Local and global control of ecological and biological networks

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Abstract
Recently, I introduced a methodological framework so that ecological and biological networks can be controlled both from inside and outside by coupling network dynamics and evolutionary modelling. The endogenous control requires the network to be optimized at the beginning of its dynamics (by acting upon nodes, edges or both) so that it will then go inertially to the desired state. Instead, the exogenous control requires that exogenous controllers act upon the network at each time step. By the way, all my previous works dealt with the goal of the global optimization of ecological and biological networks, i.e. how to drive them to a desired final state. Here I face another pivotal question: how can we locally (step-by-step) drive ecological and biological networks, so that also intermediate steps (not only the final state) are under our strict control? The ratio behind this question is that intermediate dynamics could potentially go below or above critical ecological-biological thresholds, hence invalidating the final global control. To this purpose, I introduce here a modelling solution to the complete control of ecological and biological networks that couples local and global control.

Keywords edges control; genetic algorithms; global dynamics; local dynamics; network control; nodes control.

1 Introduction
Recently, I introduced a methodological framework so that ecological and biological networks (EBN from now on) can be controlled (Ferrarini, 2011a). EBN can be efficaciously tamed from the outside (Ferrarini, 2013a) but also through the use of endogenous controllers (Ferrarini, 2013b), by coupling network dynamics and evolutionary modelling. The endogenous control requires that the network is optimized at the beginning of its dynamics (by acting upon nodes, edges or both) so that it will then go inertially to the desired state. Instead, the exogenous control requires that exogenous controllers act upon the network at each cycle.

In another paper (Ferrarini, 2013c), I faced a further important question: how reliable is the achieved solution? In other words, which is the degree of uncertainty about getting the desired result if values of edges
and nodes were a bit different from optimized ones? This is an important question, because it’s not assured that while managing a certain system we are able to impose to nodes and edges exactly the optimized values we would need in order to achieve the desired results. In order to face this topic, I have coined a 3-parts framework (network dynamics - genetic optimization - stochastic simulations).

I have further introduced a solution to the choice of the most feasible solution to network control (Ferrarini, 2013d) by introducing the concepts of control success and feasibility.

By the way, all my previous works dealt with the goal of the global optimization of EBN, i.e. how to drive them to a desired final state. Here I face another pivotal question: how can we locally (step-by-step) drive ecological and biological networks, so that also intermediate steps (not only the final state) are under our control? The ratio behind this question is that intermediate dynamics could potentially go below or above critical ecological-biological thresholds, hence invalidating the final global control. To this purpose, I introduce here a solution to the complete control of EBN by coupling local and global controls.

2 Mathematical Formulation

Given a generic ecological (or biological) dynamic system with $n$ interacting actors

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t)$$

(1)

where $S_i$ is the amount (e.g., number of individuals, total biomass, covered surface etc…) of the generic $i$-th actor, if we also consider inputs (e.g. species reintroductions) and outputs (e.g. hunting) from-to outside, we must write:

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t) + \vec{I} + \vec{O}$$

(2)

As noted by numerous authors (Liu et al., 2011; Slotine and Li, 1991) real systems’ dynamics of eq. (2) can be modelled and simulated using a system of canonical, linear differential equations

$$\begin{align*}
\frac{dS_1}{dt} &= a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 \\
\vdots \\
\frac{dS_n}{dt} &= a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n
\end{align*}$$

(3)

that can be written in the compact form

$$\frac{d\vec{S}}{dt} = A\vec{S} + \vec{I} + \vec{O}$$

(4)

with initial values

$$\vec{S}_0 = (S_1(0), S_2(0)\ldots S_n(0))$$

(5)

and co-domain limits

$$\begin{align*}
S_{1\text{min}} \leq S_1(t) \leq S_{1\text{max}} \\
\vdots \\
S_{n\text{min}} \leq S_n(t) \leq S_{n\text{max}}
\end{align*}$$

(6)
and where

\[
A = \begin{pmatrix}
    a_{11} & \ldots & a_{1n} \\
    \vdots & \ddots & \vdots \\
    a_{n1} & \ldots & a_{nn}
\end{pmatrix}
\]  

(7)

is the matrix of the per unit-of-time effect on \( S_i \) due to unitary \( S_j \).

Such systems can be tamed from outside using the following 1-external-controller model (Ferrarini, 2013a):

\[
\begin{align*}
\left( \frac{dS_1}{dt} \right)_{OPT} &= a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11}*C_1* \\
\vdots \\
\left( \frac{dS_n}{dt} \right)_{OPT} &= a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{1n}*C_1* \\
\left( \frac{dC_1}{dt} \right) &= f_{11}S_1 + \ldots + f_{1n}S_n \\
\vdots \\
\left( \frac{dC_k}{dt} \right) &= f_{k1}S_1 + \ldots + f_{kn}S_n
\end{align*}
\]

(8)

where asterisks stand for the genetic optimization (Holland, 1975) of exogenous node’s edges (i.e., coefficients of interaction with the inner system) and exogenous node’s stock, i.e. the modification of such values at the beginning of network dynamics in order to get a certain goal (e.g., maximization of the final value of a certain variable). In case 1 controller is not enough, the model in (8) must be expanded to the following \( k \)-external-controllers model (Ferrarini, 2013a):

\[
\begin{align*}
\left( \frac{dS_1}{dt} \right)_{OPT} &= a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11}*C_1* + \ldots + c_{1k}*C_k* \\
\vdots \\
\left( \frac{dS_n}{dt} \right)_{OPT} &= a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{n1}*C_1* + \ldots + c_{nk}*C_k* \\
\left( \frac{dC_1}{dt} \right) &= f_{11}S_1 + \ldots + f_{1n}S_n \\
\vdots \\
\left( \frac{dC_k}{dt} \right) &= f_{k1}S_1 + \ldots + f_{kn}S_n
\end{align*}
\]

(9)

Alternatively, an ecological or biological network can be controlled from inside using the following control model (Ferrarini, 2013b):

\[
\begin{align*}
\left( \frac{dS_1}{dt} \right)_{OPT} &= a_{11}*S_1* + \ldots + a_{1n}*S_n* + I_1* + O_1* \\
\vdots \\
\left( \frac{dS_n}{dt} \right)_{OPT} &= a_{n1}*S_1* + \ldots + a_{nn}*S_n* + I_n* + O_n*
\end{align*}
\]

(10)
where asterisks stand for the optimization of edges (i.e., coefficients of interaction among variables) or nodes (i.e., initial stocks), i.e. the modification of their values at the beginning of the network dynamics in order to get a certain goal.

The control models in (8), (9) and (10) are able to globally drive any EBN to the desired final state with an uncertainty degree that can be calculated as proposed in Ferrarini (2013c), and a feasibility degree as in Ferrarini (2013d).

Now, let’s assume that we want to tame intermediate dynamics of EBN as well, not only the final state. By introducing an intermediate control function \( ICF \), I conceive several possibilities.

a) we want the generic actor \( S_i(t) \) to stay as close as possible to a certain value during its dynamics. In this case, \( ICF \) must be as follow

\[
ICF = \int_{t=0}^{E} \left| k - S_i(t) \right| dt
\]  

(11)

where \( t \) is time (independent variable), \( E \) indicates the time at which the system goes to equilibrium, \( k \) is the value at which we want \( S_i(t) \) to be tied with, vertical lines indicates the module of the difference.

As a result we can reach our goal by optimizing our network using (8), (9) or (10) in order to achieve the minimization of \( ICF \).

b) we want the generic actor \( S_i(t) \) to stay as close as possible to a certain value but only during some steps of its dynamics.

In the previous step, I have subdued each dynamical step of \( S_i(t) \) from the beginning to the its final state. It could be possible that we want to control \( S_i(t) \) just in a certain time interval. In this case, eq. (11) becomes

\[
ICF = \int_{t=n}^{m} \left| k - S_i(t) \right| dt
\]  

(12)

where \( n \) and \( m \) are generic points along the dynamic timeline. Again, eq. (12) must be minimized using (8), (9) or (10) in order to achieve our goal.

c) we want to tie the generic actor \( S_i(t) \) to a certain dynamic \( f(t) \) (e.g. exponential). In this case, we need the following equation

\[
ICF = \int_{t=0}^{E} \left| f(t) - S_i(t) \right| dt
\]  

(13)

As a result, by minimizing \( ICF \) using (8), (9) or (10) we can constrain \( S_i(t) \) as close as possible to the desired dynamic function.

d) we want to tie the generic actor \( S_i(t) \) to another network’s actor \( S_j(t) \). In this case, we need the following equation to be minimized

\[
ICF = \int_{t=0}^{E} \left| S_j(t) - S_i(t) \right| dt
\]  

(14)

e) we want to tie the generic actor \( S_i(t) \) to a function of another network’s actor \( S_j(t) \). For instance we could want \( S_i(t) \) to be always twice as \( S_j(t) \) during network dynamics. In this case, we need the following equation to be minimized using (8), (9) or (10)
\[ ICF = \sum_{t=0}^{E} \left[ f(S_j(t)) - S_j(t) \right] dt \]

(15)

f) we want that the generic actor \( S_i \) changes from step to step of a certain value \( u \). To this aim, I use

\[ ICF = \sum_{t=0}^{E} \left[ \frac{dS(t)}{dt} - u \right] dt \]

(16)

By minimizing \( ICF \) with (8), (9) or (10), we can impose that \( S_i \) changes of exactly \( u \) at each time step.

3 An Applicative Example

Fig. 1 shows an ecological network borrowed with modification from Ferrarini (2012). Greenish nodes represent positive actors (or events) for the goal of network control, i.e. the conservation of the target species (in green at centre of the network). Reddish nodes represent ecological actors (or events) with negative impact on the target species. Blueish nodes represent resources needed by the target species. The goal is to preserve target species’ occurrence in the study area. Stocks stand for the actual amounts of individuals or biomass. Updates (indicated as \( upd \)) stand for yearly internal dynamics (i.e., intraspecific gains due to births and/or immigration rates minus losses due to deaths and/or emigration rates). Minimum and maximum values stand for lowest and highest values of stock values. For the sake of simplicity, the maximum possible value for each actor (in italic hereafter) has been set to 100. The percent value associated to links represents the percentage of the receiver that is yearly consumed by the transmitter at the beginning of the network simulation. Road mortality and re-introductions accounts for 18 and 10 individuals per year respectively (average values of the last 5 years of field observations).

Fig. 1 The ecological network on which evolutionary control has been applied.
Since data are yearly-based, I have calculated inertial dynamics and optimized the above ecological system using a system of difference recurrent equations, instead of differential ones.

Table 1 The interaction matrix relative to the ecological network of Fig. 1.

<table>
<thead>
<tr>
<th>Transmitters</th>
<th>target</th>
<th>prey1</th>
<th>prey2</th>
<th>pred1</th>
<th>pred2</th>
<th>hunters</th>
<th>var G</th>
<th>var H</th>
<th>var I</th>
<th>var J</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>1.3</td>
<td>-0.12</td>
<td>-0.15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>prey1</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>prey2</td>
<td>0</td>
<td>0</td>
<td>1.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>pred1</td>
<td>-0.35</td>
<td>0</td>
<td>0</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>pred2</td>
<td>-0.2</td>
<td>0</td>
<td>0</td>
<td>-0.9/5</td>
<td>1.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>hunters</td>
<td>-0.2</td>
<td>0</td>
<td>0</td>
<td>-0.125</td>
<td>-0.09</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>var G</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>var H</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>var I</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>var J</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The previous ecological network has the following inertial dynamics (Fig. 2), with the target species (green line) going extinction after 7 years.

![Fig. 2 Resulting dynamics for the network of Fig. 1. X-axis measures time in years.](image_url)

4 Local Network Control
Previous research revealed that the optimal solution may be to search at a high rate of crossover, a low rate of mutation and proper population size (Kuo et al., 2000). In this study, crossover was set at a probability of 60% while mutations occur with a probability of 5%. This low setting helps to avoid getting trapped local optima.
during the search (D’heygere et al., 2006). The initial population consisted of 5000 chromosomes that were evolved over minimal 10,000 generations. I applied a steady-state genetic algorithm with a one-point crossover operator (Wall, 1996) to accomplish crossover. In this case the parent genome strings are cut at some random position to produce two “head” and two “tail” segments. The “tail” segments are swapped to produce two new genomes. For parent selection the roulette wheel selection method was used (Goldberg, 1989), where the likelihood of selection is proportionate to the fitness score given by the performance criterion. After crossover and mutation, the individuals with the lowest fitness scores were removed.

![Optimized system dynamics](image)

**Fig. 3** By using both local and a global optimizations, target species has been constrained close to the stock value 50 for 20 years before going to its maximum.

Now let’s suppose we want to stabilize the target species to a steady value equal to 50 for at least 20 years before it goes to its maximum. Let’s suppose we want to reach it by acting upon target species’ intraspecific coefficient (actually equal to 1.30). To this aim, I used equations (10) and (12) and, being all other simulation parameters equal, I achieved the results of Fig. 3 where, by setting the intraspecific coefficient equal to 1.36 (solution of the optimization problem), target species is constrained to stock values close to 50 for 20 years before going to its maximum. It is not goal of this paper to discuss the ecological and biological meaning of such constrained dynamics. I just want to expose here the methodological aspects of local and global networks control.

Now let’s suppose we want to tie target species’ dynamics to pred2’s dynamics so that they are as close as possible as time goes on, and so that they both go to their maximum (i.e. 100) at equilibrium. To this aim, I have used equations (10) and (14) and achieved the optimized matrix of Table 2 and the optimized dynamics of Fig. 4. As it can be seen, I was able to achieve a perfect solution (green and red lines are overlying and they both go to 100 at equilibrium).
Table 2 Optimized matrix in order to tie target species’ dynamics to pred2’s ones, and so that they both go to their maximum. The corresponding dynamics are depicted in Fig. 4.

<table>
<thead>
<tr>
<th></th>
<th>target</th>
<th>prey1</th>
<th>prey2</th>
<th>pred1</th>
<th>pred2</th>
<th>hunters</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>1.81</td>
<td>-0.41</td>
<td>0.25</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>prey1</td>
<td>0.00</td>
<td>1.43</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>prey2</td>
<td>0.00</td>
<td>0.00</td>
<td>1.95</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>pred1</td>
<td>-0.14</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>pred2</td>
<td>-0.15</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.00</td>
<td>1.44</td>
<td>0.00</td>
</tr>
<tr>
<td>hunters</td>
<td>-0.61</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.32</td>
<td>-0.16</td>
<td>1.00</td>
</tr>
</tbody>
</table>

As last example, let’s suppose we want the target species to go to its maximum with a smooth dynamic (i.e. small increments), let’s say close to 2.5 at each time step. To this aim, I used equations (10) and (16), and I achieved the optimized matrix of Table 3 and the optimized dynamics of Fig. 5. As it can be seen, target species increases smoothly from its initial stock to the final one as it was purposed. The target species’ initial value was set to 43 instead of 50 by the optimization model because I stopped GAs simulations after 5 minutes, so the methodological framework was not able to detect a perfect solution, but only something very close to it.

It’s clear from these examples that, using the proposed methodological framework, there’s no limit to the kind of network control we can search for. It’s also clear that the same goal can be achieved via alternative and competing optimized solutions that my framework is able to detect.
Table 3 Optimized matrix that produces the corresponding dynamics depicted in Fig. 5.

<table>
<thead>
<tr>
<th></th>
<th>target</th>
<th>prey1</th>
<th>prey2</th>
<th>pred1</th>
<th>pred2</th>
<th>hunters</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>1.46</td>
<td>-0.78</td>
<td>-0.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>prey1</td>
<td>0.00</td>
<td>1.81</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>prey2</td>
<td>0.00</td>
<td>0.00</td>
<td>1.82</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>pred1</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>1.30</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>pred2</td>
<td>-0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.48</td>
<td>1.10</td>
<td>0.00</td>
</tr>
<tr>
<td>hunters</td>
<td>-0.35</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.32</td>
<td>-0.14</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Fig. 5 Optimized dynamics where target species goes smoothly to its maximum increasing by about 2.5 at each time step.

Equations from (1) top (16) are powerful enough to allow for a complete control of EBN. Not only, in fact, we can achieve global control by attaining the desired final state, but we can also constrain local network’s dynamics, i.e. intermediate states.

As a rule-of-thumb, I suggest that as the local control becomes more and more complex (e.g. we want to locally and globally control more than one variable), one should use a wider control on more switches. For instance one could start by controlling only initial stocks if the optimization goal is simple to achieve, but for tougher goals one might enlarge the control to intra-specific and inter-specific parameters or input-output flows as well. The result is an ecological or biological network with customized local and global dynamics. The framework proposed here might also be applied to semi-quantitative and qualitative networks (Ferrarini, 2011b; Ferrarini, 2011c).
5 Conclusions
The control of EBN is a pivotal topic. Its applications are unlimited, for instance: a) neutralize damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage ecological systems at the least possible cost, and d) counteract the impacts of climate change.

While in previous papers, I showed how to globally control EBN, here I introduced a modelling framework for their total control, by adding local constraints to variables’ dynamics. The ratio is that sometimes intermediate dynamics could go below or above critical ecological-biological thresholds, hence invalidating the final global control. By locally controlling EBN, this potential drawback is fixed.

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