Article

Modelling and analysis of the effects of deforestation caused by various developments on the growth of wildlife species

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Abstract
Deforestation, driven by rapid urbanization, industrialization, and other developmental activities, poses a significant threat to wildlife by disrupting natural habitats and forcing various wildlife species to migrate in search of more suitable and safer environments. This uncontrolled and often irregular migration frequently results in serious ecological imbalances, rising instances of human-wildlife conflicts, and the potential endangerment or extinction of vulnerable species. To gain a deeper understanding of the complex relationship between deforestation and emerging wildlife migration patterns, this paper presents a detailed nonlinear mathematical model that carefully examines the consequences of habitat degradation on the movement of wildlife across increasingly fragmented ecosystems. The model includes three key and interdependent variables: the cumulative density of forestry resources, the extent and intensity of developmental activity, and the population density of impacted wildlife species. It is constructed using a set of nonlinear ordinary differential equations and analyzed through stability theory to determine both equilibrium conditions and long-term dynamics. The results clearly demonstrate that as deforestation increases due to intensified developmental activities, both forest biomass and wildlife populations undergo a marked decline, resulting in elevated migration rates. Numerical simulations support these conclusions, underlining the critical need for sustainable development practices to reduce habitat destruction and preserve ecological balance.

Keywords mathematical modeling; deforestation; developments; wildlife species.

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1 Introduction

Deforestation is one of the most pressing environmental challenges in the contemporary era, primarily driven by human-induced activities such as agricultural expansion, urbanization, and infrastructure development. Forest ecosystems, covering approximately 31% of the Earth’s land area, are critical for maintaining biodiversity, regulating climate, and providing essential ecosystem services (FAO, 2020). However, the continuous loss and degradation of these ecosystems have led to significant disruptions in ecological systems, especially affecting wildlife species that depend on forest habitats for survival and reproduction (Haddad et al., 2015). The rapid rate of deforestation and its far-reaching implications necessitate an urgent need for comprehensive analysis and strategic intervention.

Forests are home to the majority of terrestrial biodiversity, supporting complex ecological interactions essential for the survival of numerous species. The destruction of these habitats results in severe consequences, including habitat loss, resource depletion, and population decline among wildlife (Newbold et al., 2016). As global demands for land and natural resources escalate, these impacts intensify, exacerbating the vulnerability of wildlife species. The consequences are particularly pronounced in biodiversity hotspots, where human activities often overlap with regions of high ecological significance. Understanding these dynamics is crucial for addressing the challenges posed by deforestation and ensuring the sustainable coexistence of human development and biodiversity conservation.

The underlying causes of deforestation are diverse and interconnected. Agricultural expansion, including crop cultivation and livestock grazing, is the leading driver of forest loss globally, accounting for approximately 77% of deforestation (Curtis et al., 2018). Infrastructure projects, such as road construction and mining, contribute to habitat fragmentation, while urban expansion transforms forested areas into residential and industrial zones (Seto et al., 2012). Additionally, logging activities, both legal and illegal, and the introduction of invasive species further exacerbate forest degradation. The interplay of these factors underscores the complexity of deforestation and its multifaceted impacts on ecosystems. Food and Agriculture Organisation provides an in-depth analysis of global deforestation trends, highlighting the primary drivers of forest loss and their impact on biodiversity (FAO, 2024). It emphasizes the need for urgent conservation strategies to mitigate habitat destruction and promote sustainable land use.

The effects of deforestation on wildlife are profound and multifaceted. Habitat loss reduces the availability of critical resources such as food and shelter, leading to population declines and, in extreme cases, species extinction (Fahrig, 2017). Fragmentation of habitats isolates wildlife populations, hindering migration and genetic exchange, which are vital for species adaptability and survival. Edge effects altered environmental conditions at the boundaries of fragmented habitats further degrade the quality of remaining habitats, making them unsuitable for many forest-dependent species (Haddad et al., 2015). The compounded effects of habitat loss, fragmentation, and climate change create a challenging environment for wildlife, necessitating targeted efforts to mitigate these impacts. Similarly, the International Fund for Animal Welfare explores the consequences of deforestation on wildlife, detailing how habitat loss forces species into smaller, fragmented areas, thereby increasing the risk of extinction (IFAW, 2024). The report underscores the importance of policy interventions and conservation efforts to protect forest-dependent species. The World Economic Forum (2024) highlights findings from a United Nations (UN) report indicating that one in five migratory species is at risk of extinction due to habitat loss, climate change, and human activities. The report underscores the urgent need for conservation efforts and policy interventions to protect migratory wildlife.

Modeling and analysis serve as essential tools for understanding the intricate relationships between deforestation and wildlife dynamics. By integrating ecological, geographical, and socioeconomic data, models can simulate various scenarios, predict long-term impacts, and identify areas at high risk of biodiversity loss (Wilson et al., 2006). Spatial models, for instance, help map critical habitats and corridors, facilitating conservation planning. Population models, on the other hand, provide insights into the demographic responses of species to habitat changes over time (Elith and Leathwick, 2009).

Recent studies have utilized mathematical modeling to examine deforestation and the depletion of forestry resources caused by various factors, as well as their detrimental effects on wildlife species (Naresh et al., 2006; Shukla et al., 2013; Sundar et al., 2017; Lata et al., 2018; Qureshi and Yusuf, 2019; Suryani et al. 2021; Kusum et al., 2023). For instance, Qureshi and Yusuf (2019) proposed a four-dimensional nonlinear mathematical model using fractional-order ordinary differential equations to study deforestation's effects on wildlife. They provided theoretical insights into the existence and uniqueness of solutions within this framework, enhancing the understanding of deforestation dynamics. Similarly, Suryani et al. (2021) presented a mathematical model incorporating Holling type-II and type-III functional responses to examine deforestation's impact on wildlife species. Their model offers a nuanced perspective on species interactions within degrading forest ecosystems. Additionally, Kusum et al. (2023) developed a nonlinear model analyzing how human population growth and associated pressures contribute to deforestation and subsequently affect both forest biomass and wildlife populations. Their findings indicate that increasing human demands lead to significant threats to forest habitats and the species that depend on them. These studies contribute to a deeper understanding of how deforestation driven by development activities affects wildlife populations, providing valuable insights for conservation efforts.

The significance of studying the effects of deforestation on wildlife lies in its implications for both ecological and societal well-being. Forests are integral to global biodiversity and provide ecosystem services that support human livelihoods, including carbon sequestration, water regulation, and soil fertility. The decline of wildlife populations and the degradation of forest ecosystems disrupt these services, posing risks to food security, climate stability, and public health. By understanding the mechanisms through which deforestation impacts wildlife, researchers and policymakers can design effective strategies to balance development needs with conservation priorities. This research contributes to the growing body of knowledge aimed at mitigating deforestation’s impacts and promoting sustainable development practices.

Despite extensive research on deforestation, critical knowledge gaps remain. While much attention has been given to habitat loss, there is limited understanding of the synergistic effects of deforestation and other environmental stressors, such as climate change, on wildlife populations. Additionally, the impacts of deforestation on ecological networks and the services they provide are not fully understood. Addressing these gaps requires interdisciplinary approaches that combine ecological, technological, and socioeconomic perspectives (Dirzo et al., 2014). However, further research is needed to refine these tools and expand their application across diverse ecological contexts.

This study aims to investigate the effects of deforestation caused by various developmental activities on the growth and survival of wildlife species. The research objectives include quantifying the extent of deforestation and habitat fragmentation, analyzing the impacts of habitat loss on wildlife populations, developing predictive models to simulate future scenarios, and proposing strategies for mitigating these impacts. By leveraging recent data and advanced modeling techniques, this study seeks to provide actionable insights that inform conservation policies and support sustainable development goals.

2 Mathematical Model

To analyze the impact of deforestation on wildlife migration and population dynamics, we develop a nonlinear mathematical model using a system of ordinary differential equations. The model incorporates three key variables:

(i) The biomass density of forest resources (B)

(ii) The level of various developmental activities (D)

(iii) The population density of wildlife species (W)

The interactions among these variables are governed by ecological and anthropogenic factors, where deforestation, driven by increasing developmental activities, leads to habitat loss and fragmentation, directly influencing wildlife migration and survival. The model accounts for the growth and depletion of forest biomass, the expansion of developmental activities, and the adaptive responses of wildlife populations to environmental changes. Stability analysis and numerical simulations are conducted to examine the system's long-term behavior under different scenarios of development intensity and conservation efforts, providing insights into sustainable strategies for mitigating deforestation's adverse effects on wildlife.

The first equation, governing the biomass density of forest resources (B), captures the dynamic interplay between natural growth, depletion due to human activities, and interactions with wildlife. The first term $sB\left(1-\frac{B}{L}\right)$ represents the logistic growth of forest biomass, where s being the intrinsic growth rate, and L is the carrying capacity of the forest. The second term $s_0 B$ accounts for the natural depletion of forest biomass over time. The third term $s_1 DB$ models the reduction of forest resources due to developmental activities such as urbanization, agriculture, and infrastructure expansion, with s_1 representing the rate of deforestation per unit of development. Lastly, the interaction term kWB represents the consumption or disturbance of biomass by wildlife species. This equation provides a foundation for understanding how forest biomass is influenced by both human-induced deforestation and ecological interactions, forming a crucial component of the overall model.

The second equation, governing the level of various developments (D), describes how human-induced activities such as urbanization, agriculture, and infrastructure expansion evolve over time. The first term $\lambda(L-B)$ represents the rate at which development increases, where λ being the growth rate of development and $L-B$ indicates that development is driven by the availability of land, which is inversely related to the remaining forest biomass (B). As forest resources decline, more land becomes available for development. The second term $\lambda_0 D$ accounts for the possible decline or stagnation of development due to economic, environmental, or regulatory constraints, where λ_0 represents the rate at which development slows down or ceases.

The third equation, governing the population density of wildlife species (W), captures the dynamics of wildlife growth and decline in response to forest biomass and developmental activities. The term πkBW represents the positive impact of forest biomass on wildlife population growth, where πk is a proportionality constant indicating how effectively the available biomass supports wildlife. The second term $rW\left(1-\frac{W}{K}\right)$ models the natural logistic growth of wildlife, where r being the intrinsic growth rate and K is the carrying capacity of the habitat. The third term r_0W accounts for natural mortality or external factors leading to a reduction in wildlife density. The last term r_1WD represents the negative effect of development on wildlife, where r_1 is the rate at which developmental activities, such as deforestation and habitat fragmentation, contribute to wildlife population decline. This equation emphasizes the balance between habitat availability and external pressures that influence wildlife sustainability.

Thus, the mathematical model describing the impact of deforestation due to developmental activities on wildlife migration consists of a system of nonlinear differential equations that govern the dynamics of forest biomass, development level, and wildlife population. The complete model is given as follows:

$$\frac{dB}{dt} = sB\left(1-\frac{B}{L}\right) - s_0B - s_1DB - kWB \tag{1}$$

$$\frac{dD}{dt} = \lambda(L-B) - \lambda_0 D \tag{2}$$

$$\frac{dW}{dt} = \pi kBW + rW\left(1-\frac{W}{K}\right) - r_0W - r_1WD \tag{3}$$

where $B(0) \geq 0,\ D(0) \geq 0,\ W(0) \geq 0$

These equations describe how forest biomass changes due to natural growth, deforestation, and interactions with wildlife; how development expands at the expense of forest resources; and how wildlife populations evolve based on habitat availability and anthropogenic pressures.

Remark 1

From Equation (1), the intrinsic growth rate of forestry resources at any time $t > 0$ is given by $s - s_0 - s_1D - kW$. This expression represents the net growth of forest biomass, considering natural growth s, natural decay s_1, deforestation due to development activities s_1D, and depletion caused by wildlife kW. For the long-term sustainability of forestry resources, this net growth rate must remain positive for all time. If this condition is not met, the forest biomass may decline to critically low levels or even become extinct, which would have severe ecological consequences, including habitat loss for wildlife. Therefore, to ensure the feasibility of the model system, we impose the constraint:

$$s - s_0 - s_1D - kW > 0,\ \forall t > 0$$

Remark 2

Similarly, from Equation (3), the intrinsic growth rate of wildlife species at any time $t > 0$ is given $\pi kB + r - r_0 - r_1D$. This term accounts for the positive effects of forest biomass availability and the natural reproduction of wildlife, while also considering the negative impacts of natural mortality and habitat loss due to development. For wildlife populations to persist and avoid extinction, this net growth rate must always remain positive. If this condition is violated, wildlife species may face population decline, leading to potential local extinctions. Thus, for the feasibility of the model system, we impose the condition:

$$\pi kB + r - r_0 - r_1 D > 0 , \quad \forall t > 0$$

Remark 3

Since wildlife species depend on forestry biomass, the impact of deforestation on them may continue worsening even after deforestation slows, making r_1 more significant over time. Therefore, in the long term r_1 may approach or exceed s_1 due to cumulative habitat loss, fragmentation, and ecological disruptions that persist even after deforestation slows. Thus, in the long term, it is reasonable to write:

$$r_1 \geq s_1$$

These constraints ensure that the forestry resources and wildlife populations remain viable over time, maintaining ecological balance in the system.

Boundedness of the system

Boundedness is necessary to analyze the model system. For this, we require the following lemma, which ensures boundedness of the model system.

Lemma

The set $\Omega = \left\{ (B, D, W) \in R^3 : 0 \leq B \leq L, 0 \leq D \leq \frac{\lambda L}{\lambda_0}, 0 \leq \frac{K}{r}(r + \pi kL) \right\}$ is the region of attraction for the model system (1) – (3) and attracts all solutions initiating in the interior of the positive orthant.

3 Equilibrium Analysis

The model system (1) – (3) has the following equilibrium points

(i) $E_1\left(0, \frac{\lambda L}{\lambda_0}, 0\right)$

(ii) $E_2\left(0, \frac{\lambda L}{\lambda_0}, \frac{K}{r}\left(r - r_0 - \frac{r_1 \lambda L}{\lambda_0}\right)\right)$

(iii) $E_3(B_3, D_3, 0)$

where $D_3 = \frac{\lambda s_0 L}{\lambda_0 s - L\lambda s_1}$ and $B_3 = L - \frac{\lambda_0 D_3}{\lambda}$ with $\lambda_0 s - L\lambda s_1 > 0$

(iv) $E^*(B^*, D^*, W^*)$

The existence of E_1, E_2 and E_3 is obvious and hence omitted. In the following, we show the existence of $E^*(B^*, D^*, W^*)$.

The solution of the equilibrium is given by the following algebraic equations

$$s\left(1-\frac{B}{L}\right)-s_0-s_1D-kW=0 \tag{4}$$

$$\lambda(L-B)-\lambda_0D=0 \tag{5}$$

$$\pi kB+r\left(1-\frac{W}{K}\right)-r_0-r_1D=0 \tag{6}$$

From equations (4) – (6), we have

$$F(B)=s\left(1-\frac{B}{L}\right)-s_0-\frac{s_1\lambda}{\lambda_0}(L-B)-\frac{kK}{r}\left(\pi kB+r-r_0-\frac{r_1\lambda}{\lambda_0}(L-B)\right)=0 \tag{7}$$

From (7), it can easily be checked that,

(i) $F(0)>0$

(ii) $F(L)<0$

(iii) $F'(B)<0$

This implies that $F(B)=0$ has a unique positive root $B=B^*$ in $0\le B\le L$. Using the value of $B=B^*$, the values of $D=D^*$ and $W=W^*$ can be determined from equations (4) and (5) respectively.

4 Stability Analysis

To analyze the stability of an equilibrium point in the model, we first consider local stability by examining the Jacobian matrix evaluated at the equilibrium. If all eigenvalues of the Jacobian have negative real parts, the equilibrium is locally asymptotically stable, indicating that small perturbations around the equilibrium will decay over time. This implies that the system will return to equilibrium under small disturbances. The local stability of the nontrivial equilibrium E^* has been examined using a suitable Lyapunov function. For global stability, we employ a suitable Lyapunov function or use a comparison principle to establish conditions under which all system trajectories converge to the equilibrium regardless of initial conditions. If such a function exists and satisfies the necessary criteria (positive definiteness and negative semi-definiteness of its derivative), the equilibrium is globally asymptotically stable, ensuring that the system tends toward the equilibrium in the long run, even under significant deviations. Thus, both local and global stability analyses provide crucial insights into the long-term behaviour and resilience of the system under external perturbations.

4.1 Local stability analysis

Jacobian matrix for the model system (1) – (3) is given as

$$J=\begin{bmatrix} s\left(1-\frac{2B}{L}\right)-s_0-s_1D-kW & -s_1B & -kB \\ -\lambda & -\lambda_0 & 0 \\ \pi kW & -r_1W & \pi kB+r\left(1-\frac{2W}{K}\right)-r_0-r_1D \end{bmatrix}$$

From this, we note the following:

(i) The equilibrium E_1 is unstable in $B-W$ plane as two eigenvalues $s-s_0-\frac{s_1\lambda}{\lambda_0}L$ and $r-r_0-\frac{r_1\lambda}{\lambda_0}L$ are positive.

(ii) The equilibrium E_2 is unstable in B direction as one eigenvalue $s-s_0-\frac{s_1\lambda L}{\lambda_0}-\frac{kK}{r}\left(r-r_0-\frac{r_1\lambda L}{\lambda_0}\right)$ is positive.

(iii) The equilibrium E_3 is unstable in W direction as one eigenvalue $\pi kB_3+r-r_0-r_1D_3$ is positive.

To establish local stability behaviour of the equilibrium E^*, we consider the following positive definite function

$$U=\frac{1}{2}\left(\frac{x_1^2}{B^*}+k_1x_2^2+k_2\frac{x_3^2}{W^*}\right) \tag{8}$$

Here, x_1, x_2 and x_3 are small perturbations about the equilibrium E^* i.e. $B=B^*+x_1$, $D=D^*+x_2$ and $W=W^*+x_3$. The constants k_1 and k_2 are positive to be chosen appropriately. The differentiation of the equation (8) with respect to $'t'$ is given as

$$\frac{dU}{dt}=\frac{x_1}{B^*}\frac{dx_1}{dt}+k_1x_2\frac{dx_2}{dt}+\frac{k_2x_3}{W^*}\frac{dx_3}{dt} \tag{10}$$

Using the linearized system of the model (1) – (3) corresponding to the equilibrium E^*, we get

$$\frac{dU}{dt}=-\frac{s}{L}x_1^2-\lambda_0k_1x_2^2-\frac{r}{K}k_2x_3^2-(s_1+\lambda k_1)x_1x_2-k(1-\pi k_2)x_1x_3-r_1k_2x_2x_3$$

Now, $\frac{dU}{dt}$ will be negative definite if the following conditions are satisfied,

$$(s_1+\lambda k_1)^2<\frac{s\lambda_0}{L}k_1 \tag{11}$$

$$k^2(1-\pi k_2)^2<\frac{sr}{LK}k_2 \tag{12}$$

$$r_1^2k_2^2<\frac{r\lambda_0}{K}k_1k_2 \tag{13}$$

Inequality (11) can further be written as

$$(s_1 - \lambda k_1)^2 + 4 s_1 \lambda k_1 < \frac{s\lambda_0}{L} k_1 \qquad (14)$$

Choosing $k_1 = \dfrac{s_1}{\lambda}$ and $k_2 = \dfrac{1}{\pi}$ with some manipulations, $\dfrac{dU}{dt}$ will be negative definite showing that U is a Lyapunov function. This result is stated in the following theorem.

Theorem 1 The equilibrium E^* is locally asymptotically stable provided the following conditions are satisfied

$$s_1 L < \frac{s\lambda_0}{4\lambda} \qquad (15)$$

$$K r_1^2 < \frac{\pi r \lambda_0 s_1}{\lambda} \qquad (16)$$

4.2 Global stability analysis

To establish global stability behaviour of the equilibrium E^*, we consider the following positive definite function

$$V(B,D,W) = \left(B - B^* - B^* \log \frac{B}{B^*} \right) + \frac{m_1}{2}(D - D^*)^2 + m_2 \left(W - W^* - W^* \log \frac{W}{W^*} \right) \quad (17)$$

where k_1 and k_2 are positive constants to be chosen appropriately.

The differentiation of the equation (17) with respect to $'t'$ is given as

$$\frac{dV}{dt} = (B - B^*) \frac{1}{B} \frac{dB}{dt} + m_1 (D - D^*) \frac{dD}{dt} + m_2 (W - W^*) \frac{1}{W} \frac{dW}{dt} \qquad (18)$$

Putting the values of the derivatives from the model system (1) – (3) in the equation (18), we get

$$\frac{dU}{dt} = -\frac{s}{L}(B - B^*)^2 - \lambda_0 m_1 (D - D^*)^2 - \frac{r}{K} m_2 (W - W^*)^2 - (s_1 + \lambda m_1)(B - B^*)(D - D^*)$$

$$- k(1 - \pi m_2)(B - B^*)(W - W^*) - r_1 m_2 (D - D^*)(W - W^*)$$

Now, $\dfrac{dV}{dt}$ will be negative definite if the following conditions are satisfied,

$$(s_1 + \lambda m_1)^2 < \frac{s\lambda_0}{L} m_1 \qquad (19)$$

$$k^2 (1 - \pi m_2)^2 < \frac{s r}{L K} m_2 \qquad (20)$$

$$r_1^2 m_2^2 < \frac{r\lambda_0}{K} m_1 m_2 \qquad (21)$$

Inequality (11) can further be written as

$$(s_1 - \lambda m_1)^2 + 4 s_1 \lambda m_1 < \frac{s\lambda_0}{L} m_1 \qquad (22)$$

Choosing $m_1 = \frac{s_1}{\lambda}$ and $m_2 = \frac{1}{\pi}$ with some manipulations, $\frac{dV}{dt}$ will be negative definite inside the region of attraction Ω showing that V is a Lyapunov function. This result is stated in the following theorem.

Theorem 2 The equilibrium E^* is locally asymptotically stable provided the following conditions are satisfied

$$S_1 = \frac{s\lambda_0}{4\lambda} - s_1 L > 0 \tag{23}$$

$$S_2 = \frac{\pi r \lambda_0 s_1}{\lambda} - K r_1^2 > 0 \tag{24}$$

Remark 4

The equilibrium point E^* is shown to be both locally and globally asymptotically stable under the same stability conditions. This implies that any small perturbation around E^* will decay over time, ensuring that trajectories starting sufficiently close to E^* return to it. Furthermore, the global asymptotic stability result confirms that all system trajectories, irrespective of their initial conditions within the biologically feasible domain, will eventually converge to E^*. This establishes E^* as a globally attracting steady state, representing the long-term behavior of the system. The result underscores the robustness of the equilibrium, indicating that the system will always stabilize at E^*, regardless of initial fluctuations or disturbances. Thus, the identical conditions for both local and global stability confirm that the equilibrium point E^* is globally asymptotically stable whenever it is locally stable. This reinforces the idea that maintaining development activities within the specified thresholds ensures long-term ecological sustainability without leading to system collapse.

Remark 5

These conditions collectively indicate that if development activities and their negative influence on forest resources and wildlife remain within controlled limits, the ecosystem can maintain stability. However, if the rate of deforestation and its cascading effects on wildlife exceed critical thresholds, the system may experience instability, potentially leading to severe ecological imbalance, biodiversity loss, or even species extinction.

Remark 6

If the impact of development on forestry resources (s_1) and the rate of development expansion due to forestry resources (λ) are significantly high, then the first stability condition will not be satisfied. This means that as s_1 increases, development activities lead to excessive deforestation, reducing forest resources beyond their capacity for natural regeneration. Similarly, a high λ implies that development is expanding rapidly, further intensifying deforestation and its cascading effects on the ecosystem. When forest resources decline at an accelerated rate, they fail to provide sufficient ecological support, leading to habitat destruction and resource scarcity for wildlife species. Moreover, if the impact of development on wildlife (r_1) is significantly high, the second stability condition will not hold. A large r_1 suggests that development activities such as urbanization, industrial expansion, and infrastructure projects directly encroach upon wildlife habitats, leading to population declines through habitat loss, food scarcity, and increased human-wildlife conflicts. Thus, these parameters have destabilizing effect on the model system. As these destabilizing parameters exceed critical thresholds, they push the ecosystem toward a state of imbalance, potentially leading to irreversible biodiversity loss and environmental degradation. Hence, effective management strategies are essential to mitigate these adverse effects and maintain ecological stability.

5 Numerical Simulation

In this section, we used Maple 18 software to conduct numerical simulations in order to validate the analytical findings. The model systems (1) - (3) were given the following set of parameter values for the analysis:

$$L = 10000, K = 1000, s = 0.08, s_0 = 0.01, s_1 = 0.000004, \lambda = 0.01, \lambda_0 = 0.1,$$

$$\pi = 0.6, r = 0.5, r_0 = 0.02, r_1 = 0.00003, \pi_1 = 0.6, k = 0.000001$$

The equilibrium values of the nontrivial equilibrium point E^* for the aforementioned set of parameter values are given below.

$$B^* = 8557.6821, D = 144.2317, W = 961.6153$$

The stability analysis of the equilibrium point E^* within the model system (1)-(3) is conducted using the Jacobian matrix. The eigenvalues -0.06, -0.10 and -0.48, obtained from this matrix, are all negative, which confirms that E^* is locally asymptotically stable. Furthermore, by evaluating the system with a specific set of parameter values, it is verified that the conditions for the existence of a nontrivial equilibrium, as established in Theorem 2, are satisfied. This ensures that E^* is nonlinearly asymptotically stable under the given parameter conditions. These findings affirm the robustness of the equilibrium, indicating that small perturbations in system variables will naturally decay over time, leading the system back to equilibrium.

Using the given parameter values, a three-dimensional graph has been generated (Figure 1) to illustrate the dynamics of biomass density of forest resources (B), the level of various developments (D), and the population density of wildlife species (W). The graphical representation reveals that all trajectories, regardless of their initial starting points within the region of attraction, converge toward the equilibrium point. This behavior confirms that the equilibrium is nonlinearly asymptotically stable, meaning that even with small perturbations, the system naturally returns to its steady-state over time. The stability of the equilibrium suggests that the interplay between forest biomass, developmental activities, and wildlife population follows a predictable and self-regulating pattern, reinforcing the importance of sustainable development policies to maintain ecological balance.

Figs 2–4 illustrate the variations in biomass density of forest resources (B), the level of various developments (D), and the population density of wildlife species (W) for different values of the growth rate λ of various developments. The results indicate that as λ increases, the biomass density of forest resources and the population density of wildlife species decline (Figures 2 and 3, respectively), while the level of various developments rises.

The results presented in Figs 2–4 highlight the significant impact of increasing development (λ) on forest biomass and wildlife populations. As developmental activities expand, the biomass density of forest resources (B) and the population density of wildlife species (W) show a clear decline. This occurs because infrastructure expansion, industrialization, and urbanization often lead to deforestation, habitat destruction, and increased human interference, which negatively affect both plant and animal life.

On the other hand, the level of various developments (D) increases with λ, showing that as economic and infrastructural activities grow, they do so at the cost of ecological resources. The findings underscore the crucial need for sustainable policies that mitigate environmental degradation while promoting economic growth. If developmental activities continue unchecked, the depletion of forest biomass and loss of wildlife species could have long-term consequences, including reduced biodiversity, climate instability, and ecosystem imbalance. Therefore, implementing conservation measures, afforestation programs, and responsible land-use planning is necessary to ensure that economic development does not come at the cost of environmental sustainability.

The stability conditions of the system, represented by S_1 and S_2, are analyzed with respect to the development growth rate parameter λ, as shown in Figs 5 and 6, respectively. From Figure 5, it is observed that the condition S_1 is satisfied when $\lambda < 0.05$, indicating that the system remains stable, meaning that its dynamics are well-regulated and bounded over time. However, for $\lambda \geq 0.05$, the stability condition S_1 is not satisfied, suggesting that the system enters an unstable state where its dynamics may diverge or become unpredictable.

Similarly, Fig. 6 illustrates the variation in stability condition S_2 with respect to λ. It is noted that S_2 is satisfied for $\lambda < 0.133325$, ensuring that the system remains stable and its dynamics are contained within a bounded region. However, for $\lambda \geq 0.133325$, S_2 is not satisfied, indicating instability and the potential for the system's behavior to become erratic or unbounded.

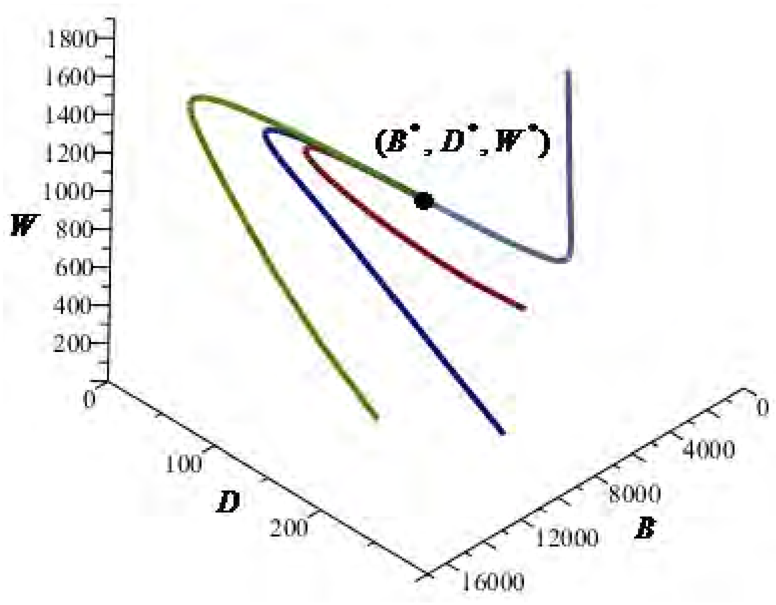

Fig. 1 Nonlinear stability in $B-D-W$ space.

These findings emphasize the critical threshold values of λ beyond which stability is lost, underscoring the need for controlled and regulated developmental activities. If λ exceeds these thresholds, the system may exhibit chaotic or unmanageable behavior, leading to adverse environmental and ecological consequences. This highlights the importance of balancing development with sustainable practices to ensure long-term stability and resilience of the system.

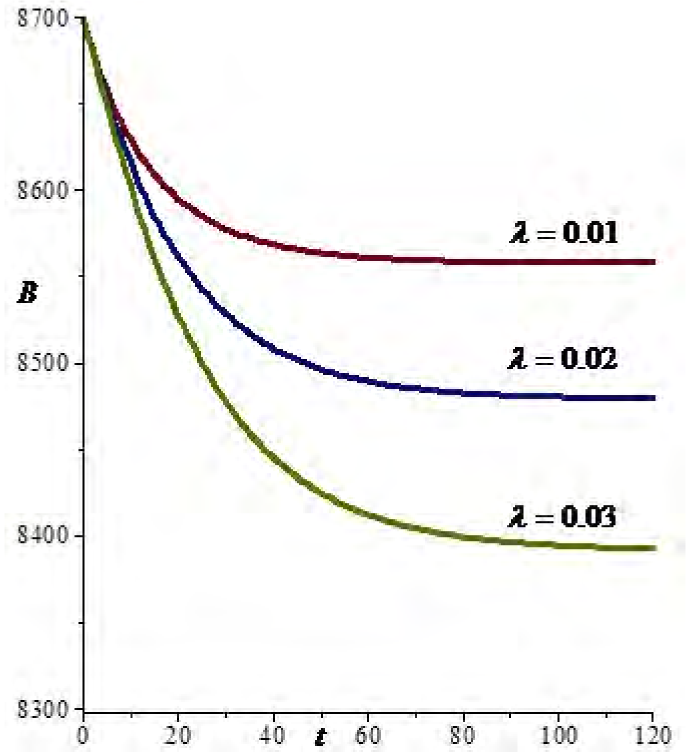

Fig. 2 Effect of λ on the biomass density B of forest resources with time t.

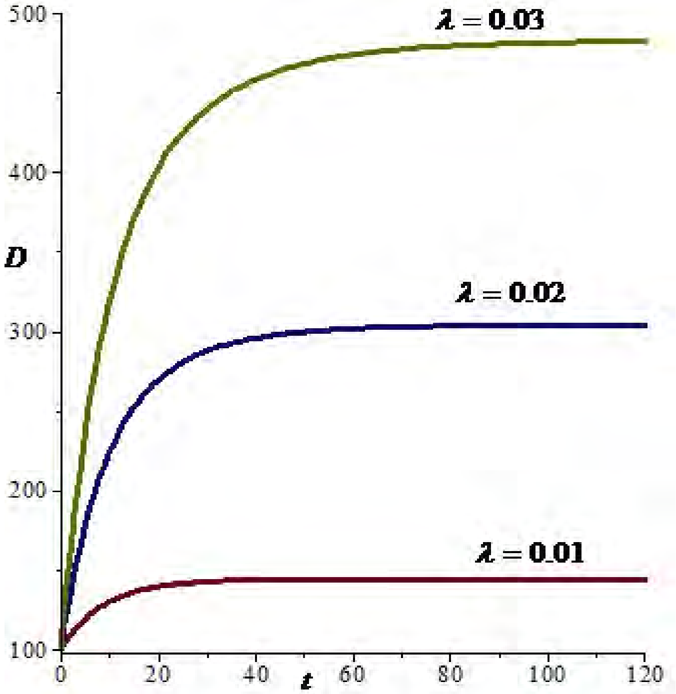

Fig. 3 Effect of λ on the level of various developments D with time t.

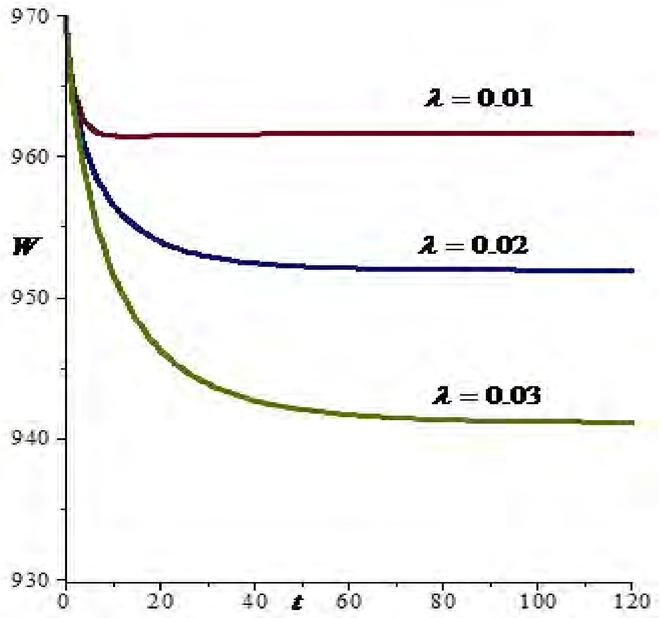

Fig. 4 Effect of λ on the population density of wildlife species W with time t.

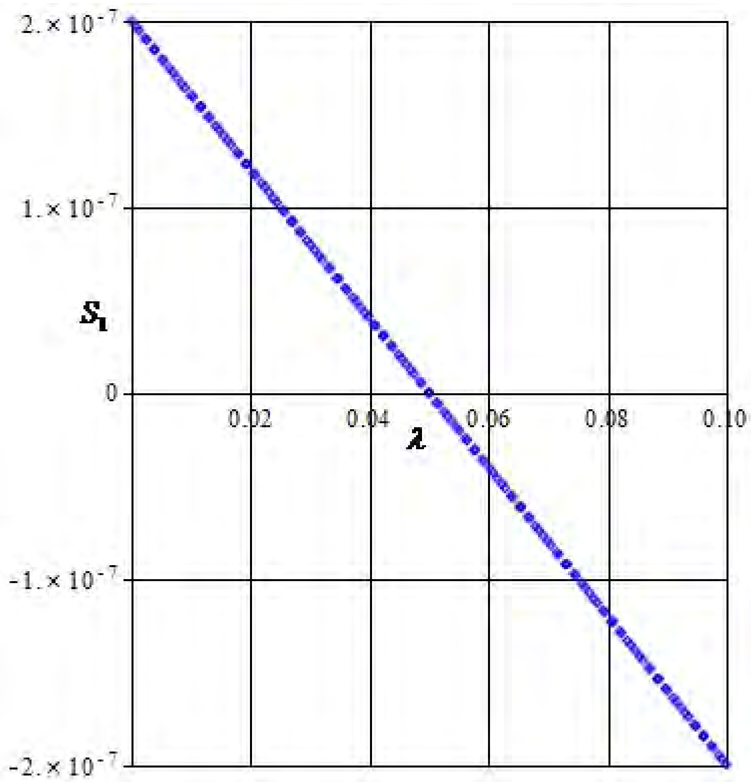

Fig. 5 Effect of λ on stability condition S_1.

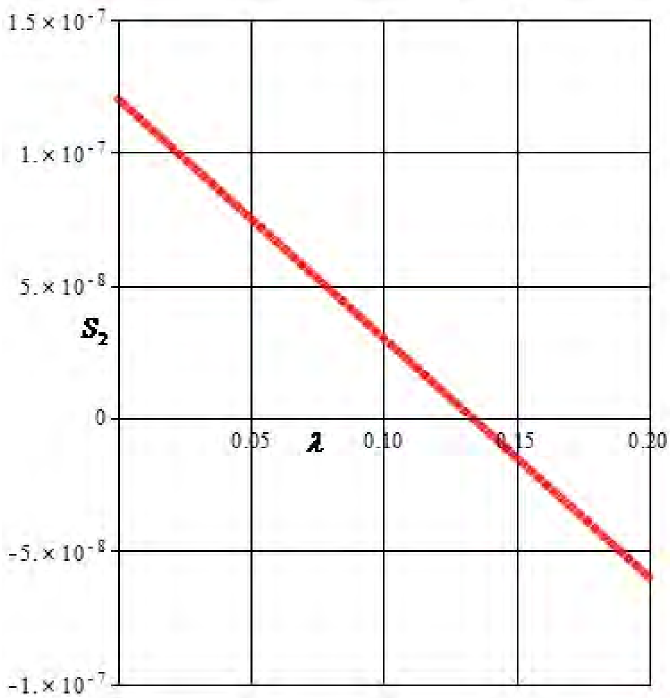

Fig. 6 Effect of λ on stability condition S_2.

6 Result and Discussion

This study presents a mathematical model to analyze the effects of deforestation caused by various development activities on the growth of wildlife species. The model incorporates forestry resources, development activities, and wildlife species to examine their interdependent dynamics. The stability analysis reveals that the system remains viable only if the negative impacts of development on forests and wildlife remain within certain limits. Specifically, when the rate of forest degradation due to development (s_1) and the growth rate of development stimulated by forestry resources (λ) exceed their critical thresholds, the system becomes unstable, leading to the depletion of forestry resources and, consequently, a significant decline in wildlife populations. Similarly, if the impact of development on wildlife species (r_1) is excessively high, the system loses stability, suggesting that uncontrolled expansion of development leads to severe biodiversity loss. These findings highlight the critical need for sustainable development practices that minimize deforestation to maintain ecological balance.

The graphical representations in Figs 1 to 6 provide a deeper insight into the dynamic interactions among forestry resources, development activities, and wildlife species under various parameter conditions. Fig. 1 confirms that the equilibrium point E^* is nonlinearly asymptotically stable, indicating that the system will tend to a steady state over time under the given stability conditions. Figs 2–4 provide insights into the impact of the development growth rate (λ) on system variables, including the biomass density of forest resources, the level of various developments, and the population density of wildlife species. As λ increases, development activities expand more rapidly, leading to a significant depletion of forest biomass due to deforestation and land conversion, which in turn adversely affects wildlife populations by reducing their natural habitat and food availability. The rise in developments also accelerates the decline of both biomass density of forest resources and the population density of wildlife species, demonstrating the interconnected nature of these variables. Furthermore, Figs 5 and 6 analyze the stability conditions S_1 and S_2 in relation to λ, revealing critical thresholds beyond which the system becomes unstable. When λ surpasses these thresholds, the negative impact of development becomes too intense for forest resources and wildlife species to recover, leading to potential ecosystem collapse. These results emphasize the necessity of controlling development rates to ensure a sustainable balance between economic expansion and environmental conservation, thereby preventing irreversible biodiversity loss and deforestation.

Furthermore, the analysis of destabilizing parameters suggests that deforestation and habitat loss due to development are major threats to wildlife sustainability. The conditions for both local and global stability are identical, implying that when stability is achieved locally, it also holds globally under the same constraints. This reinforces the importance of controlling key destabilizing factors such as excessive deforestation rates and unchecked developmental expansion. The use of Lyapunov functions to verify stability strengthens the reliability of these conclusions. Future studies can extend this model by incorporating mitigation strategies, such as afforestation initiatives and conservation policies, to assess their effectiveness in counteracting the adverse effects of development on forests and wildlife.

7 Conclusion

In this study, we developed and analyzed a nonlinear mathematical model to understand the impact of development activities on forestry resources and wildlife populations. The stability analysis revealed that the model's equilibrium depends on critical parameters, particularly the impact of development on forest resources (s_1), the growth of development due to forest exploitation (λ), and the effect of development on wildlife populations (r_1). Our findings indicate that excessive development-driven deforestation and habitat destruction can lead to ecological instability, threatening biodiversity and sustainability. The results emphasize the need for balanced development strategies that account for environmental conservation, ensuring the coexistence of economic growth and ecological integrity.

This study provides valuable insights into the long-term consequences of unregulated development on natural ecosystems. The mathematical framework can serve as a basis for policymakers and environmentalists to design interventions that mitigate deforestation and wildlife depletion. By identifying destabilizing factors, the model highlights the importance of implementing sustainable land-use policies, afforestation programs, and wildlife conservation measures to maintain ecological balance.

Future research can extend this model by incorporating climate change effects, policy-driven conservation efforts, and economic incentives for sustainable development. Additionally, incorporating spatial heterogeneity and stochastic effects may provide a more comprehensive understanding of ecosystem dynamics under varying environmental conditions.

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