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Assessing the effect of dynamics of unpredictable locust invasive behavior and its effect on food security and community livelihood

Mfano Charles Petro
Department of ICT and Mathematics, College of Business Education. P.O. Box 1968, Dar Es Salaam, Tanzania
E-mail: mfano.petro@cbe.ac.tz, mfanopetro@gmail.com

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Abstract
Locusts are a highly destructive type of pest within the grasshopper species worldwide. When there is an invasion at a specific location, they can cause severe damage to crops. This study presents a mathematical model that examines locust invasions in terms of their dynamic behavior in both the source and invaded zones. The model is formulated using differential equations and takes into account the parameters and variables identified from both zones. The parameters were estimated using the least squares method, with all parameters being normally distributed. The study thoroughly discusses the existence of solutions by identifying equilibrium points and establishing conditions for their stability using the Jacobian matrix, Routh-Hurwitz criteria, and Lyapunov function. The analytical solution suggests that the system is stable when the intrinsic growth rate (σ_{i} and σ_{s}) is higher. However, the numerical solution indicates that the presence of locust populations in the two zones is determined by the intrinsic growth rate (σ_{i} and σ_{s}) the locust population increases and stabilizes after a few days. Factors such as survival and invasion rates were found to be major contributors to the existence and dynamics of locusts in the two zones. Additionally, it was observed that the deterrent coefficient in the invaded and source zones (η_{i} and η_{s}) has a significant impact on controlling the dynamic behavior of locusts.

Keywords locust behavior; mathematical modeling; least square method (LSM); community livelihood; stability analysis.

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1Introduction

There are approximately 12,000 species of grasshoppers globally, with only 20 of these speciespossessing the ability to undergo a transformation into locusts, which are recognized as hazardous crop pests (Word Ries et al., 2024). Grasshoppers, inclusive of those capable of transitioning into locusts, are categorized under the Acrididae family, suborder Caelifera, order Orthoptera, and class Insecta (Kariuki, 2022). Their collective behavior, characterized by swarming, poses a significant threat to agricultural productivity (Babar, 2023; Hassan and Aslam, 2024; Cease et al., 2015). Among the 20 species of locusts are desert locust *(Schistocerca gregaria),* migratory locust (*Locusta migratoria*) and red locust that are extensively distributedand have extreme population changes (Rajak and Yadav, 2023). Desert locusts are mostly foundin Sahara Desert and is the most destructive locust specie in the world (Cressman, 2016; Babar, 2023). Migratory locusts are mostly found in Africa, Europe, Asia and Australia (Chen et al., 2020). The red locust, prevalent in Africa, particularly in the Lake Chad basin, Mali, CapeVerde islands, and the Great Lakes of East Africa, specifically in Tanzania, Zambia, Malawi, and Mozambique, has been observed to inflict substantial harm to agricultural produce, leadingto significant concerns regarding food security (Chen et al., 2020). The last invasion of red locust was in between 1929-1944, which affected most of African countries (Topaz et al., 2012; Price, 2023).

Approximately 20% of the world's land surface is affected by locust invasions in arid and semiarid regions, impacting nearly 10% of the global population. This causes poor crop harvests, food insecurity, loss of pasture, and environmental destruction (Rai and Chavhan, 2015; Kimathi et al., 2020; Salih et al., 2020). In Tanzania, locust invasions are a recurrent problem with severe implications for food security (Ahmad et al., 2022). The desert locust, in particular, has caused significant agricultural losses in recent years, exacerbated by climate change and increasingly erratic weather patterns (Babar, 2023). These invasions disrupt the livelihoods of farmers and pastoralists, leading to food shortages, economic instability, and increased poverty levels (Thomson and Miers, 2002).

Understanding the dynamic nature of locust behavior, particularly the unpredictable patterns of their invasions, presents a formidable challenge in effectively mitigating their impact is of economical importance. The comprehension of these dynamics holds paramount importance in the development of strategies aimed at safeguarding crops and ensuring food security. The application of mathematical modeling serves as a potent instrument in the assessment and anticipation of locust behavior, against invasions. Traditional methods of locust control, including chemical pesticides, often thereby facilitating enhanced readiness and responsive measures struggle to keep pace with the rapid movements and large scale of infestations. Moreover, these methods can have detrimental environmental and health impacts, prompting the need for more sustainable and effective management strategies (Zhang, 2025). At low density, they have short wings solitarious, they remain in breeding areas while doing less harm to vegetation cover. As population increases, they become gregarious, smaller body size, long winged, change their colour and can leave their breeding areas and fly up to a height of 1800 m above the ground to greater distances (Brazdil et al., 2014). Wind movement influences the locust speed and direction. Locusts feed on all vegetation without selecting. Grasshoppers can swarm when they are able to fly but gregarisation phase can skip between generations either one or two generations (Rai and Chavhan, 2015).

To address this threat, the Tanzanian government, along with regional and international partners, implements various strategies to manage and control locust populations. These efforts include monitoring and surveillance of locust breeding grounds, especially after periods of rain, as well as using early warning systems to predict outbreaks and mobilize rapid response efforts. Control measures involve the use of pesticides and biological agents that target locusts without harming other wildlife. About USD 138,000,000/= were used in 2020 to control the desert locustand support affected people for food in the horn of Africa countries whereby in Tanzania about USD 505,000/= were required to control the locust and support the affected people (Mamo and Bedane, 2021). Despite the efforts made by the government and international organizations locust are still dangerous to agriculture sector, food security and environment. Different studies have been conducted on locust outbreak but none of them have developed a mathematical model of locust invasion. A mathematical model is the most powerful scientific model for studying locust population dynamics and its control. Therefore, a mathematical model for studying locust dynamical behavior and control of locust invasion is necessary. The model is used to study the behaviour of locusts so that the information obtained help to take important preventions and efficient control tactics before causing massive damage. This study intends to use mathematical model to study the locust invasion and understand its dynamical behavior.

To do this we first formulate a mathematical model of locust invasion, analyse the model for feasibility of the system, existence and stability of equilibrium states and explore locust invasion behavior in the system using numerical simulations.

2 Methods

2.1 Model description

The model is divided into two settings, the Source zone - region where the loust are comingfrom and the Invaded zone - region where the locust invasion has occurred. Locust from the Source zone are denoted by $L_s(t)$ while those from the Invaded zone are denoted by $L_i(t)$.

2.2 Population dynamics

Locust from the source zone grow at intrinsic growth rate σ_s depending on their population and environment's carrying capacity k_1. From the source zone locust are removed through control methods with effort E_s and control coefficient η_s. The removal rate is hindered by the environment and locust's dynamical and survival mechanism at the removal deterrent coefficient ρ_s. When the environmental and other factor are favourite, locust from the source move toinvade at the rate δ which is monitored by whether and to what extent the factors are favouriteor unfavourite at invasion deterrent coefficient λ. Those from invaded zone grow at intrinsic growth rate σ_i depending on their population and environment's carrying capacity k_2. They are removed through control methods with effort E_i and control coefficient η_i. The removal rate of locust from invaded zone is hindered by the environment and locust's dynamical and survival mechanism at the removal deterrent coefficient ρ_i. Locust from both zones die naturally at a rate α.

2.3 Variable and parameters and their description

Table 1 Parameters and their description for locust invasion model.

Parameters	Description
L_s	Biomass density of Locust from source zone
L_i	Biomass density of Locust from invaded zone
σ_s	Intrinsic growth rate of locust from source zone
σ_i	Intrinsic growth rate of locust from invaded zone
k_1	Carrying capacity of locust in source zone
k_2	Carrying capacity of locust in invaded zone
ρ_s	Removal deterrent coefficient in source zone
ρ_i	Removal deterrent coefficient in invaded zone
η_s	Control coefficient of the control method in source zone
η_i	Control coefficient of the control method in invaded zone
E_s	Locust removal effort applied by control method in source zone
E_i	Locust removal effort applied by control method in invaded zone
δ	Locust invasion rate (from source zone to invaded zone)
α	Locust natural death rate
λ	Invasion deterrent coefficient

2.4 Model assumption

- The locust invasion model rely on the following assumptions;
- Locust can grow independently in the two zones and their population sizes are bounded.
- Locust from the two zones have ecological interactions in their respective zones.
- The control efforts applied to the source and invaded zone are different and proportional to the level of the invasion.
- Locust recruitment depends on the carrying capacity of the respective zone.
- Locust in both zones die naturally at the same rate.
- All other removal factors are ignored except the natural and those aided by applied control strategies.
- Only adult locust with full invasive properties are considered in this model

2.5 Model equations

The dynamics of locust populations in Source zone and Invaded zone may be governed by the autonomous system of differential equations as represented by system (1)

$$\frac{dL_s(t)}{dt} = \sigma_s L_s(t)\left(1 - \frac{L_s(t)}{k_1}\right) - (1-\rho_s)\eta_s E_s L_s(t) - \left(\frac{\delta}{1+\lambda} + \alpha\right) L_s(t)$$

(1a)

$$\frac{dL_s(t)}{dt} = \sigma_i L_i(t)\left(1 - \frac{L_i(t)}{k_2}\right) + \left(\frac{\delta}{1+\lambda}\right) L_s(t) - (1-\rho_i)\eta_i E_i L_i(t) - \alpha L_i(t)$$

(1b)

ρ_s and ρ_i describe the rates at which the environment and locust's dynamical and survival mechanism reduce the efficient of the removal method in source and invaded zones respectively, are named as removal deterrent coefficients.

The parameters σ, k_1, ρ_s, η_s, δ, λ, α, ρ_i, η_i and η_i are assumed to be positive constants and $0 < \rho_s < 1$ and
$0 < \rho_i < 1$

Rearranging the system and using the method by Edelstein-Keshet (2001) we get;

$$\frac{dL_s(t)}{dt} = -L_s(t)\left(-\sigma_s + \sigma\frac{L_s(t)}{k_1} + (1-\rho_s)\eta_s E_s + \frac{\delta}{1+\lambda} + \alpha\right), \text{(2a)}$$

$$\frac{dL_i(t)}{dt} = -L_i(t)\left(-\sigma_i + \sigma_i\frac{L_i(t)}{k_2} + (1-\rho_i)\eta_i E_i + \alpha\right) + \left(\frac{\delta}{1+\lambda}\right) + L_s(t). \quad \text{(2b)}$$

Considering (2a) if $-\sigma_s + (1-\rho_s)\eta_s E_s + \frac{\delta}{1+\lambda} + \alpha > 0$ then $\frac{dL_s(t)}{dt} < 0$, similarly if we consider (2b) we can observe that if there is no locust invasion from the source $(\delta = 0)$ and

$-\sigma_i + (1-\rho_i)\eta_i E_i + \alpha + \left(\frac{\delta}{1+\lambda}\right) > 0$ then $\frac{dL_i(t)}{dt} < 0$.

Considering the condition above, for $\frac{dL_i(t)}{dt}$ and $\frac{dL_s(t)}{dt}$ to be positive we need $(1-\rho_s)\eta_s E_s + \frac{\delta}{1+\lambda} + \alpha < \sigma_s\delta$ 1+λ + α <σ_s and $(1-\rho_i)\eta_i E_i + \alpha < \sigma_i$ respectively. This means that the existence of locust population and their dynamics in the two zones is possible if the intrinsic growth rate of locust from source zone and that of the invasion zone is greater that the natural and/or human induced removal rate of locust from the source zone and invasion zone respectively. This condition holds throughout our analysis in this work.

If there is no invasion and no efforts invested to control the locust in the two zones; for $\frac{dL_i(t)}{dt}$ and $\frac{dL_s(t)}{dt}$ to be positive we need $\alpha < \sigma_s$ and $\alpha < \sigma_i$ respectively. This means that the existenceand the sustainable dynamics of locust population will be possible if natural removal of locustis less than the intrinsic growth rate of locust in the two source zone and invasion zone.

3 Existence of the Equilibria

To find the equilibrium points $E(L_S^*, L_i^*)$ of the model system (1), we set $\frac{dL_i(t)}{dt} = 0$ and $\frac{dL_s(t)}{dt} = 0$. From equation (1) we take

$$\sigma_s L_s(t)\left(1 - \frac{L_s(t)}{k_1}\right) - (1-\rho_s)\eta_s E_s L_s(t) - \left(\frac{\delta}{1+\lambda} + \alpha\right) L_s(t) = 0$$

(3a)

$$\sigma_i L_i(t)\left(1 - \frac{L_i(t)}{k_2}\right) + \left(\frac{\delta}{1+\lambda}\right) L_s(t) - (1-\rho_i)\eta_i E_i L_i(t) - \alpha L_i(t) = 0$$

(3b)

Such that (3b) and (3a) yields to;

$$-\sigma_s + \sigma_s \frac{L_S^*}{k_1} + (1-\rho_s)\eta_s E_s + \frac{\delta}{1+\lambda} + \alpha = 0,$$

$$-\sigma_i + \sigma_i \frac{L_i^*}{k_2} + (1-\rho_i)\eta_i E_i + \alpha = 0.$$

3.1 Absence of locust in the two zones

When we consider the assumption that there is no locust in the source zone and the invaded zone then the equilibrium point is as given in (4)

$$E_{(L_s=0, L_i=0)}(L_S^*, L_I^*) = (0,0). \tag{4}$$

3.2 Absence of locust in the source zone

When we consider the assumption that the locust are only in the invaded zone, then the stationary point in this scenario is as in (5)

$$E_{(0,L_i)}(L_S^*, L_I^*) = \left(0, \frac{K_2}{\sigma_i}\left(\sigma_i - (1-\rho_i)\eta_i E_i - \alpha\right)\right) \tag{5}$$

The stationary point $E_{(0,L_i)}$ is positive if $\sigma_i > \left((1-\rho_i)\eta_i E_i + \alpha\right)$

3.3 Absence of locust in the invaded zone

Here we consider the assumption that locust are available in the source zone only, the equilibrium point is as given in (6)

$$E_{(l_i,0)}(L_S^*, L_I^*) = \left(\frac{K_1}{\sigma_s}\left(\sigma_s - (1-\rho_s)\eta_s E_s - \alpha\right), 0\right) \tag{6}$$

The stationary point $E_{(L_s,0)}$is positive if$\sigma_s > \left((1-\rho_s)\eta_s E_s + \alpha\right)$.

3.4 Presence ot locust in the two zones

Here we consider the assumption that the locust are both in the source and invaded zones.

Solving system of equation (3) we obtain the the stationary points as given in (7)

$$E(L_S^*, L_i^*) = \left(\frac{k_i}{\sigma_s}(\sigma_s - \theta_1), k_2\left(1 + \frac{k_1\delta}{\sigma_i(1+\lambda)}\right) - \left(\frac{k_2}{\sigma_i}\theta_2 + \frac{k_1\delta}{\sigma_s(1+\lambda)}\right)\theta_3\right). \tag{7}$$

where

$\theta_1 = (1-\rho_s)\eta_s E_s + \alpha + \frac{\delta}{(1+\lambda)}$,

$\theta_2 = \left((1-\rho_i)\eta_i E_i + \alpha\right)$,

$\theta_3 = (1-\rho_s)\eta_s E_s + \alpha + \frac{\delta}{(1+\lambda)}$,

The stationary point E is positive if$\sigma_s > \theta_1$ and $k_2\left(1+\frac{k_1\delta}{\sigma_i(1+\lambda)}\right) > \left(\frac{k_2}{\sigma_i}\theta_2 + \frac{k_1\delta}{\sigma_s(1+\lambda)}\theta_3\right)$

4 Stability of Equilibria

In this section we check the stability of equilibrium points. we investigate the local and global stability of the equilibrium point E, assuming that the locust are present in source and invaded zone.

4.1 Local stability

We consider the variational matrix for the model system (1) around the equilibrium point (L_s^*, L_i^*) such that:

$$\begin{aligned} \dot{L}_s &\approx \left.\frac{\partial(\dot{L}_s)}{\partial L_s}\right|_{(L_s^*,L_i^*)} (L_s - L_s^*) + \left.\frac{\partial(\dot{L}_s)}{\partial L_i}\right|_{(L_s^*,L_i^*)} (L_s - L_s^*) \\ \dot{L}_i &\approx \left.\frac{\partial(\dot{L}_i)}{\partial L_s}\right|_{(L_s^*,L_i^*)} (L_s - L_s^*) + \left.\frac{\partial(\dot{L}_i)}{\partial L_i}\right|_{(L_s^*,L_i^*)} (L_i - L_i^*) \end{aligned} \tag{8}$$

by computing the partial derivatives of (1a) and (1b) using the notation in (8) it yields to:

$$\left.\begin{aligned} \left.\frac{\partial(\dot{L}_s)}{\partial L_s}\right|_{(L_s^*,L_i^*)} &= \sigma_s\left(1-\frac{2L_s^*}{k_1}\right) - (1-\rho_s)\eta_s E_s - \frac{\delta}{(1+\lambda)} - \alpha \\ \left.\frac{\partial(\dot{L}_s)}{\partial L_i}\right|_{(L_s^*,L_i^*)} &= 0 \\ \left.\frac{\partial(\dot{L}_i)}{\partial L_s}\right|_{(L_s^*,L_i^*)} &= \frac{\delta}{(1+\lambda)} \\ \left.\frac{\partial(\dot{L}_i)}{\partial L_i}\right|_{(L_s^*,L_i^*)} &= \sigma_i\left(1-\frac{2L_i^*}{k_2}\right) - (1-\rho_i)\eta_i E_i - \alpha \end{aligned}\right\} \tag{9}$$

Thus, the variational matrix $J(L_s^*, L_i^*)$ as computed in (9) of the locust invasion model is given by;

$$\boldsymbol{J(L_s^*, L_i^*)} = \begin{pmatrix} \sigma_s - \frac{2\sigma_s L_s^*}{k_1} - (1-\rho_s)\eta_s E_s - \frac{\delta}{(1+\lambda)} - \alpha & 0 \\ \frac{\delta}{(1+\lambda)} & \sigma_i - \frac{2\sigma_i L_i^*}{k_2} - (1-\rho_i)\eta_i E_i - \alpha \end{pmatrix} \tag{10}$$

We compute characteristics equation of a variational matrix in the form

$$\lambda_2 + P_1\lambda + P_2 \tag{11}$$

The coefficients P_1 and P_2 of the characteristics equation of variational matrix (10) is as given in (12) and (13) respectively

$$P_1 = 2\alpha + \sigma_i + \frac{2\sigma_i L_i}{k_2} + (1-\rho_i)\eta_i E_i + \frac{\delta}{(1+\lambda)} + (1-\rho_s)\eta_s E_s + \frac{2\sigma_s L_s}{K_1} - \sigma_s \tag{12}$$

$$P_2 = \frac{2\sigma_s L_s}{K_1}\left(\sigma_i + \frac{2\sigma_i L_i}{k_2} + (1-\rho_i)\eta_i E_i + \alpha\right) + (1-\rho_s)\sigma_i\eta_s E_s\left(1+\frac{2L_i}{K_1}\right)$$
$$+(1-\rho_s)\eta_s E_s\left((1-\rho_i)\eta_i E_i + \alpha\right) + \frac{\delta}{(1+\lambda)}\left(\sigma_i + \frac{2\sigma_i L_i}{k_2} + (1-\rho_i)\eta_i E_i + \alpha\right)$$

$$+\alpha\left(\sigma_i+\frac{2\sigma_i L_i}{k_2}+\alpha\right)+(1-\rho_i)\eta_i E_i\alpha-\left(\sigma_s\sigma_i\left(\frac{2L_i}{k_2}\right)\right)$$
$$+\sigma_s\big((1-\rho_i)\eta_i E_i+\alpha\big) \tag{13}$$

We apply the Routh-Hurwitz criteria by Mathebula (2012) to prove the the stability of equilibrium point $E(L_s^*, L_i^*)$. In our case, using Routh-Hurwitz criteria the $E(L_s^*, L_i^*)$ is locally asymptotically stable (LAS) if and only if $P_1 > 0$ and $P_2 > 0$. The Routh array is constructed as follows:

$$\begin{array}{c|cc} \lambda^2 & 1 & 0 \\ \lambda^1 & P_1 & 0 \\ \lambda^0 & P_0 & 0 \end{array} \tag{14}$$

Now analysing $P_1 > 0$ and $P_2 > 0$ in (12) and (13) we can see that, the equilibrium point $E(L_s^*, L_i^*)$ is LAS if and only if it satisfies the condition in (15) and (16).

$$\alpha-\sigma_s-\frac{2L_i}{k_2}(\alpha-\sigma_s)>0 \tag{15}$$

$$\frac{2L_s}{k_1}>1 \tag{16}$$

4.2 Global stability

Here we determine the conditions under which the equilibrium points E is globally stable. In this case we need to prove whether the solution starting sufficiently close to the equilibrium remains close to the equilibrium and approaches the equilibrium as $t \longrightarrow \infty$, or if there aresolutions starting arbitrary close to the equilibrium which do not approach it respectively.

Theorem 4.1 (Lyapunov's Stability Theorem)

Given a continuously differentiable function $V : \mathbb{R}^n \longrightarrow \mathbb{R}$ the equilibrium point $x = 0$ of thesystem $x^{\cdot} = f(x)$ is globally asymptotically stable if:

- $V(0) = 0$ an $V(x) > 0$ for all $x \neq 0$ and $\dot{V}(0) = \nabla V(x) \cdot f(x) \leq 0$ for all $x \in \mathbb{R}^n$.

where, $V(x) >$ serves as a Lyapunov function, ensuring positive definiteness around $x = 0.\dot{V}(x)$ represents the derivative of $V(x)$ along the trajectories of $\dot{x} = f(x)$, determining the stability properties of the equilibrium.

As applied by past research (Drazin, 1992; Chen, 2004; Karafyllis and Jiang, 2011), choose the Lyapunov function $\mathrm{V}(L_s, L_i)$ as:

$$\mathrm{V}(L_s, L_i)=\frac{1}{2}\left(\frac{L_s^2}{k_1}+\frac{L_i^2}{k_2}\right) \tag{17}$$

where k_1 and k_2 are positive constants.

Compute the time derivative of $\mathrm{V}(L_s, L_i)$ in equation (17) such that

$$\dot{\mathrm{V}}(L_s, L_i)=L_s\frac{dL_s}{dt}+L_i\frac{dL_i}{dt}, \tag{18}$$

by substituting (1) and (2) in (18) we have:

$$\dot{V}(L_s, L_i) = L_s\left[\sigma_s L_s\left(1 - \frac{L_s}{k_1}\right) - (1-\rho_s)\eta_s E_s L_s - \left(\frac{\delta}{1+\lambda} + \alpha\right) L_s\right]$$
$$+L_i\left[\sigma_i L_i\left(1 - \frac{L_i}{k_2}\right) + \left(\frac{\delta}{1+\lambda}\right) L_s - (1-\rho_i)\eta_i E_i L_i - \alpha L_i\right] \quad (19)$$

After simplification (19), we obtain:

$$\dot{V}(L_s, L_i) = -(1-\rho_s)\eta_s E_s L_s^2 - (1-\rho_i)\eta_i E_i L_i^2 \quad (20)$$

Since $\eta_s, \eta_i, E_s, E_i > 0$ and $\rho_s, \rho_i \in [0, 1]$in (20), we have:

$(1-\rho_s) \geq 0$ and $(1-\rho_i) \geq 0$ and $\eta_s, \eta_i, E_s, E_i > 0$.

Therefore, $\dot{V}(L_s, L_i) \leq 0$ for all $L_s, L_i \geq 0$.

• **Positive Definiteness**: $V(L_s, L_i)$ is positive definite for $L_s, L_i > 0$ and $V(0, 0) = 0$.

• **Decreasement Condition**: $\dot{V}(L_s, L_i) \leq 0$ with equality $\dot{V}(L_s, L_i) = 0$ if and only if $L_s = 0$ and $L_i = 0$.

Therefore, by the Lyapunov theorem, the equilibrium point $(L_s, L_i) = (0, 0)$ of the system described by equations (1a) and (1b) is globally asymptotically stable. This implies that, under the given conditions and parameter values, the locust population dynamics will converge tozero in the absence of external perturbations, ensuring the control and stabilization of the locust populations in both the source and invaded zones.

5 Numerical Analysis of the Model

In this section, we illustrate the invasion behavior of the locust through numerical simulation. The chosen parameter values align with those documented in relevant literature, and in cases where specific parameters are not explicitly delineated; estimations are derived from secondary statistical data, as delineated in the accompanying Table 2.

Table 2 A table showing the invasion of locust swarms in Tanzania (for Eastern Africa, 2022).

S/N	Year	Estimated Swarms Controlled
1	2009	650
2	2010	700
3	2011	650
4	2012	650
5	2013	350
6	2014	-
7	2015	-
8	2016	-
9	2017	-
10	2018	-
11	2019	-
12	2020	700
13	2021	820

Table 2 shows estimates of locust swarms for thirteen years from 2009 to 2021. The records show that locust invasion in the years 2009 to 2013, 2020 and 2021 was an invasion of locustswarms whereby from 2014 to 2019 there was no invasion of locust swarms in Tanzania. Ingeneral, the information implies that locusts are dangerous and Tanzania is a vulnerable country.

5.1 Parameter values

5.1.1 Model fittings and parameter estimation

After conducting model analysis of the dynamics and qualitative outcomes of the locust model, it becomes essential to accurately determine the model's parameters for making quantitative predictions within a limited time frame using real-world data (Charles et al., 2024). In this study, we employed the non-linear least squares method (NLSM) to estimate the parameters of model equation (1a) and (1b). To achieve this, we generated synthetic data that represented the expected locust invation patterns at various time points, denoted as t_i (Capaldi et al., 2012; Charles et al., 2024). These patterns were computed by numerically solving equation (1a) and (1b) with a fifth-order Runge-Kutta method in the MATLAB environment, initializing the parameters with value from literature denoted as Θ_i and inintial condition for the number of $L_s(0) = 700$, $L_i(0) = 500$. In order to generate the locust dataset $RD(t_i\ \Theta_i)$ we added random Gaussian noise $\eta_i(t_i\ \Theta_i)$ measurements to the data simulate real-world dynamics where measurement errors are common. Thus the observed dependent data were given as;

$$Y_i = RD(t_i\ \Theta_i) + \eta_i(t_i\ \Theta_i) \text{ for each time } t_i \in [1, n] \qquad (21)$$

The parameter values $\overline{YY}(Table.3)$ were determined by minimizing the sum of squared residuals expressed as;

$$\overline{YY}(\Theta) = min \sum_{k=1}^{n} (Y_i - Y)^2 \qquad (22)$$

between the model solutions (Y) obtained through solving the locust 1b model using the real parameters from the generated data and the synthetic data Y_i generated by introducing random Gaussian noise to the model output $RD\ (t_i\ \Theta_i)$ (Herrera et al., 2022). Estimated parametervalues were then used to fit the data Y_i, and the resulting best fits were depicted in Figs. 1 (a)-(b) Table 3 presents the values of parameters used in the simulation

Fig. 2 depicts the normal distribution of ecological parameters, such as growth rates $(\sigma_s \sigma_i)$, carrying capacities (k_1, k_2), and removal deterrent rate (ρ_s, ρ_i, δ, α), signifies natural variability within ecosystems. This variability, driven by genetic diversity, environmental factors, and stochastic events, shapes population dynamics and species interactions. Parameters like resource conversion efficiencies (η_s, η_i), environmental carrying capacities (E_s, E_i), and invasion deterrent coefficient (λ) further reflect how species respond to varying resource availability and environmental conditions. Understanding these distributions provides insights into species resilience, community structure, and ecosystem stability, crucial for effective conservation and management strategies in dynamic ecological systems.

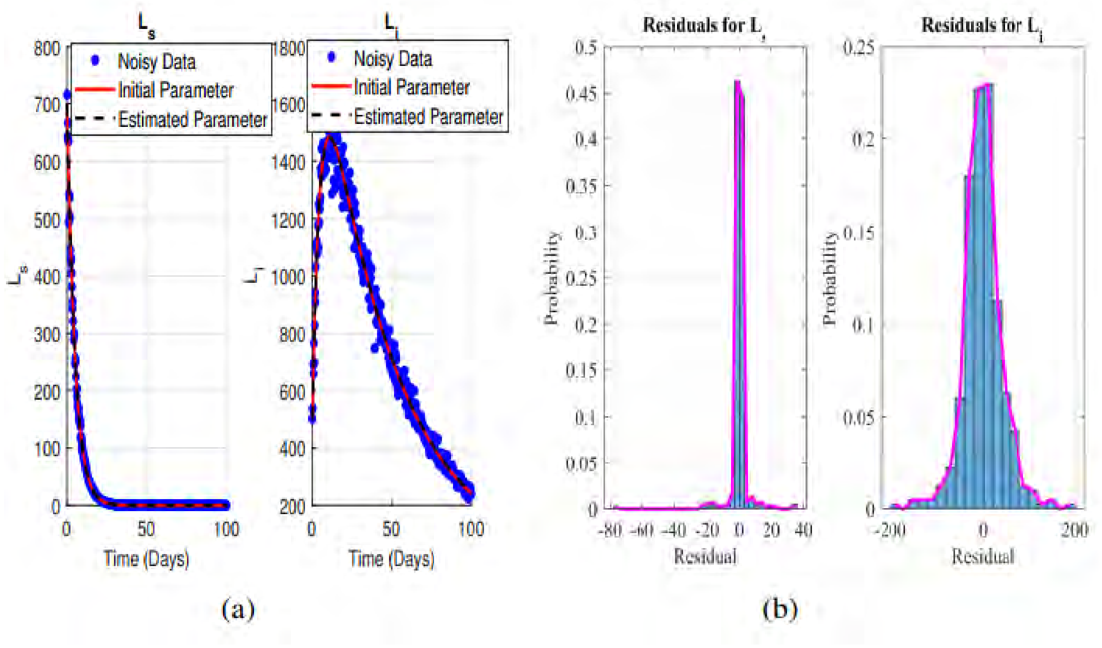


Fig. 1 Scatter estimated with standard deviation of 0.05 and numerical simulation (sold) with confidence interval of 95%.

5.1.2 Evaluation of error metrics and model performance

In order to quantifies the average absolute difference between observed and estimated values, measures the average relative deviation between observed and estimated values. The Mean Absolute Error (MAE) and Mean Relative Error (MRE) metrics were used to evaluate the accuracy of estimated values $\hat{y}_i$ compared to observed values y_i in the context of estimating L_s and L_i (population sizes) using the formula below

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i| \text{ and } MRE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right|. \quad (25)$$

where;

y_i: Observed values of L_s or L_i, $\hat{y}$: Estimated values of L_s or L_i, N : Total number of data points.

Using the advantage of MATLAB along with equations (1a), (1b) and (23) the results were dipicted in Table 4 and Fig. 3. Table 4 shows that the MAE for $L_s$is 1.599081, indicating that, on average, the predictions for L_s differ from the observed noisy data by 1.599 units. In contrast, the MAE for L_i is significantly higher at 28.317970, suggesting that the predictions for $L_i$are, on average, about 28.318 units away from the observed values.

When examining the MRE, we find that for L_s, the MRE is 0.209269, meaning that the absolute error constitutes approximately 20.93% of the observed values for L_s. For L_i, the MRE is substantially lower at 0.038030, indicating that the absolute error is only about 3.80% of the observed values.

These metrics suggest that while the model has higher precision in absolute terms for L_s due toits lower MAE, the predictions for L_i are more accurate in relative terms, as indicated by the lower MRE. Therefore, the model demonstrates better relative accuracy for L_i compared to L_s (see Fig. 3).

Table 3: Baseline Parameters values and their Estimated for locust invasion model.

Parameter	Baseline Value	Source	Estimated value	$\mathcal{N}$ (Mean μ, Stardard deviation σ)
σ_s	0.7	Estimated	0.699114	0.699557, 6.26×10^{-4}
σ_i	0.7	Estimated	0.700203	0.700101, 1.44×10^{-4}
k_1	3000000	Estimated	2909183.860000	2954591.930000, 6.4216708×10^{4}
k_2	3000000	Estimated	4713251.980000	3856625.990, 1.211452×10^{6}
ρ_s	0.5	Estimated	0.459371	0.479685, 2.8729×10^{-2}
ρ_s	0.5	Estimated	0.499051	0.499526, 6.71×10^{-4}
η_s	0.267	Estimated	0.270013	0.2685065, 2.131×10^{-3}
η_i	0.67	Estimated	0.671928	0.670964, 1.1363×10^{-3}
E_s	0.5	Estimated	0.472896	0.486448, 1.9165×10^{-2}
E_i	0.9	Estimated	0.896766	0.898383, 2.2287×10^{-3}
δ	0.75	Estimated	0.738174	0.744087, 8.362×10^{-3}
α	0.42	Estimated	0.419836	0.434918, 2.1329×10^{-2}
λ	0.6	Estimated	0.784268	0.79214, 1.1124×10^{-2}

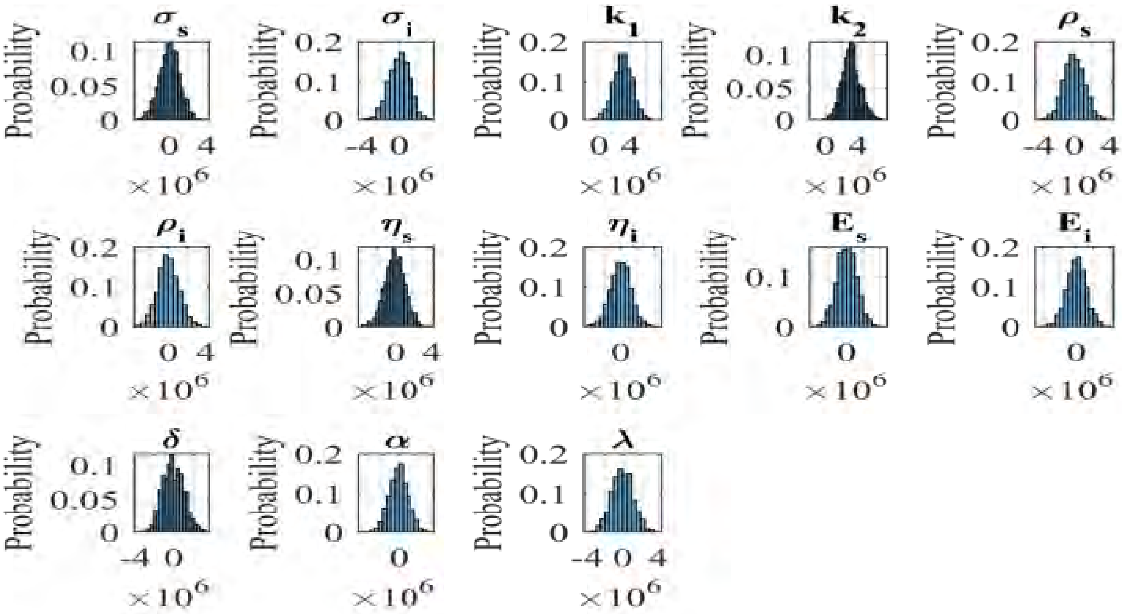


Fig. 2 Normal distribution of parameters.

Table 4: Error Metrics for Evaluation of Model Performance

Error Metric	**Value for L_s**	**Value for L_i**
Mean Absolute Error (MAE)	1.599081	28.317970
Mean Relative Error (MRE) (%)	20.93%	3.80%

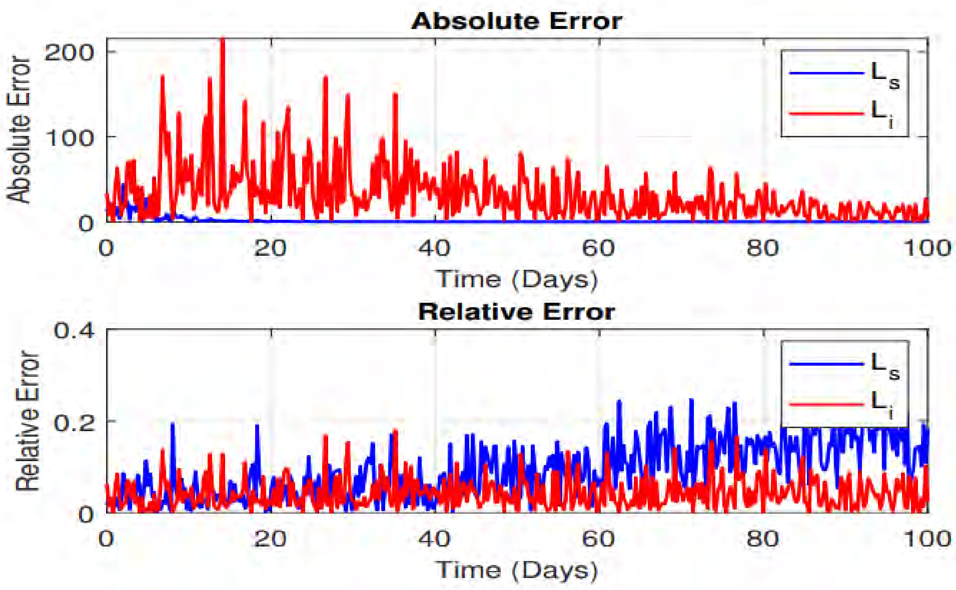
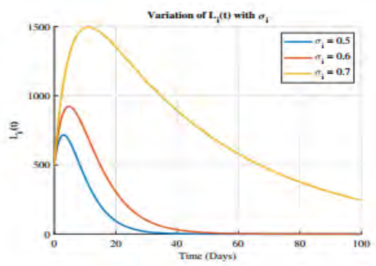


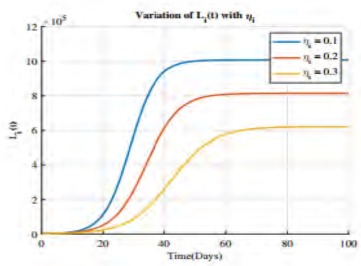
Fig. 3 Model's performance, Mean Absolute Error (MAE) amd the Mean Relative Error (MRE) for two variables, L_s and L_i.

5.2 Numerical results

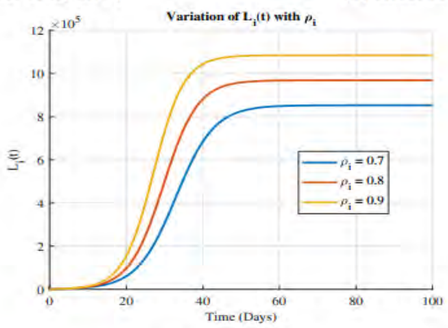
In Fig. (4) (a), it is observed that the intrinsic growth rate σ_i of locusts from the invaded zone L_i initially increases with the rise in L_i for the first 20 days, and subsequently decreases after 21 days. A 70% increase in σ_i results in 1500 units of L_i, while a 60% increase yields 800 unitsof L_i, and a 50% of σ_i increase leads to 700 units of L_i. Furthermore, Fig. (4)(b) Indicates that an increase in the removal deterrent coefficient in the invaded zone η_i leads toa decrease in L_i, suggesting that η_i is an effective control measure for L_i. Finally, Fig. 4 c illustrates that an increase in ρ_i results in an increase in L_i.



(a) effect of L_i on σ_i



(b) effect of L_i on η_i



(c) effect of L_i on ρ_i

Fig. 4 Effect of parameters on Locust invaded *Li*.

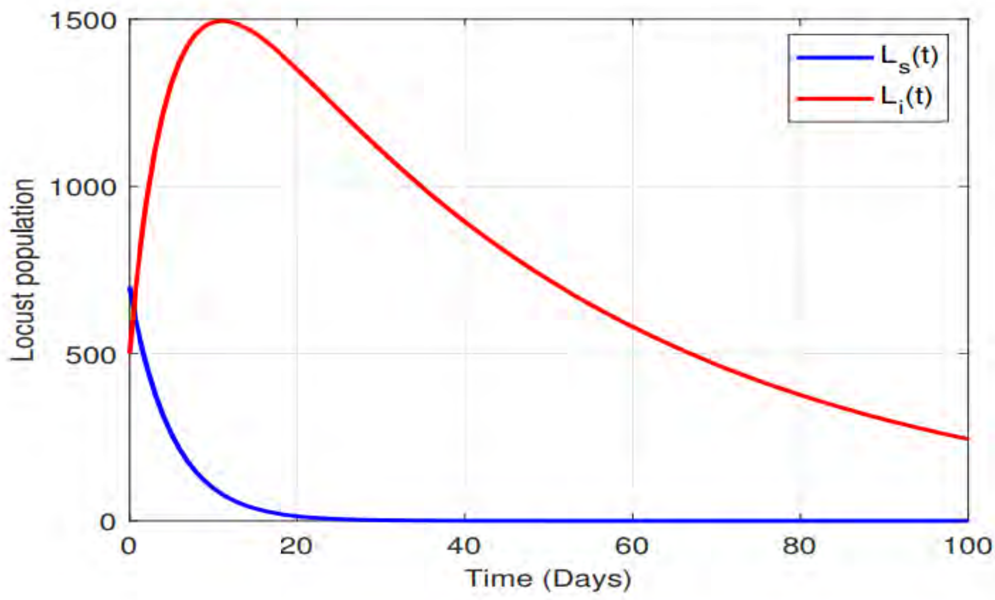


Fig. 5 Dynamics of locust in Source and Invasion zones with baseline parameters.

When there is effort applied to control the locust population increase in the population tend to decrease at high rate as the large biomass of locust will be removed through the control mechanism applied. Fig. 5 shows the rapid decrease of the locust population both from theinvasion zone and source zone. It can also be seen that the invasion period is short as the time locust population reach and stay at its highest level is very short before it decreases to the level where they are not chancy. Efforts applied to control the locust population in the source zones reduce the possibility of the invasion to occur. When there is an active control strategies inboth zones the locust invasion possibility become even thinner as the locust population will be maintained at a level that won't be ample for invasion. The importance of control is vivid as depicted in Fig. 5, but when no control is applied to both zones (Source and invasion zone) the condition become uncontrollable and the possibility of occurrence of invasion becomes high to the two zones. Fig. 6 demonstrates that the locust removal effort (E_i) causes a reduction in L_i: with a 30% (0.3) increase in E_i, a decrease of 780000 units of L_i ensues, and with a 60% (0.6) increase of E_i, a decline in L_i occurs; a 90% (0.9) increase of E_i eliminates L_i altogether.

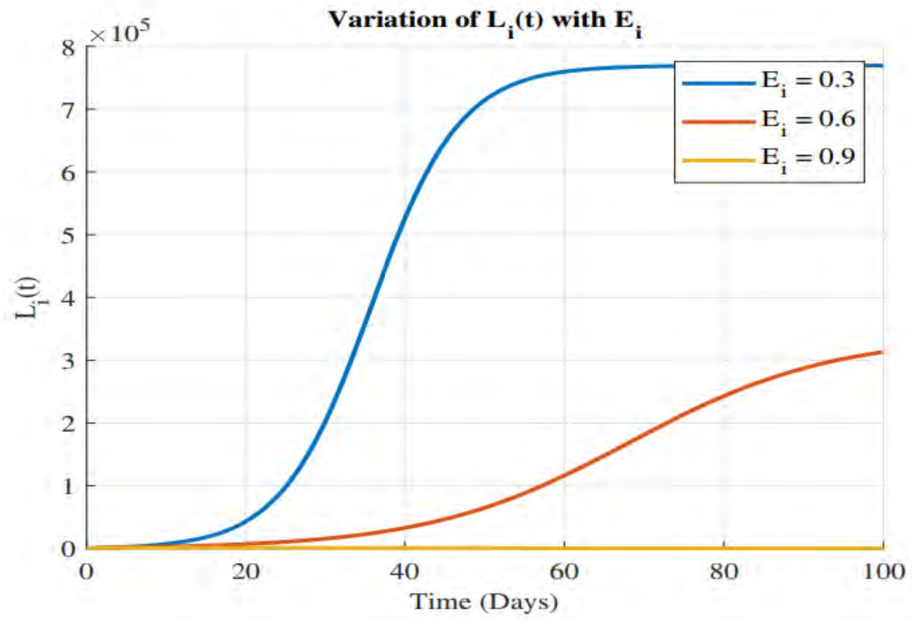


Fig. 6 Effect of variation of level of control in invasion zone.

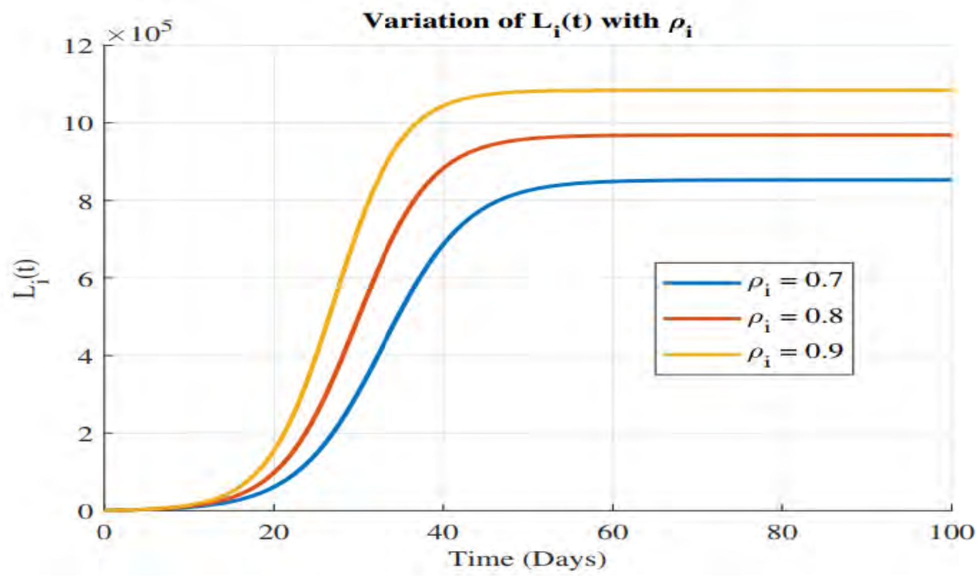
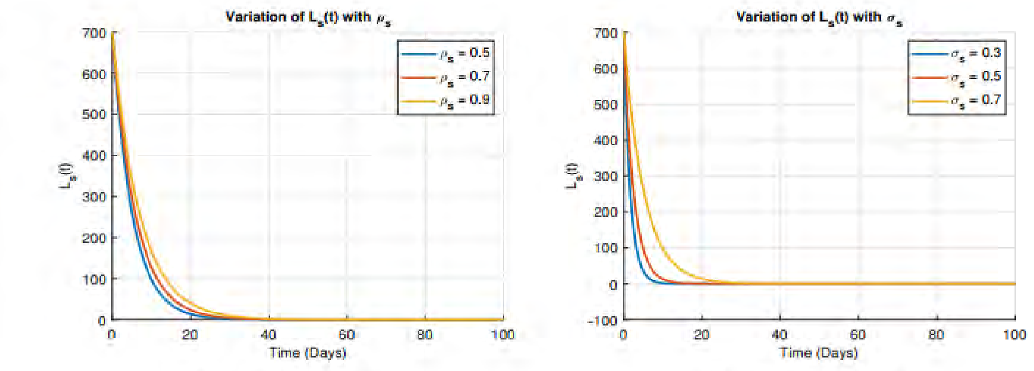


Fig. 7 Locust population of dynamics without control efforts.

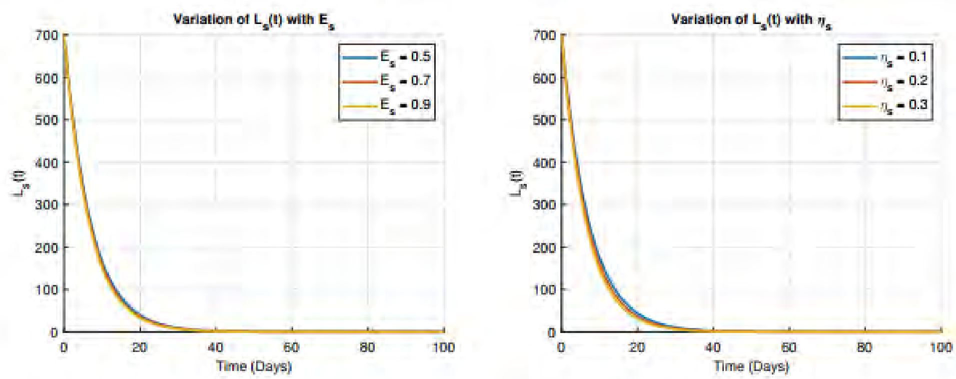
The results depicted in Fig. 7 highlight the fact that efforts invested to control the locust invasion dictate the time the locust spend in the invaded area and as a result the effect caused bythe locust invasion in the particular area. The choice of the control method is also an important factor to consider when planning for control of locust, this is due to the fact that the control mechanisms varies in the control efficiency of locust and the cost. Therefore for a fruitful control results, one should think of the optimal control of locust both in the source zone and the invasion zone. The next section thoroughly discuses the optimal control of locust that will suggest the kind of control mechanism and the related costs for optimal control results. In Fig. 8 (a), it is observed that the intrinsic growth rate ρ_s of locusts from the source zone L_s initially decrease with a minimal increase in magnitude of L_s for the first 20 days, and subsequently decline to zero after 21 days. Furthermore,

Fig. 8 (b) indicates that an increase in the removal deterrent coefficient in the locust source zone σ_s leads to a decrease in L_s, suggesting that η_i is an effective control measure for L_i. Finally, Fig. 9 (c) illustrates that an increase in ρ_s results in an increase in L_s.



(a) effect of L_s on ρ_s **(b)** effect of L_s on σ_s

Fig. 8 Effect of parameters on Locust source L_s.



(a) effect of L_s on E_s **(b)** effect of L_s on η_s

Fig. 9 Effect of parameters on Locust source L_s.

6 Conclusions

The application of Mean Absolute Error (MAE) and Mean Relative Error (MRE) metrics has illustrated that while the model provides reasonably accurate predictions for the population size L_s with a low MAE, it demonstrates superior relative accuracy for L_i as indicated by a lower MRE. These metrics underscore the model's effectiveness in quantifying predictionerrors and highlight its variable-specific performance characteristics. Factors such as growth rates and removal coefficients have a significant influence on population sizes (Zhang et al., 2020), emphasizing the need for early intervention and adaptive management. Effective control measures offer hope for reducing locust outbreaks and ensuring food security. Coordinated efforts are crucial for reducing population sizes and preventing widespread invasions, highlighting the necessity of strategic interventions to improve resilience against locust threats in the future.

Data availability

The data used in this study is available from the corresponding author upon request.

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