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# Networks control: Introducing the degree of success and feasibility

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**Abstract**
Taming ecological and biological networks is a key-issue. It could be used to: a) neutralize damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage ecological systems at the least possible cost, and d) counteract the impacts of climate change. While I recently showed that ecological and biological networks can be efficaciously controlled both from inside (inside-control model) and outside (outside-control model), here I propose a solution to the choice of the most feasible solution to network control. To do this, I introduce the concepts of control success and feasibility.

**Keywords** edges control feasibility; control success; control uncertainty; genetic algorithms; network control; stochastic simulations.

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## 1 Introduction

Recently, I proposed that ecological and biological networks can be controlled by coupling network dynamics and evolutionary modelling (Ferrarini, 2011). They can be efficaciously tamed from outside (Ferrarini, 2013a), but also through the use of endogenous controllers (Ferrarini, 2013b). These two approaches are different from both a theoretical and methodological viewpoint. The endogenous control requires that the network is optimized at the beginning of its dynamics (by acting upon nodes, edges or both) so that it will then go inertially to the desired state. Instead, the exogenous control requires that exogenous controllers act upon the network at each cycle. *A priori*, it's hard to say which of the two approaches is more effective, it mainly depends on the kind of ecological or biological network one is dealing with.

In another paper (Ferrarini, 2013c), I have faced a further important question: how reliable is the achieved solution? In other words, which is the degree of uncertainty about getting the desired result if values of edges and nodes were a bit different from optimized ones? This is a pivotal question, because it's not assured that while managing a certain system we are able to impose to nodes and edges exactly the optimized values we would need in order to achieve the desired results. In order to face this topic, I have coined a 3-parts framework (network dynamics - genetic optimization - stochastic simulations).

Here I propose a solution to the choice of the most feasible solution to network control. To do this, I introduce the concepts of control success and feasibility.

## 2 Mathematical Formulation

Most real systems' dynamics can be modelled and simulated as follows (Liu et al., 2011; Slotine and Li, 1991):

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11}S_1 + ... + a_{1n}S_n + I_1 + O_1 \\ ... \\ \dfrac{dS_n}{dt} = a_{n1}S_1 + ... + a_{nn}S_n + I_n + O_n \end{cases} \quad (1)$$

where $S_i$ is the number of individuals (or the total biomass, or the covered surface in case of plant species) of the generic *i-th* species, while *I* and *O* represent inputs and outputs from/to outside.

Biological and ecological systems can be tamed from outside using the following 1-external-controller model (Ferrarini 2013a):

$$\begin{cases} \left(\dfrac{dS_1}{dt}\right)_{OPT} = a_{11}S_1 + ... + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} \\ ... \\ \left(\dfrac{dS_n}{dt}\right)_{OPT} = a_{n1}S_1 + ... + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} \\ \dfrac{dC_1}{dt} = f_1S_1 + ... + f_nS_n \end{cases} \quad (2)$$

where asterisks stand for the genetic optimization (Holland 1975) of exogenous node's edges (i.e., coefficients of interaction with the inner system) and exogenous node's stock, i.e. the modification of such values at the beginning of network dynamics in order to get a certain goal (e.g., maximization of the final value of a certain variable). There's 1 controller $C_1$ that, in some cases, can also receive feedbacks from the network, It's clear that also the feedback $dC_1/dt$ to the controller could be subject to genetic control by taming $<f_1 \dots f_n>$.

In case 1 controller is not enough, the model in (2) must be expanded to the following *k*-external-controllers model (Ferrarini 2013a):

$$\begin{cases} \frac{dS_1}{dt} = a_{11}S_1 + ... + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} + ... + c_{1k*}C_{k*} \\ ... \\ \frac{dS_n}{dt} = a_{n1}S_1 + ... + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} + ... + c_{nk*}C_{k*} \\ \frac{dC_1}{dt} = f_{11}S_1 + ... + f_{1n}S_n \\ ... \\ \frac{dC_k}{dt} = f_{k1}S_1 + ... + f_{kn}S_n \end{cases} \tag{3}$$

Alternatively, an ecological or biological network can be controlled from inside using the following control model (Ferrarini, 2013b):

$$\begin{cases} \left(\frac{dS_1}{dt}\right)_{OPT} = a_{11*}S_1^{\ *} + ... + a_{1n*}S_n^{\ *} + I_{1*} + O_{1*} \\ ... \\ \left(\frac{dS_n}{dt}\right)_{OPT} = a_{n1*}S_1^{\ *} + ... + a_{nn*}S_n^{\ *} + I_{n*} + O_{n*} \end{cases} \tag{4}$$

where asterisks stand for the optimization of edges (i.e., coefficients of interaction among variables) or nodes (i.e., initial stocks), that is the modification of their values at the beginning of the network dynamics in order to get a certain goal.

After optimization is reached, the degree of uncertainty of (2), (3) and (4) about getting the desired result can be computed as (Ferrarini, 2013c):

$$\begin{cases} \left(\frac{dS_1}{dt}\right)_{OPT} = \underline{a_{11*}}\,\underline{S_1^{\ *}} + ... + \underline{a_{1n*}}\,\underline{S_n^{\ *}} + \underline{I_{1*}} + \underline{O_{1*}} \\ ... \\ \left(\frac{dS_n}{dt}\right)_{OPT} = \underline{a_{n1*}}\,\underline{S_1^{\ *}} + ... + \underline{a_{nn*}}\,\underline{S_n^{\ *}} + \underline{I_{n*}} + \underline{O_{n*}} \end{cases} \tag{5}$$

where:

$$\begin{cases} 0.95 * a_{ij*} \le \underline{a_{ij}} \le 1.05 * a_{ij*} \\ 0.95 * S_j^{\ *} \le \underline{S_j} \le 1.05 * S_j^{\ *} \end{cases} \tag{6}$$

or alternatively:

$$\begin{cases} 0.9 * a_{ij*} \le \underline{a_{ij}} \le 1.1 * a_{ij*} \\ 0.9 * S_j^{\,*} \le \underline{S_j} \le 1.1 * S_j^{\,*} \end{cases} \tag{7}$$

Hence, $\underline{a_{ij}}$ represents a 5% (or 10%) uncertainty about $a_{ij*}$, while $\underline{S_j}$ represents a 5% (or 10%) uncertainty about $S_j^*$. If, after genetic optimization, we stochastically vary *n* times (e.g. 10,000 times) $a_{ij*}$ and $S_j^*$, we are able to compute how many times such uncertainty makes the optimization procedure useless. Hence, uncertainty about network control can be computed as (Ferrarini, 2013c):

$$U_{\%} = 100 * \frac{k}{n} \tag{8}$$

where *k* is the number of stochastic simulations acting upon optimized parameters that make the optimization procedure useless (i.e. the goal of optimization is not reached).

Now, let's assign to each species a weight of importance $\sigma_i$:

$$\sigma_i : \begin{cases} > 0 & \text{for benefit species (or network actors)} \\ 0 & \text{for species (or network actors) of no interest} \\ < 0 & \text{for cost species (or network actors)} \end{cases} \tag{9}$$

I suggest here that the degree of success $DS_i$ of network control for each *i-th* species can be computed as the weighted difference between the optimized dynamic of the species ($S_i^{opt}$: how it goes, at equilibrium, after optimization) and the inertial one ($S_i^{in}$: how it would go, at equilibrium, without optimization):

$$DS_i = \sigma_i * \Delta_i = \sigma_i * (S_i^{opt} - S_i^{in}) \tag{10}$$

$DS_i$ is positive if a benefit species has increased thanks to the network control or a cost species has decreased. Instead it's negative in case a benefit species has decreased due to the network control or a cost species has increased. The overall degree of success of network control for *n* species (or network actors) can be hence calculated as:

$$DS = \sum_{i=1}^{n} \sigma_i * \Delta_i \tag{11}$$

Now I define the degree of feasibility *F* of network control as:

$$F = \frac{DS}{1 + U_{\%}} \tag{12}$$

where the constant *1* has been added to avoid that the denominator goes to 0. As a first approximation, I suggest that the weight of importance $\sigma_i$ should go from -1 to +1. But, in order to give *DS* and $1+U_{\%}$ the same order of magnitude, I suggest that $\sigma_i$ should be set so that:

$$\begin{cases} 1 \le \dfrac{DS}{1+U_{\%}} \le 10 \\ or \\ 1 \le \dfrac{1+U_{\%}}{DS} \le 10 \end{cases} \tag{13}$$

It's clear that *F* is a 3D surface equation in the form:

$$Z = \frac{x}{1+y} \tag{14}$$

Hence the feasibility surface is like in Fig. 1.

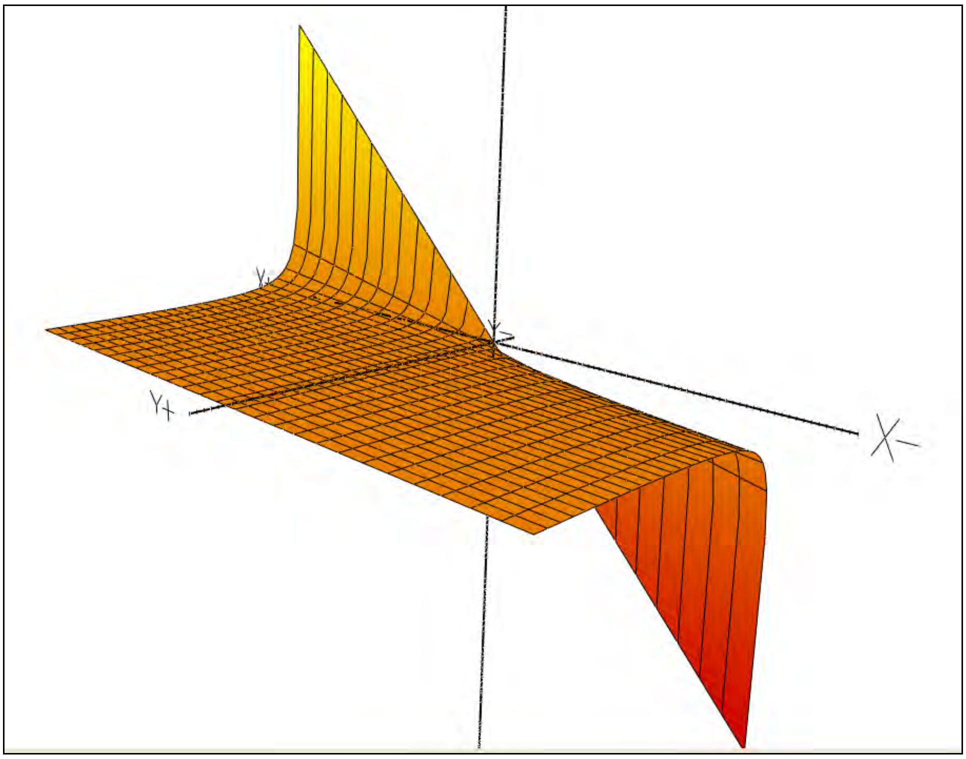


**Fig. 1** Feasibility surface as a function of control success (X-axis) and control uncertainty (Y-axis).

Since many solutions to network control can be found using the previous control models (2), (3) and (4), each *j-th* solution will receive its degree of feasibility

$$F_j = \frac{DS_j}{1+U_{j\%}} \tag{15}$$

and the best solution to network control will be the one with

$$max(F_j) \tag{16}$$

## 3 Conclusions

Taming ecological and biological networks is a key-issue. It could be used to: a) neutralize damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage ecological systems at the least possible cost, and d) counteract the impacts of climate change.

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