Article

A dynamic model for balanced biomass growth under management effort and technological capability

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Abstract
The prolonged improvement of resource biomass density has become increasingly rigorous due to booming environmental stress and the limited capacity of natural systems to renew. This work creates a nonlinear mathematical framework to examine how technological advancement combined with continual management effort can support biomass growth while ensuring sustainability. The proposed model captures the interactions among biomass density, technological capability, and management effort through a system of coupled ordinary differential equations. The analysis identifies more than one equilibrium states, including demolition, partial persistence, and a positive interior equilibrium that corresponds to the justifiable coexistence of biomass, technology, and effort. Under realistic parameter conditions, numerical simulation shows that consistent technological capability, when applied through controlled management effort, plays a critical role in increasing biomass density and reducing the danger of resource degradation. In addition, sensitivity analysis using Partial Rank Correlation Coefficient (PRCC) highlights the proportional impact of key parameters. Overall, the model provides significant theoretical insights into technology-effort interactions in resource management and serves as a useful tool for informing sustainable strategies targeted at raising biomass density through well planned technological interventions.

Keywords biomass density; technological capability; management effort; sensitivity.

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1 Introduction

The worldwide reduction in biomass density has become an urgent environmental concern, driven by the multifaceted impacts of the ecological stress, overexploitation of natural resources, and climate change. Sifts in climatic conditions, such as rising temperatures and altered precipitation patterns, together with elevated human activities, are disrupting the natural functioning of ecosystems. Moreover, hazardous utilization of resources lowers the regenerative capacity of ecosystems, resulting in a gradual depletion of biomass over time. Together, these elements lead to reduction in biomass at global scale and the deterioration of ecological stability (IPCC, 2021; ME Assessment, 2001; Chapin III et al., 2002).

Biomass is a renewable organic resource that is extracted from plants and animals that can be used directly for heat generation or transformed into liquid and gaseous fuels through variety of processes (Bonechi et al., 2017). Biomass, representing the living portion of vegetation, plays an essential role in the global carbon cycle and climate management. Approximately 50% of biomass consists of carbon, and thus, the removal or degradation of vegetation-particularly through deforestation-leads to massive emissions of carbon dioxide into the atmosphere, contributing significantly to the creation of greenhouse gases (IPCC, 2021; Pan et al., 2011). On the other hand, developing vegetation absorbs atmospheric carbon dioxide through photosynthesis and deposit it in the form of biomass, acting as a natural carbon sink (Bonan, 2008). Therefore, biomass dynamics, particularly those associated with forest ecosystems, serve both as a source and a sink of carbon, making them essential to comprehending climate change and developing effective mitigation strategies (Houghton, 2012).

Furthermore, by continuously accumulating and decomposing organic matter, high biomass density improves soil health by enriching the soil with essential nutrients and strengthening its structure. Dense vegetation also plays a substantial role in regulating local and regional climate by moderating temperature, maintaining humidity, and facilitating carbon sequestration (Sherly and Veeresha, 2024). Additionally, it contributes to efficient water cycling by lowering surface runoff, encouraging groundwater recharge, and halting soil erosion. Because of their structural complexity and biodiversity, these ecosystems are more resilient to environmental disruptions. Consequently, regions with high biomass density provide vital ecosystem services that promote long-term environmental sustainability and human well-being in addition to maintaining ecological integrity (Odum, 1971).

Although natural ecosystems possess innate ability to regenerate, enabling them to recover from disturbances through mechanisms like biological interactions, food cycling, and ecological succession. This regenerative capacity enables ecosystems to revive biomass, maintain productivity, and preserve ecological equilibrium over time. These self-regulating mechanisms are generally sufficient to sustain ecosystem stability and resilience under natural settings. However, in recent decades, this regenerative ability has been increasingly outpaced by intensified anthropogenic pressures. Activities such as rapid industrialization, deforestation, and excessive exploitation of natural resources have severely disrupted ecosystem processes. Extensive deforestation reduces green cover and carbon storage, while industrial activities contribute to pollution and environmental degradation (Dubey et al., 2009; Dubey and Narayanan, 2010; Mishra et al., 2014; Lata et al., 2014; Shukla et al., 1989; Agarwal et al., 2010; Swaroop et al., 2026). Additionally, excessive resource use exceeds the natural rate of regeneration, leading to reduction of biomass and loss of ecosystem functionality. As a result, ecosystems are experiencing reduced resilience and diminished ability to recover from disturbances. The imbalance between natural regeneration and human-induced degradation has accelerated ecosystem decline, threatening biodiversity, ecological stability, and the provision of essential ecosystem services (Jatav et al., 2026; Gicheru et al., 2026). This highlights the urgent need for sustainable management practices to restore and preserve the regenerative capacity of natural systems.

Modern technological advancements have emerged as valuable tools for addressing the challenges associated with biomass management and ecosystem conservation. Technologies such as advanced monitoring systems, sustainable harvesting practices, and ecological restoration techniques enable more efficient assessment, utilization, and recovery of natural resources. Monitoring tools, including remote sensing and data-driven models, help in tracking changes in biomass and ecosystem health, while improved harvesting methods aim to minimize environmental impact. Additionally, restoration approaches support the recovery of degraded ecosystems and enhance their productivity.

However, the effectiveness of these technologies depends largely on their proper implementation and governance. Technological solutions alone cannot ensure sustainability unless they are supported by appropriate policies, regulatory frameworks, and responsible management practices. Without careful planning and enforcement, even advanced technologies may fail to prevent resource depletion and environmental degradation. Therefore, a balanced approach that integrates technological innovation with effective regulation and sustainable practices is essential for long-term ecological stability.

Several studies have emphasized the importance of sustainable management and conservation of forest biomass using both practical and analytical approaches. The role of alternative resources in reducing pressure on forest biomass has been highlighted, suggesting that minimizing dependency on traditional forest products can significantly aid in conservation efforts (Agarwal and Pathak, 2015). In addition, mathematical modelling has been increasingly applied to understand the dynamics of depleted forestry resources and to evaluate strategies for their restoration and long-term sustainability (Goshu and Endalew, 2022). Furthermore, modelling approaches have been used to design effective frameworks for sustainable forest management by analyzing the balance between resource extraction and natural regeneration processes (Mishra and Lata, 2015). Recent studies have highlighted the impact of technological efforts on forestry biomass dynamics, showing that appropriate technological measures can significantly contribute to biomass conservation and regeneration (Marwar et al., 2025). Collectively, these studies demonstrate that integrating resource management practices with mathematical modeling provides a powerful tool for ensuring the conservation and sustainable utilization of forest ecosystems.

The primary objective of this study is to develop a comprehensive framework for understanding biomass dynamics under the combined influence of environmental variability and anthropogenic pressures. The specific objectives of the study are as follows:

• To formulate and analyze a mathematical model describing the growth and depletion of biomass in response to ecological stress and human-induced activities.

• To assess the effectiveness of sustainable management strategies and technological interventions in the conservation and restoration of biomass resources.

• To investigate the long-term behavior of the system and evaluate ecological stability through qualitative and quantitative analysis of the proposed model.

This study integrates ecological principles with mathematical modelling to provide a systematic framework for analyzing biomass dynamics under environmental and anthropogenic influences. It identifies key factors responsible for biomass depletion and facilitates the evaluation of technical and management strategies for sustainable resource utilization. The findings can assist policymakers and environmental managers in developing effective approaches for conservation, restoration, and long-term ecological sustainability.

2 Mathematical Model

In this section, we develop a mathematical model comprising three coupled dynamic variables, each corresponding to an essential component of the system and describing its temporal evolution and interactions. These variables are biomass density $B(t)$, technological capability $T(t)$, and management effort $F(t)$. Here, we assume a constant environment independent of time during constructing mathematical model. In the proposed model, the resource biomass density $B(t)$ is described by logistic growth with a finite carrying capacity L. The parameter s characterizes the inherent growth rate of the biomass, whereas s_0 accounts for losses due to natural depletion processes. s_1 is the rate of contribution to the intrinsic growth of $B(t)$ due to management effort $F(t)$ and θ_1 is the efficiency of this contribution. The term s_2 represents rate of contribution to carrying capacity of resource biomass due to management effort. As a result, the time evolution of the biomass density is described by the following differential equation:

$$\frac{dB}{dt} = s(B - \frac{B^2}{L}) - s_0 B + s_1 \theta_1 BF + s_2 \theta_2 B^2 F.$$

It is assumed that technology is applied to the remaining biomass capacity, represented by $(L - B)$, with the objective of enhancing biomass density. Accordingly, the rate of technological growth is taken to be directly proportional to $(L - B)$, where λ denotes the rate of technology implementation and λ_0 is natural depletion in its implementation. The evolution of the technological component is therefore described by the following dynamics:

$$\frac{dT}{dt} = \lambda(L - B) - \lambda_0 T.$$

Furthermore, it is assumed that management effort is required for the effective implementation of technology. Consequently, the growth rate of effort is taken to be directly proportional to the level of technology, with θ denoting the rate of implementation cost. The parameter θ_0 represents inefficiencies or losses in the application of effort, while the terms $\theta_1 BF$ and $\theta_2 B^2 F$ quantify the contributions to the intrinsic growth and carrying capacity of biomass density $B(t)$ associated with effort interactions. Based on the above assumptions, the dynamics of management effort is represented by the following differential equation:

$$\frac{dF}{dt} = \theta T - \theta_0 F - \theta_1 BF - \theta_2 B^2 F.$$

Based on the above assumptions and interactions, the resulting system of differential equations describing the model is given as follows:

$$\begin{aligned} \frac{dB}{dt} &= s(B - \frac{B^2}{L}) - s_0 B + s_1 \theta_1 BF + s_2 \theta_2 B^2 F, \\ \frac{dT}{dt} &= \lambda(L - B) - \lambda_0 T, \\ \frac{dF}{dt} &= \theta T - \theta_0 F - \theta_1 BF - \theta_2 B^2 F. \end{aligned} \tag{1}$$

where, $B(0) \geq 0,\ T(0) \geq 0,\ F(0) \geq 0$. The initial conditions are assumed to be strictly positive, and all model parameters are taken as positive constants. The interrelationships among the three variables of the system are illustrated in Fig. 1 and the description of all the parameters and variables are provided in the Table 1.

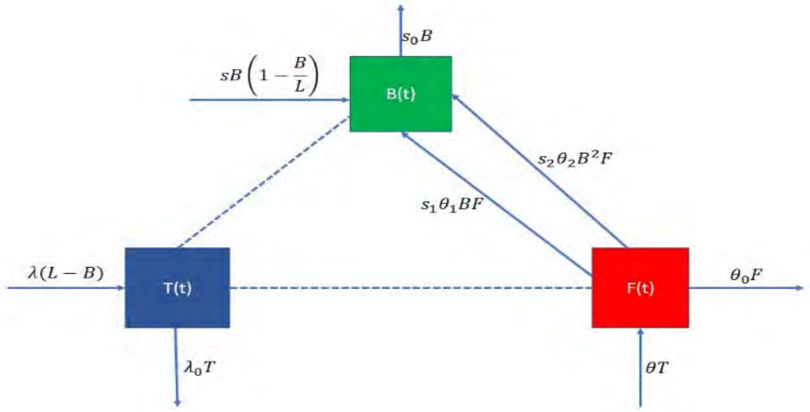

Fig. 1 Schematic representation of state variables.

Table 1 Description table for various parameters and variables.

Description of parameter	Symbol	Units and values
Intrinsic growth rate of resource biomass density	s	1.2 per year
Carrying capacity of resource biomass density	L	100 tons
Natural depletion in resource biomass density	s_0	0.3 per year
Intrinsic growth rate of resource biomass due to management effort	s_1	0.04 tons per thousand USD
Rate of contribution to carrying capacity of resource biomass density due to effort	s_2	0.01 tons per thousand USD
Technology implementation rate	λ	0.8 thousand USD per tons per year
Lack in technology implementation	λ_0	0.4 per year
Rate of implementation cost for management effort	θ	1.5 per year
Lack in management effort implementation	θ_0	0.02 per year
Rate at which effort increases intrinsic growth of B	θ_1	0.003 per tones per year
Rate at which management effort effect logistic growth of B	θ_2	0.001 per $(tons)^2$ per year
Time	t	year
Density of resource biomass	$B(t)$	tons
The measure of technological capability	$T(t)$	USD in thousands
The measure of management effort	$F(t)$	USD in thousands

Lemma 1 For any strictly positive initial conditions, the solutions of the proposed dynamical system (1) remain non-negative and bounded for all $t \geq 0$.Then there exists a positively invariant and bounded region

$$\Omega=\{(B,T,F)\in R^3_+:\ 0 \leq B \leq B_{max},\ 0 \leq T \leq T_{max},\ 0 \leq F \leq F_{max}\},$$

where, $B_{max} = L(\frac{s-s_0}{s})$, $T_{max} = \frac{\lambda L}{\lambda_0}$, $F_{max} = \frac{\theta\lambda L}{\theta_0\lambda_0}$ such that every solution starting in Ω remains in Ω for all $t \geq 0$. Moreover, Ω is a region of attraction for the system.

3 Equilibrium Analysis

In this section, we determine the admissible equilibrium states of the system. These equilibria are obtained by setting the right-hand sides of the governing differential equations equal to zero. The model admits two feasible equilibrium points, which are presented below:

1. $E_0(0, \frac{\lambda L}{\lambda_0}, \frac{\theta\lambda L}{\theta_0\lambda_0})$
2. $E^*(B^*, T^*, F^*)$

3.1 The existence of E_0

To establish the existence of the equilibrium state E_0, we set $B = 0$ in first equation of the system (1). E_0 can be obtained solving below equations:

$$\lambda(L - B) - \lambda_0 T = 0, \tag{2}$$
$$\theta T - \theta_0 F = 0. \tag{3}$$

We get $T = \frac{\lambda L}{\lambda_0}$ and $F = \frac{\theta\lambda L}{\theta_0\lambda_0}$. Hence, the equilibrium point is $E_0(0, \frac{\lambda L}{\lambda_0}, \frac{\theta\lambda L}{\theta_0\lambda_0})$.

3.2 The existence of E^*

Take $B \neq 0$ from the first equation of the model (1) and solve the following algebraic expressions:

$$s(1 - \frac{B}{L}) - s_0 + s_1\theta_1 F + s_2\theta_2 BF = 0, \tag{4}$$
$$\lambda(L - B) - \lambda_0 T = 0, \tag{5}$$
$$\theta T - \theta_0 F - \theta_1 BF - \theta_2 B^2 F = 0. \tag{6}$$

From (5) and (6), we get $T = \frac{\lambda(L-B)}{\lambda_0}$ and $F = \frac{\theta\lambda(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)}$ respectively. Using these values of T and F in (4), we get

$$(s - s_0) - s\frac{B}{L} + (s_1\theta_1 + s_2\theta_2 B)\frac{\lambda\theta(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)} = 0. \tag{7}$$

Now, define a function

$$F(B) = [(s - s_0) - s\frac{B}{L}](\theta_0 + \theta_1 B + \theta_2 B^2) + (s_1\theta_1 + s_2\theta_2 B)\frac{\lambda\theta(L-B)}{\lambda_0}. \tag{8}$$

From (8), we have

$$F(0) = (s - s_0)\theta_0 + s_1\theta_1\frac{\lambda\theta L}{\lambda_0} > 0 \tag{9}$$
$$F(L) = -s_0(\theta_0 + \theta_1 L + \theta_2 L^2) < 0 \tag{10}$$

Therefore, there exists a root $B = B^*$ in the interval $0 < B < L$. For uniqueness we have to show that $\frac{dF(B)}{dt} < 0$.

$$\frac{dF(B)}{dt} = -\frac{s}{L}(\theta_0 + \theta_1 B + \theta_2 B^2) + (s - s_0 - \frac{sB}{L})(\theta_1 + 2\theta_2 B) - (s_1\theta_1 + s_2\theta_2 B)\frac{\lambda\theta}{\lambda_0} + \frac{\lambda\theta(L-B)s_2\theta_2}{\lambda_0} \tag{11}$$

Now, from (7) we have

$$(s-s_0)-\frac{sB}{L}=\frac{-(s_1\theta_1+s_2\theta_2 B)\lambda\theta\,(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)} \tag{12}$$

and also we have $\frac{s}{L}-s_2\theta_2 F>0$. Hence, $\frac{s}{L}-\frac{s_2\theta_2\lambda\theta\,(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)}>0$.

$$-\frac{s}{L}+\frac{s_2\theta_2\lambda\theta\,(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)}<0. \tag{13}$$

Using (12) and (13) in (11), we get

$$\frac{dF(B)}{dt}=(\theta_0+\theta_1 B+\theta_2 B^2)\left(-\frac{s}{L}+\frac{s_2\theta_2\lambda\theta\,(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)}\right)$$
$$-\frac{(s_1\theta_1+s_2\theta_2 B)\lambda\theta\,(L-B)}{\lambda_0(\theta_0+\theta_1 B+\theta_2 B^2)}(\theta_1+2\theta_2 B)-(s_1\theta_1+s_2\theta_2 B)\frac{\lambda\theta}{\lambda_0}. \tag{14}$$

Clearly, $\frac{dF(B)}{dt}<0$. Hence, in the region $0<B<L$ we have unique root B^*. After calculating B^* we can easily obtain T^* and F^*. This shows the existence of equilibrium point E^*.

4 Stability analysis

In this section, we examine the qualitative behavior of the dynamical system in the vicinity of its equilibrium points. Stability of the system is governed by the local and global stability of the equilibrium point.

4.1 Local stability analysis

Here, we analyze how small perturbations around an equilibrium influence the system dynamics, which provides insight into the local stability properties of the model. Now, the jacobian matrix J_0 corresponding to E_0 is defined as follows:

$$J_0=\begin{pmatrix} s-s_0+s_1\theta_1\frac{\theta\lambda L}{\theta_0\lambda_0} & 0 & 0 \\ -\lambda & -\lambda_0 & 0 \\ -\frac{\theta_1\theta\lambda L}{\theta_0\lambda_0} & \theta & -\theta_0 \end{pmatrix}$$

It is observed that one of the eigen value i.e. $s-s_0+s_1\theta_1\frac{\theta\lambda L}{\theta_0\lambda_0}>0$ of J_0 matrix. Hence, the equilibrium point E_0 is unstable in $B-$ direction. Now, the jacobian matrix corresponding to E^* is J^* defined as follows:

$$J^*=\begin{pmatrix} -\frac{sB^*}{L}+s_2\theta_2 F^*B^* & 0 & (s_1\theta_1 B^*+s_2\theta_2 B^{*2}) \\ -\lambda & -\lambda_0 & 0 \\ -\theta_1 F^*-2\theta_2 F^*B^* & \theta & -\theta_0-\theta_1 B^*-\theta_2 B^{*2} \end{pmatrix}$$

The characteristic equation for this matrix is as follows:

$$x^3+A_1x^2+A_2x+A_3=0 \tag{15}$$

where,

$$A_1=\lambda_0+\left(\frac{s}{L}B^*-s_2\theta_2 F^*B^*+\theta_0+\theta_1 B^*+\theta_2 B^{*2}\right)$$

$$A_2 = \lambda_0(\frac{s}{L} - s_2\theta_2 F^* B^* + \theta_0 + \theta_1 B^* + \theta_2 B^{*2}) + (\frac{s}{L}B^* - s_2\theta_2 F^* B^*)(\theta_0 + \theta_1 B^* + \theta_2 B^{*2})$$

$$A_3 = \lambda_0(\frac{s}{L}B^* - s_2\theta_2 F^* B^*)(\theta_0 + \theta_1 B^* + \theta_2 B^{*2}) + \lambda\theta(\theta_0 + \theta_1 B^* + \theta_2 B^{*2})$$

All A_1, A_2 and A_3 are positive and $\frac{s}{L} - s_2\theta_2 F^* > 0$. Hence, there is no positive root of equation (15). Therefore, all three roots of cubic equation are negative, showing $E^*(B^*, T^*, F^*)$ is locally stable equilibrium point.

4.2 Global stability

In this part, we investigate the global dynamics of the equilibrium point E^* by constructing an appropriate Lyapunov function. The resulting stability properties are presented in the following theorem:

Theorem 1 The equilibrium point $\mathrm{E}^*(\mathrm{B}^*, \mathrm{T}^*, \mathrm{F}^*)$ is globally asymptotically stable provided that the following conditions hold:

$$\theta^2 < (\theta_0 + \theta_1 B^* + \theta_2 B^{*2})\lambda_0 \tag{16}$$

$$\lambda^2(s_1\theta_1 + s_2\theta_2 B_{\max}) < \lambda_0 F_{\max}(\theta_1 + 2\theta_2 B^*)(\frac{s}{L} - s_2\theta_2 F^*) \tag{17}$$

Proof. Take a positive definite function

$$W = m_1(B - B^* - B^* \ln\frac{B}{B^*}) + \frac{m_2}{2}(T - T^*)^2 + \frac{m_3}{2}(F - F^*)^2, \tag{18}$$

where, $m_1 > 0,\ m_2 > 0,\ m_3 > 0,$ to be chosen properly. Now, differentiating equation (18) of function W with respect to t we get,

$$\dot{W} = m_1(1 - \frac{B^*}{B})\frac{dB}{dt} + m_2(T - T^*)\frac{dT}{dt} + m_3(F - F^*)\frac{dF}{dt}. \tag{19}$$

using model system (1) equations in the above equation (19) we get,

$$\begin{aligned} \dot{W} = & -m_1(\frac{s}{L} - s_2\theta_2 F^*)(B - B^*)^2 - m_2\lambda_0(T - T^*)^2 \\ & -m_3(\theta_0 + \theta_1 B^* + \theta_2 B^{*2})(F - F^*)^2 \\ & +(m_1(s_1\theta_1 + s_2\theta_2 B) - m_3(\theta_1 + 2\theta_2 B^*)F)(B - B^*)(F - F^*) \\ & -m_2\lambda(B - B^*)(T - T^*) + m_3\theta(F - F^*)(T - T^*). \end{aligned} \tag{20}$$

$\dot{W}$ comes out to be negative definite if the following inequalities hold

$$(m_1(s_1\theta_1 + s_2\theta_2 B_{\max}) - m_3(\theta_1 + 2\theta_2 B^*)F_{\max})^2 < m_1 m_3(\theta_0 + \theta_1 B^* + \theta_2 B^{*2})(\frac{s}{L} - s_2\theta_2 F^*),$$

(21)

$$m_3\theta^2 < m_2\lambda_0(\theta_0 + \theta_1 B^* + \theta_2 B^{*2}), \tag{22}$$

$$m_2\lambda^2 < m_1\lambda_0(\frac{s}{L} - s_2\theta_2 F^*). \quad (23)$$

Solving equations (21)- (23) and taking $m_1 = \frac{(\theta_1+2\theta_2 B^*)F_{\max}\, m_3}{(s_1\theta_1+s_2\theta_2 B_{\max}\,)}$, $m_2 = m_3 = 1$ inside the region of attraction Ω , we derive

$$\theta^2 < (\theta_0 + \theta_1 B^* + \theta_2 B^{*2})\lambda_0 \quad (24)$$

$$\lambda^2(s_1\theta_1 + s_2\theta_2 B_{\max}) < \lambda_0 F_{\max}\,(\theta_1 + 2\theta_2 B^*)(\frac{s}{L} - s_2\theta_2 F^*) \quad (25)$$

If the above inequalities holds then $\frac{dW}{dt}$ is negative definite. Hence, the interior equilibrium point E^* is globally asymptotically stable inside Ω. Hence, the proof.

5 Numerical Simulation

To validate and illustrate the analytical findings, numerical simulations of the proposed system are performed using MATLAB. These simulations are carried out for various sets of admissible parameter values and positive initial conditions in order to examine the qualitative behavior of the system and to support the theoretical stability results. The numerical simulations are conducted using the set of parameter values specified below:

$s = 1.2,\quad L = 100,\quad s_0 = 0.3,\quad s_1 = 0.04,\quad s_2 = 0.01,\quad \lambda_0 = 0.4,\quad \lambda = 0.8,\quad \theta = 1.5,$
$\theta_0 = 0.02,\quad \theta_1 = 0.003,\quad \theta_2 = 0.001.$

A comprehensive description of all model parameters is given in Table 1. Under this set of parameter values, the system possesses a unique positive (non-trivial) equilibrium state

$$E^*(B^*, T^*, F^*) = (75.88,\ 48.23,\ 12.04)$$

and the eigenvalues are $-0.894022, -0.4290083, -5.9837865$. Thus, at the equilibrium point E^*, none of the eigenvalues are positive. Hence, the equilibrium $E^*(B^*, T^*, F^*)$ is locally asymptotically stable.

Fig. 2 illustrates how changes in the technology implementation rate λ influence the behavior of different state variables over time. It can be seen from Fig. 2(A), (B) and (C) that as the value of λ increases, the biomass density $B(t)$ rises, along with corresponding increases in technological capability $T(t)$ and management effort $F(t)$.

The impact of changes in s_1 on different state variables over time is illustrated in Fig. 3. Fig. 3(A) indicates that biomass density $B(t)$ increases with a rise in s_1, whereas from Figs 3(B) and 3(C) we observed that technological capability $T(t)$ and management effort $F(t)$ decrease as s_1 becomes larger.

The influence of variations in θ_1 on different state variables over time is presented in Fig. 4. Figure 4(A) shows that biomass density $B(t)$ increases with an increase in θ_1, while technological capability $T(t)$ and management effort $F(t)$ decrease as θ_1 rises shown in Fig. 4(B) and 4(C) respectively.

The three-dimensional global stability in $B - T - F$ space is illustrated in the Fig. 5, where all solution trajectories are observed to converge toward the equilibrium point E^*.

Phase portraits and time series plots illustrating the emergence of limit cycles and the occurrence of Hopf bifurcation in the system. In Fig. 6, the temporal evolution of the biomass variable demonstrates that the system does not always converge to a steady state. Instead, for certain parameter values, persistent oscillations are observed, indicating the presence of dynamic instability in the system.

Further, the phase portrait for $s = 0.002$ is presented in the Fig. 7. It can be observed that the solution trajectory gradually moves toward the interior equilibrium point over time. This consistent convergence indicates that the system stabilizes at this equilibrium state, thereby confirming its stability.

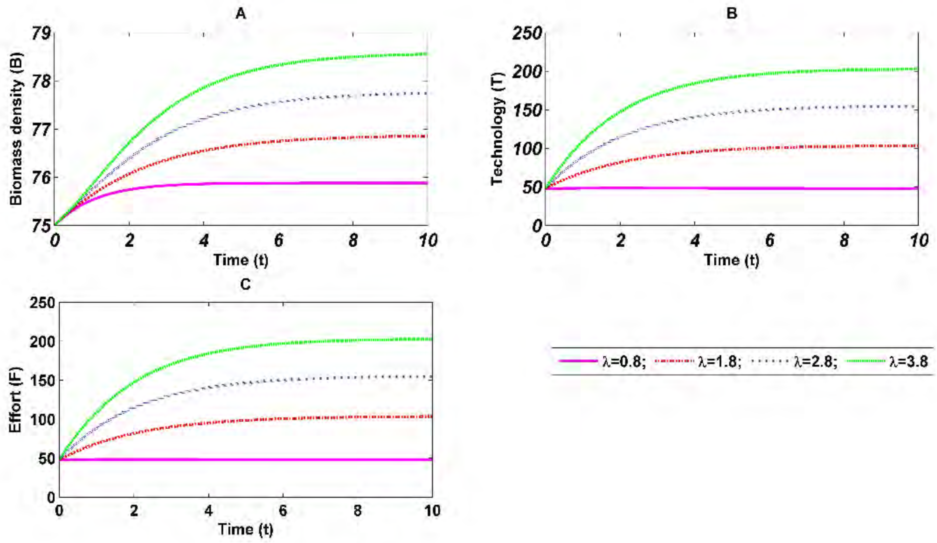

Fig. 2 Effect of variation in technology implementation rate λ on various state variables over time. Figures (A), (B), (C) shows that biomass density $B(t)$, technological capability $T(t)$, management effort $F(t)$ increase as the value of λ increases.

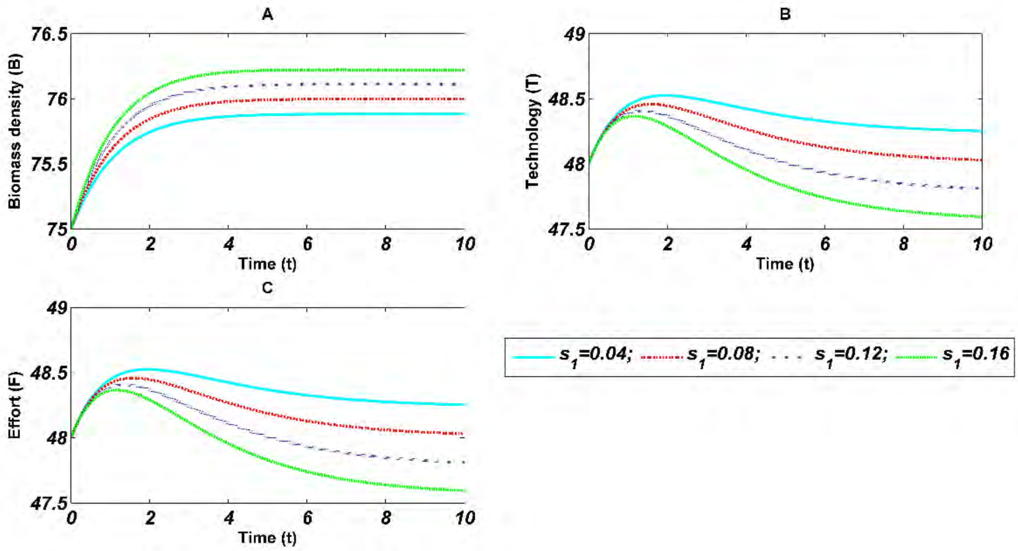

Fig. 3 Effect of variation in s_1 on various state variables over time. Figures shows that biomass density $B(t)$ increases as s_1 increases but technological capability $T(t)$, management effort $F(t)$ decrease as the value of s_1 increases.

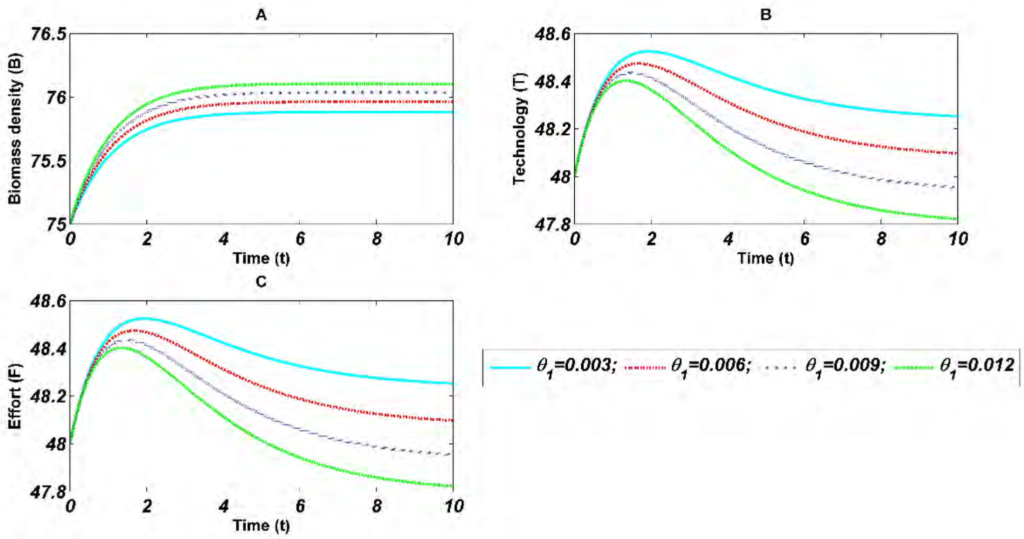

Fig. 4: Effect of variation in θ_1 on various state variables over time. Figures shows that biomass density $B(t)$ increases as θ_1 increase but technological capability $T(t)$, management effort $F(t)$ decrease as the value of θ_1 increases.

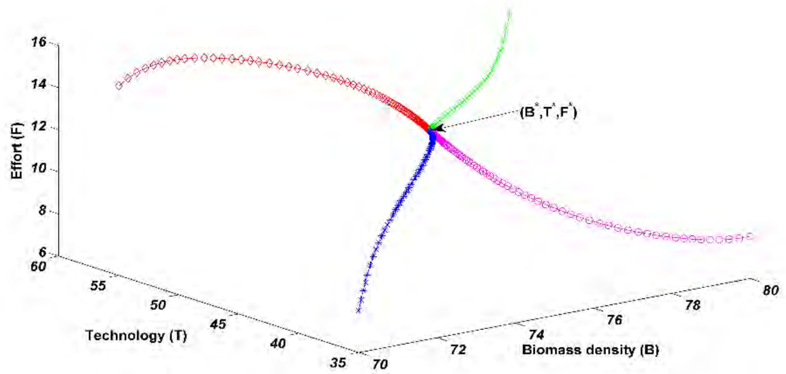

Fig. 5 Global stability in $B-T-F$ space.

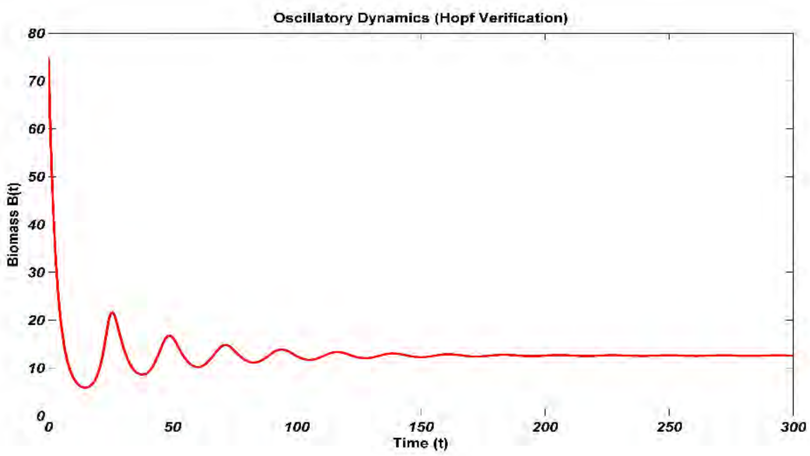

Fig. 6 Equilibrium curve of biomass density $B(t)$ concerning parameter s.

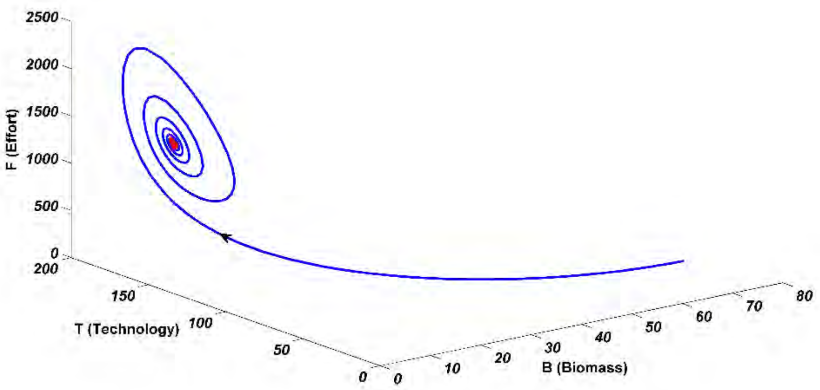

Fig. 7 Phase portraits in $B-T-F$ space for $s=0.002$.

6 Sensitivity Analysis

Global sensitivity analysis is used to determine which parameters have the greatest impact on system dynamics and to minimize uncertainty. In this study, Latin Hypercube Sampling (LHS) is combined with Partial Rank Correlation Coefficients (PRCCs) to assess parameter sensitivity. LHS provides an efficient way to explore the multidimensional parameter space, while PRCCs help reveal monotonic relationships between input parameters and model outputs. The parameters s, s_0, θ, λ and θ_0 are varied within a range provided in the Table 2, and a total of 1000 LHS simulations are conducted. Here, PRCCs are calculated for biomass density $B(t)$ illustrated in Figure 8. The parameter s, θ and λ are strong positive correlations with $B(t)$, whereas s_0 and θ_0 demonstrate negative effect. Thus, it is observed that intrinsic growth rate of resource biomass density, the rate of implementation cost for effort and the technology implementation rate are the influencing the system's performance, whereas the natural depletion coefficient of resource biomass density and lack in effort implementation declines the growth.

Table 2 Parameter ranges used for PRCC sensitivity analysis.

Parameter	s	s_0	θ	λ	θ_0
Range	0.5–2.0	0.2–1.0	0.3–2.0	0.3–1.0	0.01–0.1

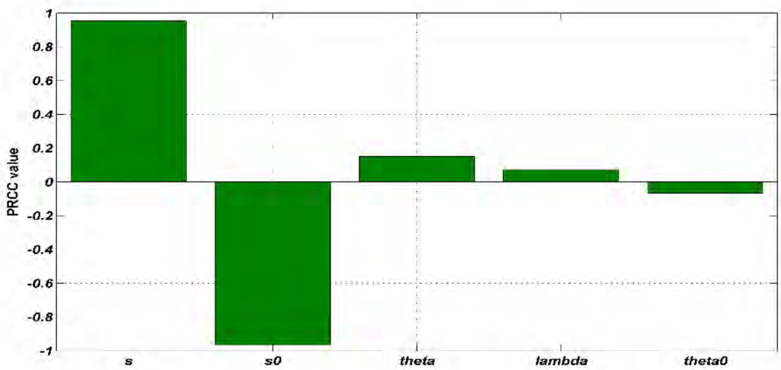

Fig. 8 Partial rank correlation coefficients (PRCCs) for $B(t)$ with respect to s, s_0, θ, λ, θ_0.

7 Result and Discussion

The proposed nonlinear model provides a comprehensive framework to examine the interplay between resource biomass density, technological capability, and management effort. The interaction among these components governs the overall system dynamics and determines whether the ecological system evolves toward sustainability or degradation.

The equilibrium analysis reveals the existence of two steady states. One of these corresponds to a lower biomass regime and is found to be unstable in the biomass direction, indicating that small perturbations can drive the system away from this state. The second equilibrium represents a desirable ecological condition with higher biomass density and is locally asymptotically stable under appropriate parameter conditions. This coexistence of stable and unstable equilibria suggests the presence of threshold behavior in the system.

The stability results further highlight the critical role of management effort in regulating system dynamics. It is observed that sufficient management effort stabilizes the system and supports biomass recovery by enhancing the effectiveness of technological intervention. On the other hand, inadequate effort leads to instability and prevents the system from achieving a sustainable state, even in the presence of technological inputs. This demonstrates that technology alone cannot guarantee resource restoration unless it is supported by consistent and efficient management practices.

Figs 2-4 illustrate the influence of key model parameters on the system dynamics, particularly the technological implementation rate λ, the interaction coefficient s_1, and the technological efficiency parameter θ_1. The results indicate that variations in these parameters significantly affect biomass density, the measure of technology capability, and the measure of management effort. An increase in the technological implementation rate enhances the responsiveness of the system, while higher values of s_1 and θ_1 strengthen the positive interaction between biomass and supporting factors, thereby promoting overall system stability and growth.

Fig. 5 demonstrates the convergence of all solution trajectories toward the equilibrium point, confirming the local asymptotic stability of the system under the chosen parameter set. Regardless of the initial conditions, the trajectories gradually approach the steady state, indicating that the system possesses a strong tendency to stabilize when appropriate conditions are maintained.

Figs 6 and 7 depict the bifurcation behavior of the system, highlighting how qualitative changes in dynamics arise due to parameter variation. The results reveal the presence of critical thresholds beyond which the system transitions from stable equilibrium to more complex dynamical states. In particular, the emergence of oscillatory patterns suggests the occurrence of bifurcation phenomena, emphasizing the sensitivity of the system to parameter changes.

Furthermore, Fig. 8 presents the results of the Partial Rank Correlation Coefficient (PRCC) analysis, which quantifies the sensitivity of the model output with respect to different parameters. The analysis identifies the most influential parameters governing biomass dynamics, indicating that growth-related and technology-driven parameters have a strong positive impact, whereas depletion-related parameters exert a negative influence. This highlights the relative importance of each parameter in shaping the long-term behavior of the system.

Overall, the findings underscore that sustainable biomass management is governed by a delicate balance between natural growth processes, technological support, and management effort. The results suggest that while technological advancement plays a significant role in improving system performance, its success is strongly dependent on the level of management effort applied. Effective coordination between these factors is essential to ensure long-term ecological stability and efficient resource utilization.

8 Conclusion

This study presents a nonlinear mathematical model to examine the combined influence of technological capability and management effort on biomass dynamics. The analysis shows that the system possesses multiple equilibria, and its stability is strongly governed by the interplay between growth processes, depletion effects, and technological support. It is observed that adequate management effort significantly enhances the effectiveness of technological implementation, leading to stable and sustainable biomass levels, whereas insufficient effort limits its impact and may result in instability.

From a broader perspective, the results highlight that real-world resource management systems are inherently complex and require a balanced integration of multiple factors rather than reliance on a single intervention. The model emphasizes that technology acts as a supportive mechanism whose success depends on consistent and well-planned management strategies.

Furthermore, the bifurcation analysis reveals the presence of critical thresholds where small parameter changes can alter system behavior, including the emergence of oscillatory dynamics. These findings underline the importance of careful parameter regulation in avoiding undesirable fluctuations. Overall, the study provides useful insights into sustainable resource management and underscores the need for coordinated technological and managerial efforts.

The study suggests several directions for future work. The model can be extended to include spatial heterogeneity for region-specific analysis and stochastic effects to capture environmental uncertainties. It may also be enhanced by incorporating optimal control strategies for sustainable resource management. Additionally, integrating policy-oriented aspects could improve its relevance in balancing ecological sustainability with economic development.

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