^*)^2+\left(\frac{\gamma_2r_2}{\delta_2}-r_1-\gamma_1F^*\right)G^*\right]>0$ if $\frac{\gamma_2r_2}{\delta_2}>r_1+\gamma_1F^*$,

4 Stability Analysis of the System (1)

The system of equations (1-4) can be represented in the Jacobian matrix as

$$M=\begin{bmatrix}e_{11} & e_{12} & e_{13} & e_{14}\\ e_{12} & e_{22} & e_{23} & e_{24}\\ e_{31} & e_{32} & e_{33} & e_{34}\\ e_{41} & e_{42} & e_{43} & e_{44}\end{bmatrix}$$

where

$e_{11}=r_1+r_1^*F^*-r_2p^*;$

$e_{12}=r_3;\ e_{13}=-\gamma_2G^*;\ e_{14}=r_3$

$e_{21}=\delta_1;e_{22}=0;e_{23}=0;e_{24}=0$

$e_{31}=\frac{-\mu_1p^*}{\alpha^2};e_{32}=0;e_{33}=\frac{-h_3}{k_1}-\frac{(\beta^2-p^2)F_3^*}{(\beta^2+p^2)^2};e_{34}=-\mu_2p^{7*}$

$e_{41}=-\frac{\eta_2}{\alpha^2}F^*;e_{42}=-\eta_3F^*;e_{43}=2\eta_1\mu_3F^*p^*/\beta^2$

$e_{44}=\frac{-r_4}{k_2};e_{33}=\frac{-r_3}{k_1}-\frac{r_3}{\beta^4}(\beta^2-p^{x^2})f^*$

Therefore, the characteristic equation is$\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0$ (A)

$\lambda^4+\lambda^3+\lambda^2+A_3\lambda+=0$

$A_1=-e_{33}-e_{44}-e_{11}$

$A_2=e_{33}e_{44}+e_{11}e_{33}+e_{11}e_{44}-e_{34}e_{43}+e_{12}e_{21}-e_{31}e_{13}-e_{14}e_{31}e_{43}$

$A_3=-e_{11}e_{33}e_{44}+e_{11}e_{34}e_{43}-e_{12}e_{21}e_{33}+e_{31}e_{13}e_{44}-e_{41}e_{13}e_{34}-e_{14}e_{21}e_{42}e_{33}-e_{14}e_{31}e_{14}+e_{14}e_{41}e_{33}$

$A_4=-e_{12}e_{21}e_{33}e_{44}-e_{13}e_{21}e_{42}e_{34}+e_{44}e_{21}e_{42}e_{33}$

Using the Routh–Hurwiz criterion to equation (A) which is the characteristic Equation.The sufficient and necessary requirement of the positive equilibrium $E_4(G^*,T^*,P^*,F^*)$ for the local stability is that all the determinants $D_1=A_1$, $D2=A_1A_2-A_3$,$D_3=A_1(A_1A_3-A_1A_4)-A_3^2$ and $D_4=A_4D_3$ are positive. Thus, we have the following theorem:

Theorem 4.1 The endemic equilibrium points$E_4(G^*,T^*,P^*,F^*)$ of system (1) is asymptotically stable, If $D_i>0,i=1,2,3,4$

5 Numerical Simulations

The model (1) was simulated numerically using the ode45 solver in the MATLAB programming, and using parameters given below:

$r_1=0.031;\gamma_1=0.028;\gamma_2=0.038;\gamma_3=0.035;\ r_2=0.031;\delta_1=0.002;\delta_2=0.407;$
$r_3=0.868;k_1=0.045;\mu_1=0.885;\alpha=0.25;\mu_2=0.712;\ \mu_3=0.401;\beta=0.542;\mu_4=0.448;$
$r_4=0.287;k_2=0.177;\eta_1=0.341;\eta_2=0.042;\eta_3=0.721$

The simulations attempt to validate the study's analytical conclusions while also describing the species and their behaviors dynamically, particularly fish populations and plankton, in marine systems under rapid climate change.The authors discussed the possible impacts of climate change on fisheries resources and the planktonic in direct proportion to the constant increase in the concentration of GHGs in atmosphere. The swift accumulation of greenhouse gases leads to the introduction of acidity in ocean water, resulting in the devastation of marine biodiversity and fisheries resources. In addition, the fast warming of the planet is mostly to blame for the decline and threat to marine fishing resources.

However, due to a lack of food, the population of fish drops proportionately as plankton variety declines. As a result, rising GHG concentrations are contributing to global warming, which in turn causes a sharp decline in planktonic population through the introduction of acidity and warmth. This ultimately leads to a sharp decline in fisheries resources. Fig.2 depicts the variations in the dynamic species' development.The authors discussed various rates of GHG absorption under consideration bydynamic species from their dynamic behaviors by the planktonic population, as shown in Figs.3–6. When illustrated in Fig. 3, the growing concentration of GHGs slows down when the planktonic population absorbs nitrous oxide (N_2O) and saturated

CO_2. Since concentration of environmental GHGs changes proportionally with temperature, as demonstrated in Fig. 4, the absorption increments in GHGs produced by environment and the planktonic species plays role in decrement of temperature of atmosphere.

Plankton population's ability to photosynthesize depends on CO_2, and its ability to store O_2 for respiration depends on N_2O. As a result, when saturated CO_2 and N_2O are absorbed at higher rates, marine plankton develops faster. The enhancing process of the planktonic population as GHG absorption rises is seen in Fig. 5.However, because fish eat plankton, the rise in fish population is related to that of the population of plankton species.In Fig. 6, the planktonic population grows due to absorption of CO_2which is dissolved, then population of fishes also grows by obtaining enough food from the plankton, obtaining enough dissolved O_2, and experiencing less warming of the sea water. The models come to the conclusion that when marine plankton's rate of GHG absorption rises, global warming is mitigated by a decrease in GHG concentration and an improvement in plankton diversity density and fishing resources.

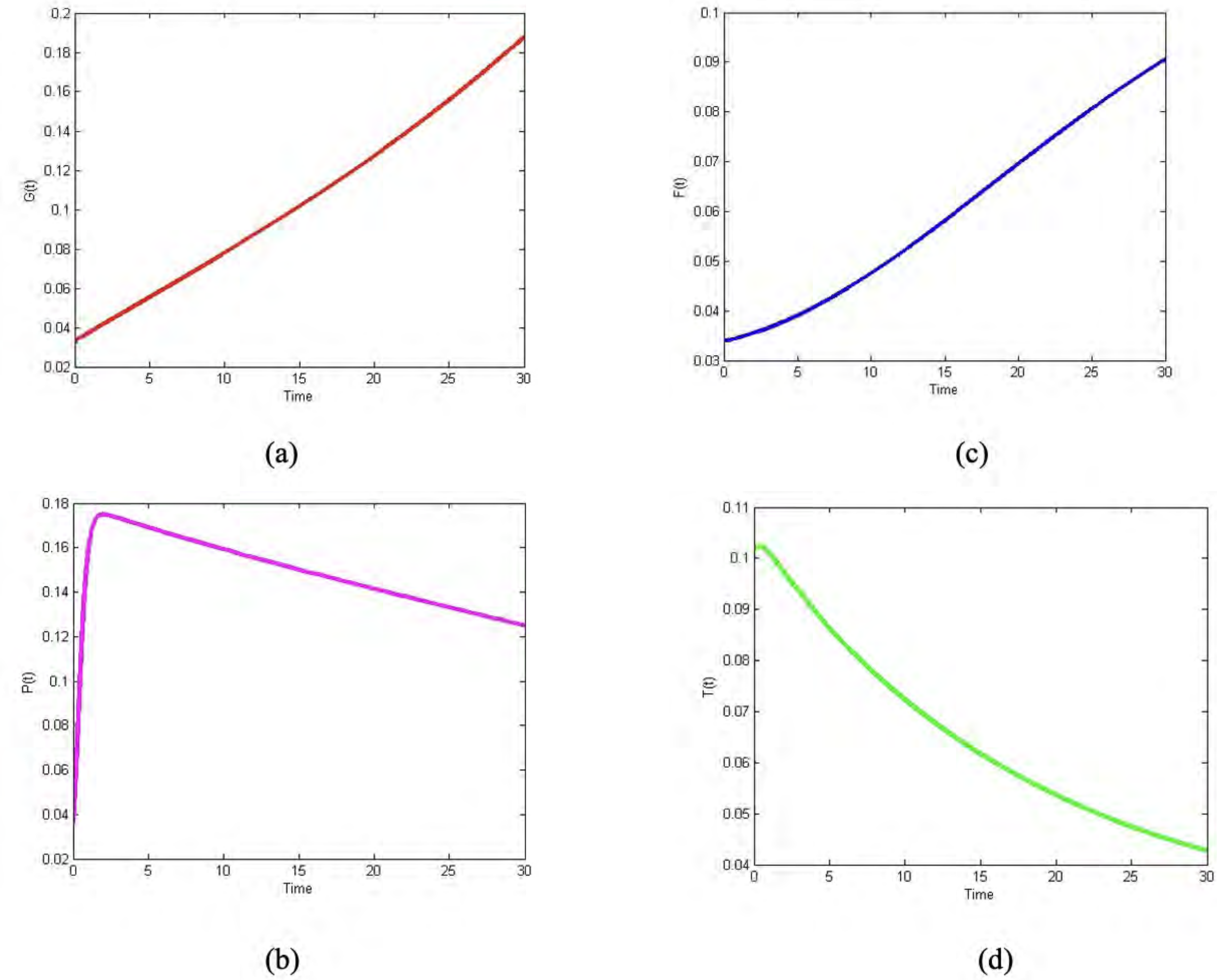

Fig 2. Increment in GHGs and their impacts temperature in on atmosphere and marine ecosystems.

The Fig. 2 presents the impacts of increment in GHGs on marine ecosystems and temperature of atmosphere which includes the phytoplankton and fish populations. Part (a) shows the increase in greenhouse gases G(t) over time. The slow rise in shows increment in concentration of greenhouse gases over time due to continuous emissions from natural and anthropogenic sources. The graph in Part (b) indicates the phytoplankton population density P(t) over time. The initial increment can be seen in the curve which start decreasing after the peak. This implies that although the initial situation may support phytoplankton growth, the rising temperature or additional factors eventually result in a decline in the population, potentially due to unfavourable atmospheric conditions or increased predation. The graph in part (c) presents the fish population density F(t) over time. The population of fish indicates a slow and consistent decline over a period of time. Possible causes for this drop may include elevated temperatures impacting their natural surroundings, reduced abundance of food (phytoplankton), or additional stresses associated with environmental shifts. Part (d) shows |the atmospheric temperature T(t) as a function of time. The temperature steadily rises over time due to increased greenhouse gases, indicating their influence on global warming.

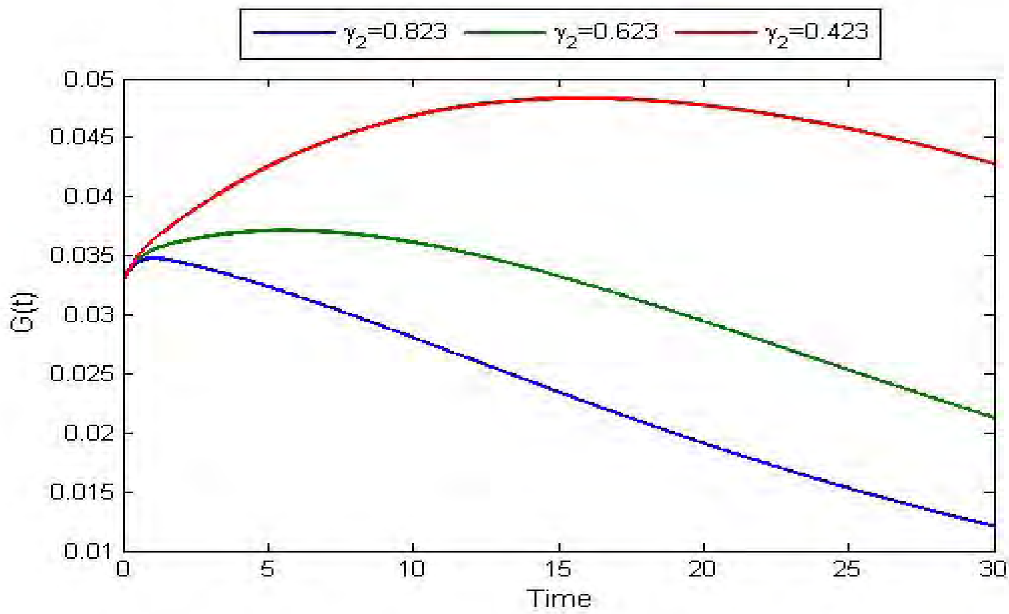

Fig. 3 Decrement rate of GHGs by Increment in γ_2.

The graph represents the temporal variation of the level of GHGs for many different values of the parameter γ_2. The value γ_2 reflects how greenhouse gases are absorbed or eliminated from the atmosphere, potentially through natural processes or synthetic interventions. Fig. 3 indicates that when the value of γ_2 increases, the system becomes increasingly efficient in decreasing the levels of greenhouse gases over time. The graph highlights the significance of these strategies in mitigating climate change by demonstrating how they can expedite the reduction of atmospheric greenhouse gases.

The graph illustrates the temporal variation of the temperature of the atmosphere T(t)for different values of the parameter γ_2. It presumably indicates the rate at which greenhouse gases are absorbed from the atmosphere, impacting the total temperature.The graph demonstrates that when the value of γ_2 increases, there is a more pronounced reduction in air temperature. This implies that improving the processes or technologies that eliminate or absorb greenhouse gases can significantly reduce global warming. The slower decline in temperature for smaller $\gamma_2$values highlight the difficulties of reducing global temperatures if greenhouse gas absorption rates are insufficiently high.Fig. 4 illustrates that the air temperature declines faster with an increasing value of γ_2. This underscores the significance of enhancing mechanisms for absorbing greenhouse gases to address climate change and mitigate global temperatures effectively. The graph effectively illustrates the possible advantages of increasing γ_2 to achieve greater temperature reductions over a period of time.

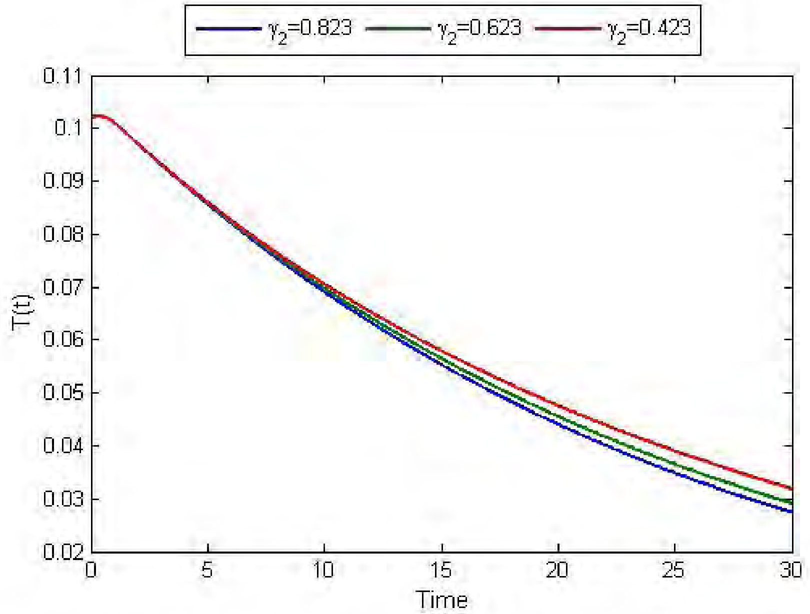

Fig. 4 Decrement rate of the atmospheric temperature caused by increment in γ_2 values.

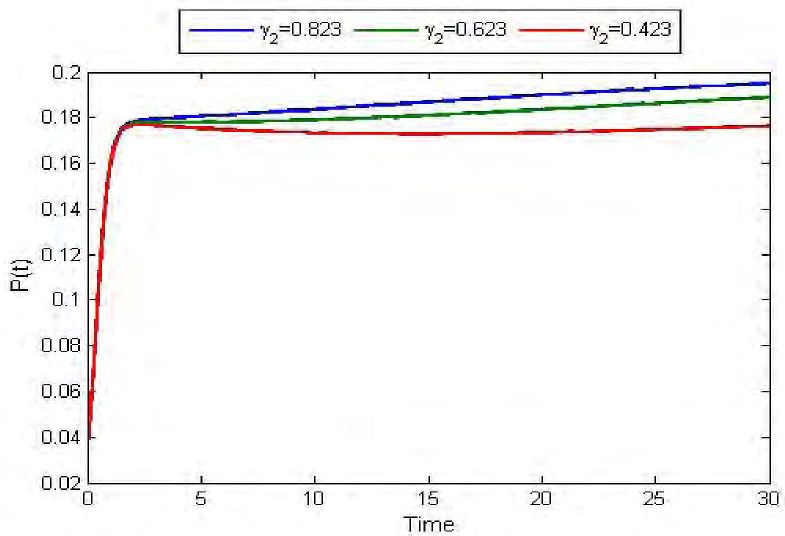

Fig. 5 Increment in rate of the planktonic population caused by increment in γ_2 values.

The graph in Fig. 5 illustrates the temporal variations of the planktonic population density P(t) for various values of the parameter. Similar to prior data, it is probable that the figure represents the rate at which greenhouse gases are taken in from the atmosphere, which indirectly impacts environmental variables like temperature, subsequently influencing the number of plankton. The graph demonstrates that when the γ_2 grows, the plankton population experiences a positive impact, exhibiting accelerated growth and ultimately attaining greater equilibrium. These findings indicate that enhancing the processes by which greenhouse gases are absorbed could benefit marine ecosystems, specifically for primary producers such as plankton, which play a crucial role in the food chain. Fig. 5 shows that the planktonic population experiences a rise and then reaches a stable state at higher values. This suggests that improved absorption of greenhouse gases can result in more advantageous environmental conditions, promoting the growth and stability of the planktonic population, which is crucial for preserving the well-being of marine ecosystems. The graph emphasizes the significance of reducing greenhouse emissions to maintain and improve marine life.

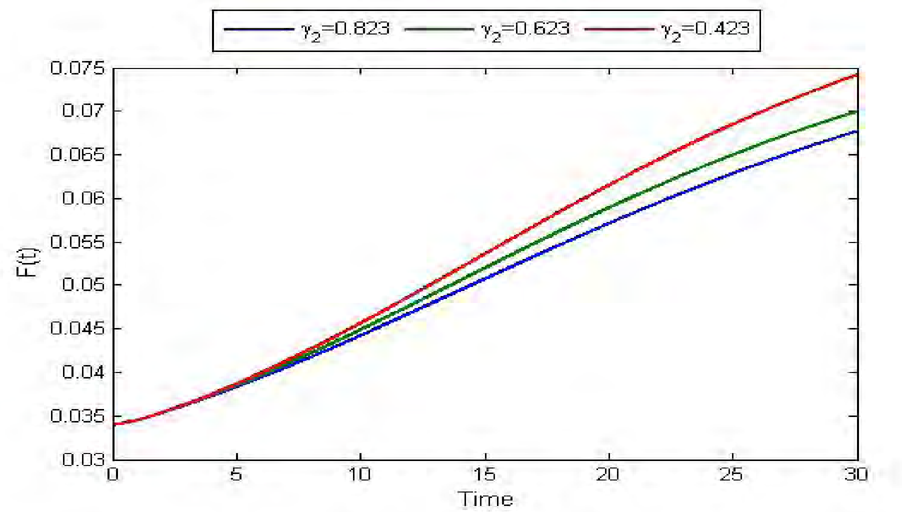

Fig. 6 Increment infish population rate caused by γ_2 values increment.

The graph demonstrates the temporal variations in fish population density, denoted as F(t), for various amounts of the parameter γ_2. Similar to the preceding data, γ_2 most likely indicates the rate at which greenhouse gases are taken in or eliminated from the atmosphere, affecting environmental factors like temperature and consequently impacting the marine ecosystem, including the fish population. The graph illustrates that the fish population experiences a more rapid growth when the efficiency of greenhouse gas absorption is lower. This phenomenon may indicate intricate interconnections within the ecosystem, wherein specific levels of greenhouse gases or related circumstances transiently promote the proliferation of fish populations. Nevertheless, it is crucial to consider that rapid expansion resulting from reduced γ_2 values may not be viable in the extended period, as it could be linked to additional adverse ecological consequences. Fig. 6 demonstrates that the fish population experiences a more pronounced growth when the value of γ_2 drops. This implies that reduced efficiency in absorbing greenhouse gases may lead to favourable conditions for an increase in the accelerated fish population. Nevertheless, the long-term viability of this expansion in the face of these circumstances may be uncertain, contingent upon additional variables within the ecosystem. The graph illustrates the intricate nature of biological reactions to alterations in climatic conditions caused by greenhouse gas dynamics.

6 Concluding Remarks

In this study, we provide a novel model (1-4) for environment management with time-varying parameters. The authors explored the effects of climate change/global warming on marine ecosystem caused by the fast accumulation of ambient GHGs using the model. The authors validated the model through analysis, and numerical simulations are used to test the study's analytical findings. For the first time, we look at how changing ambient GHG concentrations would affect marine ecosystems and global warming. In this regard, we related the rise of air’s temperature, fish population and planktonic population to the variation of GHG concentrations.The authors highlighted that climate change is fast growing as a result of rising GHG concentrations, which increase the temperature and acidity of ocean water, resulting in a steady decline in fish and plankton populations in marine environments. Simulations show that increasing marine plankton diversity while lowering GHG concentrations can improve marine fisheries resources. This study also forecasts that if current trends continue, marine habitats will become ecological deserts in upcoming fifty yearsdue to the ongoing decline in fisheries resources and plankton diversity. The current research serves as a conceptual framework combining environmental variables and real-life situations based mathematical equations. As a result, the study's findings may alter if there is significant change in conditions and parameters related to marine ecosystem and environment in future.

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