Article

Evolutionary Network Control also holds for nonlinear networks: Ruling the Lotka-Volterra model

Alessandro Ferrarini
Department of Evolutionary and Functional Biology, University of Parma, Via G. Saragat 4, I-43100 Parma, Italy
E-mail: sgtpm@libero.it, alessandro.ferrarini@unipr.it

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Abstract
The proof of our understanding of ecological and biological systems is measured by our skill to rule them, i.e. to channelize them towards a desired state. Control is a cardinal issue in most complex systems, but because a general theory to apply it in a quantitative manner has been absent so far, little was known about how we can rule weighted, directed networks that represent the most common configuration of real systems. To this purpose, Evolutionary Network Control (ENC) has been developed as a theoretical and methodological framework aimed to the control of ecological and biological networks by coupling network dynamics and evolutionary modelling. ENC is a tools to address controllability for arbitrary network topologies and sizes. ENC has proven to cover several topics of network control, e.g. a) the global control from inside and b) from outside, c) the local (step-by-step) control, and the computation of: d) control success, e) feasibility, and f) degree of uncertainty. Taken together, these results indicate that many aspects of controllability can be explored exactly and analytically for arbitrary networks, opening new avenues to deepening our understanding of complex systems. As yet, I have applied ENC only to linear ecological and biological networks. In this work, I show that ENC also holds for any kind of nonlinear networks, and provide an applicative example based on the nonlinear, widely-used, Lotka-Volterra model.

Keywords Evolutionary Network Control; genetic algorithms; global dynamics; nonlinear networks; predator-prey model; sensitivity analysis; stochastic simulations.

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1 Introduction

I have recently introduced a methodological framework, named Evolutionary Network Control (ENC; Ferrarini, 2013a; Ferrarini, 2013b), so that ecological and biological networks can be controlled from the outside (Ferrarini, 2013a) but also through the use of endogenous controllers (Ferrarini, 2013b), by coupling network dynamics and evolutionary modelling (Holland, 1975; Goldberg, 1989). The endogenous control requires that the network is optimized at the beginning of its dynamics, by acting upon nodes, edges or both, so that it will inertially go to the desired state. On the contrary, the exogenous control requires that one or more exogenous controllers act upon the network at each time step (Ferrarini, 2011a; Ferrarini, 2011b).

In another paper (Ferrarini, 2013c), I have examined a further issue: how reliable is the achieved solution? This is an important question, because it's not assured that while managing a network-like system we are able to impose to nodes and edges exactly the optimized values we would need in order to achieve the desired control. In order to face this topic, I have coined a 3-parts (network dynamics - genetic optimization - stochastic simulations) solution.

I have further proposed a solution to the choice of the most feasible solution to network control by introducing the concepts of control success and feasibility (Ferrarini, 2013d).
Later, I have faced another pivotal question, i.e. how to locally (step-by-step) drive ecological and biological networks so that also intermediate steps (and not only the final state) are under our control (Ferrarini, 2014). The ratio behind this question is that intermediate dynamics could potentially go below or above critical ecological or biological thresholds, hence invalidating the final global control. To this purpose, I have introduced a solution to the complete (local + global) control of ecological and biological networks by making use of an intermediate control function.

As yet, I have applied ENC only to linear ecological and biological networks. While it is not goal of this paper to discuss the implications of the ENC of ecological and biological networks, I show here that ENC also holds for any kind of nonlinear networks, and provide an applicative example based on the nonlinear, widely-used, Lotka-Volterra model (Lotka, 1925; Volterra, 1926).

2 Subduing the Lotka-Volterra Model via ENC: Mathematical Formulation

It's given a generic ecological (or biological) dynamical system with *n* interacting actors

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t) \tag{1}$$

where S_i is the amount (e.g., number of individuals, total biomass, density, covered surface etc…) of the generic *i-th* actor.

If we also consider inputs (e.g. species reintroductions) and outputs (e.g. hunting) from-to outside, we must write

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t) + \vec{I}(t) + \vec{O}(t) \tag{2}$$

with initial values

$$\vec{S}_0 = \langle S_1(0), S_2(0) \ldots S_n(0) \rangle \tag{3}$$

and co-domain limits

$$\begin{cases} S_{1\min} \le S_1(t) \le S_{1\max} \\ \ldots \\ S_{n\min} \le S_n(t) \le S_{n\max} \end{cases} \quad \forall t \tag{4}$$

The Lotka-Volterra equations (Lotka, 1925; Volterra, 1926), otherwise known as the predator-prey equations, are a combination of two first-order, non-linear, differential equations frequently used to describe the dynamics of biological systems with two interacting species, one as a prey and the other as a predator.

The Lotka-Volterra model makes five assumptions about the environment and the dynamics of the two interacting species: 1) the prey population finds food at any times; 2) the food supply for the predator depends on the size of the prey population; 3) the rate of change of each population is proportional to its size; 4) while interacting, the environment does not change; 5) predators have unbounded appetency. As differential equations are used, the solution is deterministic and continuous so that the generations of both the predator and prey persistently overlap.

The nonlinear Lotka-Volterra model with logistic grow of the prey S_1 is a particular case of eq. (1) and it reads as follows

$$\begin{cases} \frac{dS_1}{dt} = \alpha S_1 (1 - \frac{S_1}{\kappa}) - \beta S_1 S_2 \\ \frac{dS_2}{dt} = \beta\gamma S_1 S_2 - \delta S_2 \end{cases} \tag{5}$$

with initial values

$$\vec{S}_0 = <S_1(0),\ S_2(0)> \tag{6}$$

and co-domain limits

$$\begin{cases} S_{1\min} \le S_1(t) \le S_{1\max} \\ S_{2\min} \le S_2(t) \le S_{2\max} \end{cases} \quad \forall t \tag{7}$$

In order to get the global control of such model, ENC can act upon the previous Lotka-Volterra model as follows

$$\begin{cases} \frac{d\tilde{S}_1}{dt} = \tilde{\alpha} S_1 (1 - \frac{S_1}{\tilde{\kappa}}) - \tilde{\beta} S_1 S_2 \\ \frac{d\tilde{S}_2}{dt} = \tilde{\beta}\tilde{\gamma} S_1 S_2 - \tilde{\delta} S_2 \\ \tilde{\vec{S}}_0 = <\tilde{S}_1(0),\ \tilde{S}_2(0)> \end{cases} \tag{8}$$

with steady values at

$$\frac{dS_1}{dt} = \frac{dS_2}{dt} = 0 \tag{9}$$

where the tilde symbol means that the ENC is active over such actors by controlling equation parameters and initial values.

The control equations in (8) are able to globally drive any ecological and biological network, and the nonlinear Lotka-Volterra as a particular case, to the desired final state with an uncertainty degree that can be calculated as proposed in Ferrarini (2013c).

$$\begin{cases} \frac{d\tilde{S}_1}{dt} = \underline{\tilde{\alpha}} S_1 (1 - \frac{S_1}{\underline{\tilde{\kappa}}}) - \underline{\tilde{\beta}} S_1 S_2 \\ \frac{d\tilde{S}_2}{dt} = \underline{\tilde{\beta}}\underline{\tilde{\gamma}} S_1 S_2 - \underline{\tilde{\delta}} S_2 \\ \tilde{\tilde{S}}_0 = < \underline{\tilde{S}}_1(0), \underline{\tilde{S}}_2(0) > \end{cases} \quad (10)$$

where the underscores represent 1%, 5% or 10% uncertainties about the optimized parameters. Thus for example:

$$\begin{cases} 0.99 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.01 * \tilde{\alpha} \\ or \\ 0.95 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.05 * \tilde{\alpha} \\ or \\ 0.90 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.10 * \tilde{\alpha} \end{cases} \quad (11)$$

If we stochastically vary *n* times (e.g. 10,000 times) the parameters that have been optimized via ENC, we can compute how many times such uncertainty makes the optimization procedure useless. Hence, uncertainty about network control can be computed as in Ferrarini (2013c)

$$U_{\%} = 100 * \frac{k}{n} \quad (12)$$

where *k* is the number of stochastic simulations acting upon the optimized parameters that make the optimization procedure useless (i.e. the goal of the optimization procedure is not reached).

In order to apply the ENC framework, the software Control-Lab (Ferrarini, 2013e) has been developed using Visual Basic (Balena, 2001; Pattinson, 1998).

3 An Applicative Example

Let’s consider the Lotka-Volterra system of eq. (5) with the following parameters and constants:

$$\begin{cases} S_1(0) = 100 \\ S_2(0) = 10 \\ \alpha = 4 \\ \beta = 0.05 \\ \gamma = 1 \\ \delta = 4 \\ \kappa = 500 \\ dt = 0.01 \end{cases} \quad (13)$$

Fig. 1 shows its dynamical behaviour. Figure 2 depicts its phase plot.

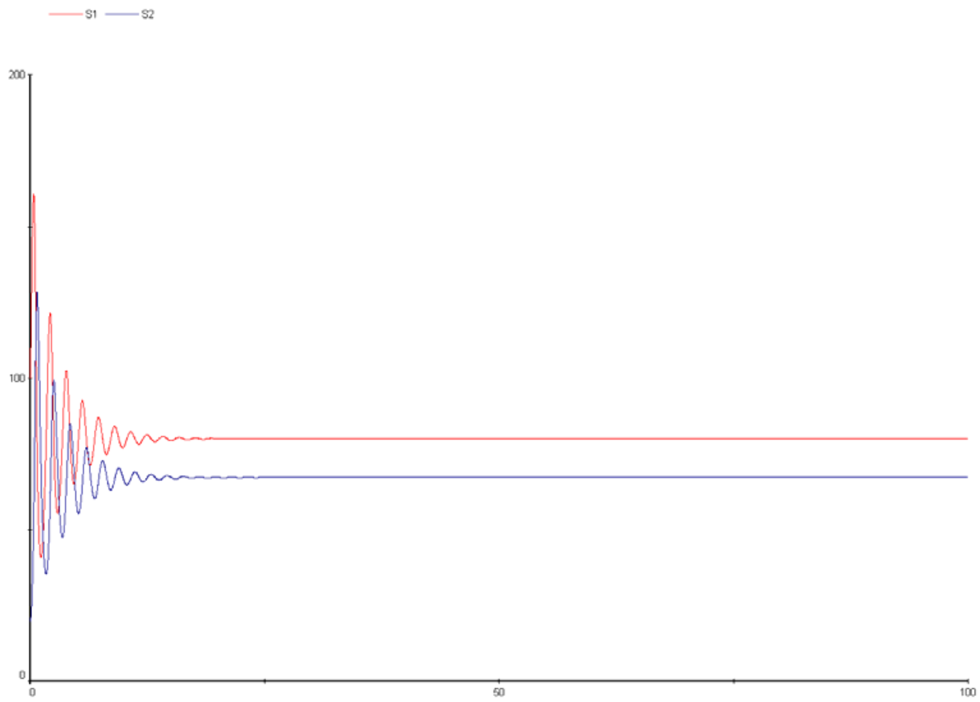

Fig. 1 Time plot of the above-depicted nonlinear Lotka-Volterra dynamical system.

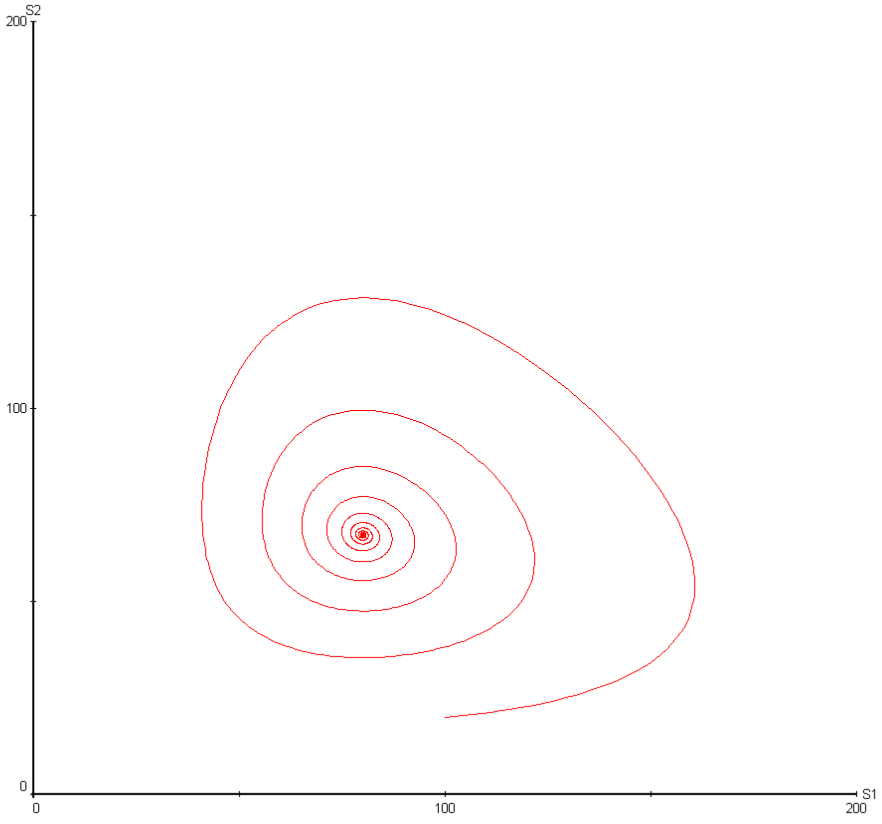

Fig. 2 Phase plot of the above-depicted nonlinear Lotka-Volterra dynamical system.

The previous nonlinear system goes at the steady state with S_1= 80.00 and S_2= 67.20.

Let's suppose we want that both the prey and the predator go to equilibrium with values close to 100. Fig. 3 shows the optimized solution (S_1= 99.45 and S_2= 99.91) detected via ENC by acting upon alpha, beta, gamma and delta. The steady state happens at t= 42.15.

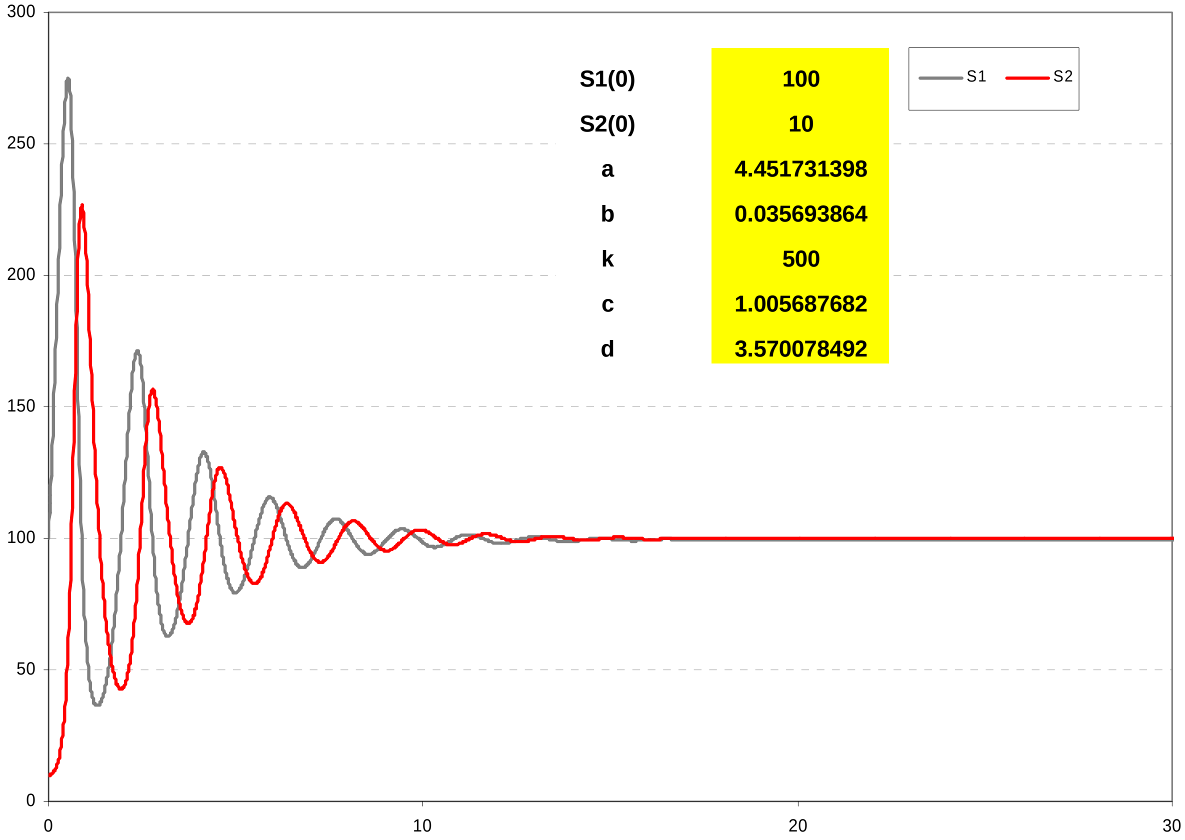

Fig. 3 The ruled Lotka-Volterra system with equilibrium values S_1= 99.45 and S_2= 99.91. The optimized parameters detected via Evolutionary Network Control are indicated above the two curves. The solution has been found through Control-Lab (Ferrarini, 2013e).

After 10,000 simulations (1% uncertainty) on the optimized parameters (alpha, beta, gamma and delta), we achieve the results of Table 1. It is evident that the solution depicted in Fig. 3 is robust with regard to a 1% sensitivity analysis, as mean and median values of the 10,000 simulations are very close to the desired solution S_1= S_2= 100.

Table 1 Sensitivity analysis (1% uncertainty) on the optimized parameters of Fig. 3.

Statistics	S_1	S_2
min	96.73	97.98
max	102.38	101.72
mean	99.52	99.92
median	99.57	99.92
std. dev.	1.02	0.74

Now let's suppose we want that both the prey and the predator go to equilibrium with values close to 50. Fig. 4 shows the optimized solution (S_1= 50.96 and S_2= 50.00) found via ENC by acting upon alpha, beta, gamma and delta. The steady state happens at t= 74.48.

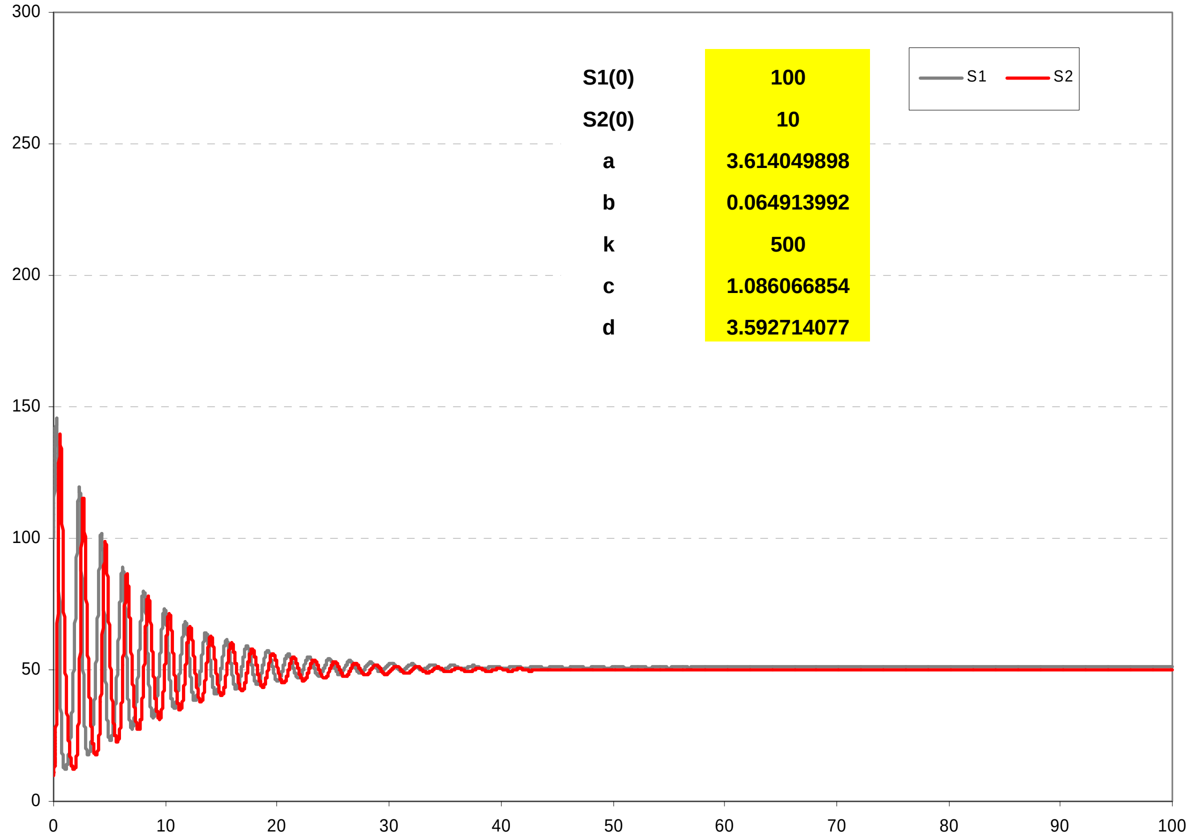

Fig. 4 Ruled Lotka-Volterra system with equilibrium values S_1= 50.96 and S_2= 50.00. The optimized parameters detected via Evolutionary Network Control are indicated above the two curves. The solution has been found through the software Control-Lab (Ferrarini, 2013e).

After 10,000 simulations (5% uncertainty) on the optimized parameters (alpha, beta, gamma and delta), we achieve the results of Table 2. It is clear that the solution depicted in Fig. 4 is robust with regard to a 5% sensitivity analysis, as mean and median values of the 10,000 simulations are very close to the desired solution S_1= S_2= 50.

Table 2 Sensitivity analysis (5% uncertainty) on the optimized parameters of Fig. 4.

Statistics	S_1	S_2
min	44.55	45.50
max	58.45	55.12
mean	50.94	49.95
median	50.85	49.91
std. dev.	2.52	1.95

Last, let's suppose we require that the prey and the predator go to equilibrium with $S_1+S_2= 90$. Fig. 5 shows the optimized solution ($S_1= 54.92$ and $S_2= 35.23$) detected via ENC by acting upon alpha, beta, gamma and delta. The steady state happens at $t= 95.00$.

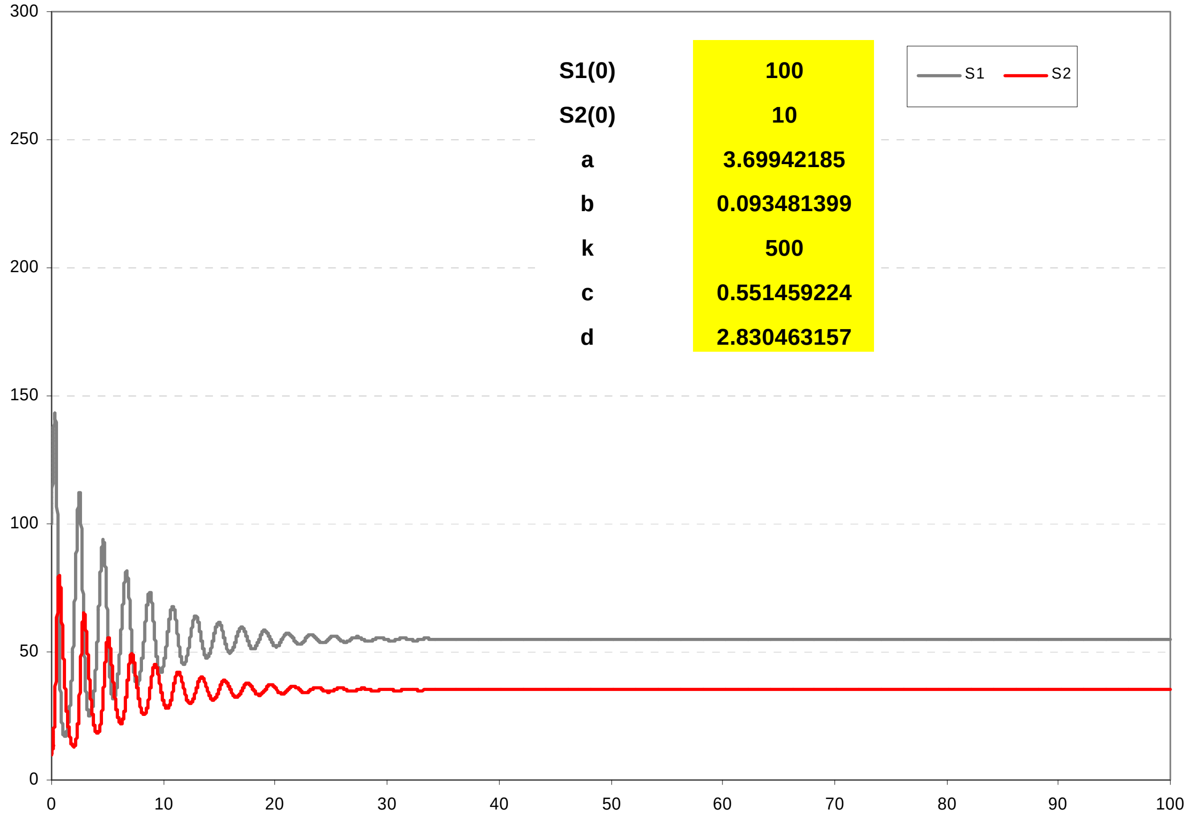

Fig. 5 Ruled Lotka-Volterra system with equilibrium values $S_1= 54.92$ and $S_2= 35.23$ and sum equal to 90.15. The optimized parameters detected via Evolutionary Network Control are indicated above the two curves. The solution has been found through the software Control-Lab (Ferrarini, 2013e).

After 10,000 simulations (10% uncertainty) on the optimized parameters (alpha, beta, gamma and delta), we achieve the results of Table 3. The solution depicted in Fig. 5 is robust with regard to a 10% sensitivity analysis, as mean and median values of the 10,000 simulations are approximately equal to the desired solution $S_1+S_2= 90$.

Table 3 Sensitivity analysis (10% uncertainty) on the optimized parameters of Fig. 5.

Statistics	S_1	S_2
min	42.12	29.52
max	71.29	42.14
mean	55.22	35.27
median	54.91	35.13
std. dev.	5.66	2.65

Any other kind of network control is feasible using ENC, including the control of Lotka-Volterra models with $n>2$ actors. ENC can also be employed to impose early (or late) stability to Lotka-Volterra models in particular, but also to any arbitrary ecological and biological networks more in general (Ferrarini, 2015).

4 Conclusions

The control of ecological and biological networks has unlimited applications: a) neutralize damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage ecological systems at the least possible cost, d) counteract the impacts of climate change.

While in previous papers I have showed how to globally and locally rule linear ecological and biological networks, here I have showed that Evolutionary Network Control is on top of the control of nonlinear networks as well.

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