Article

Centrality measures for immunization of weighted networks

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Abstract
Effective immunization of individual communities with minimal cost in vaccination has made great discussion surrounding the realm of complex networks. Meanwhile, proper realization of relationship among people in society and applying it to social networks brings about substantial improvements in immunization. Accordingly, weighted graph in which link weights represent the intensity and intimacy of relationships is an acceptable approach. In this work we employ weighted graphs and a wide variety of weighted centrality measures to distinguish important individuals in contagion of diseases. Furthermore, we propose new centrality measures for weighted networks. Our experimental results show that Radiality-Degree centrality is satisfying for weighted BA networks. Additionally, PageRank-Degree and Radiality-Degree centralities showmoreacceptable performance in targeted immunization of weighted networks.

Keywords centrality measure; epidemic threshold; largest connected components; targeted immunization; weighted graphs.

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1 Introduction

Epidemic is 'the occurrence of more cases of disease than expected in a given area or among a specific group of people over a particular period of time' (Parthasarathy, 2013). Nowadays, nearly 13 million people die annually by infectious diseases which imposes high costs to society. Between 2009 and 2010, 14,000 people lost their lives as a result of an influenza epidemic (Jain et al., 2009). Since 1981, AIDS is known as an epidemic disease and according to estimates by WHO and UNAIDS only in 2013, it was the cause of death of 1.5 million people in the world. As a result, prediction and control of epidemics among populations has attracted many researchers.

Immunization prevents diseases from spreading and saves many people from suffering and death (Parthasarathy, 2013). Vaccination is one of the immunization techniques that protects the vaccinated person as well as prevents the spread of disease, simultaneously (Cornforth et al., 2011). In traditional methods (mass vaccination) the entire population will be vaccinated, which is not affordable due to high costs (Gallos et al., 2007; Vidondo et al., 2012). A more effective method is targeted vaccination in which people are clustered by some known criteria. In these categories people with more influence are identified and vaccinated (Chen et al., 2008; Cornforth et al., 2011; Eames et al., 2009; Hartvigsen et al., 2007; Miller and Hyman, 2007; Peng et al., 2010; Schneider et al., 2012; Shams and Khansari, 2014; Vidondo et al., 2012).The purpose of targeted vaccination is that people are selected in such a way that unsafe clusters are as small as possible (Schneider et al., 2012). In other words, the work is done by immunization with intercommunal individuals.

Complex networks has become a suitable tool for modeling, representing and describing the characteristics of many natural phenomena (Zhgang and Zhan, 2011; Zhang, 2012). Among these networks, social networks can be outlined in which each node represents an individual while communication between a pair of individuals is represented as a link. As a result, many studies have been done on disease distributions and immunization within these networks (Cornforth et al., 2011; Hartvigsen et al., 2007; Shams and Khansari, 2014). In these studies different centralities have been defined and nodes with high centrality have been vaccinated and the result of the vaccination has been analyzed.

Although representation of social networks with simple graphs leads to suitable models, adopting weighted networks can be more appropriate to include detailed information. In weighted social networks, the quality of communication or relationship between a pair of nodes is considered as the weight of the edge between them. Some works which have studied centrality measures in weighted networks are (Abbasi and Hossain, 2013; Abbasi et al., 2011; Wang et al., 2008; Zhai et al., 2013).

Shams and Khansari have proposed an evaluation framework for targeted immunization algorithms in which some centrality measures are utilized (Shams and Khansari, 2014). These centrality measures have been used to distinguish effective nodes, though other meaningful and powerful ones are not considered.

To our knowledge, a complete study in capability of various centrality measures in detection of effective nodes in weighted networks has not been done. Thus, we conduct a comprehensive study to determine the effectiveness of different centrality measures in detecting important nodes within weighted networks. In this study we propose new centrality measures while comparing them with the current existing measures. All the evaluations are done on real and artificial networks.

The rest of this paper is organized as follows: section 2 presents current existing centrality measures for weighted networks, following four new centrality measures which are introduced in section 3. Model evaluation methods and analysis are introduced in section 4 while experimental results are presented in the following section 5. Finally, Section 6 concludes this work.

2 Centrality Measures for Weighted Networks

Node centrality is one of the most analyzed concepts which determines the node effect on network flows. Centrality of a node represents its prominence based on the definition of centrality. In a social network node centrality exhibits the individuals' communication pattern and his/her position in the network.

Leavitt defined this concept for the first time in 1951 (Leavitt, 1951)and Freeman defined three important and applicable centralities: *degree*, *closeness* and *betweenness* in 1978 (Freeman, 1978). In the following subsections current existing centrality measures are reviewed with emphasis on weighted networks.

2.1 Degree centrality

Degree centrality is a simple local centrality which is defined based on neighborhood concept. In a social network it exhibits a node popularity and in our examinations it shows the node influence in spreading and suppression of diseases. In weighted networks, a node degree centrality is the sum of the weights of the edges attached to that node which is also nominated as the node *strength*. This measure represents the whole involvement of a node in the network (Opsahl et al., 2010):

$$C_D^W(i) = \sum_{j=1}^{N} W(i,j) \tag{1}$$

where, W is the weighted adjacency matrix and $w(i,j)$ is the weight of edge which ties node $i$to j.

2.2 Distance-based Centralities

Other centralities are distance-based measures and have a close relationship with the concept of "path"in weighted networks. In contrast to binary networks, different values can be assigned to different links in weighted networks. Edge weight can represent cost (Dijkstra, 1959) as well as strength of connection (Newman, 2001). In social networks edge weight represents strength of physical connection or sense of intimacy between individuals. Hence, distance between two adjacent nodes is the inverse of the weight of their corresponding link. Thus to calculate the geodesic path we use the method proposed in (Newman, 2001). Distance-based centralities are as follows:

2.2.1 Closeness centrality

Closeness centrality is a global centrality which represents the independence of a node in the network (Freeman, 1978). The most central node has the minimum sum of the 'geodesic' distances to all other nodes in the network:

$$C_C^W(i) = \left(\sum_j d^w(i,j)\right)^{-1} \tag{2}$$

where, $d^w(i,j)$ is the weighted geodesic path between nodes$i$and j.

2.2.2 Betweenness centrality

Betweenness centrality represents the node's ability to control the data flow in the network (Freeman, 1978). This measure is the proportion of number of geodesic paths that pass through the given node to number of geodesic paths between any pair of nodes in the network:

$$C_B^W(i) = \frac{g_{jk}^w(i)}{g_{jk}^w} \tag{3}$$

where, $g_{jk}^w(i)$is the number of weighted geodesic paths which pass through node i and g_{jk}^w is the number of weighted geodesic paths between two nodes.

2.2.3 Radiality centrality

Radiality represents the extent of access into the network privided by the node's neighbors (Valente and Foreman, 1998). High radiality means that it takes less time for the infectious node to reach others in the network:

$$C_{Rad}^W(i) = \frac{\sum_{j\in V}(\Delta_G^W + 1 - d^w(i,j))}{g_{jk}^w} \tag{4}$$

where, Δ_G^W is the diameter of graph G, V is the set of nodes in graph G, $d^w(i,j)$ is the weighted geodesic path between nodes$i$ and j and $|V| = n$.

2.2.4 Katz centrality

Katz centrality is the generalization of degree centrality. In other words, a node's degree centrality is the number of its neighbors while Katz centrality is the number of nodes which are accessible through a specific path. However, contribution of distant nodes is reduced:

$$C_{Katz}^{W}(i) = \alpha \sum_j W(i,j) \,.\, C_{Katz}^{W}(j) + \beta \quad (5)$$

where, $W(i,j)$ is the element of row i and column j of weighted adjacency matrix.α, β are positive constants where, α is an attenuation factor,$0 \leq \alpha \leq 1$(Katz, 1953). By $\beta > 0$, nodes with zero in-degree (in directed networks) get positive centrality.

2.2.5 Subgraph centrality

This measure is based on spectral properties and characterizes the contribution of a node in different possible subgraphs. Subgraph centrality of a node is the sum of closed paths with different lengths which launch from and terminate in that node. Shorter path lengths have more influence in the calculations (Estrada and Rodriguez, 2005):

$$C_{SG}^{W}(i) = \sum_{k=0}^{\infty} \frac{\mu_k^W(i)}{k!} \quad (6)$$

where, $\mu_k^w(i)$ is the number of closed walks of cost k starting and ending at node i.

2.2.6 Eccentricity centrality

A node's eccentricity centrality is the inverse of the longest geodesic path from that node to other nodes. Hence, a node with the smallest longest geodesic path is the most important node in network. In social networks this centrality exhibits how distant, at most, is each person from others.

$$C_{Ecc}^{W}(i) = max\{d^w(i,j)\}^{-1} \quad (7)$$

where, $d^w(i,j)$ is the weighted shortest path between nodes$i$and j.

2.2.7 Communication centrality

Communication centrality represents the capability of a node to communicate with other nodes. This measure depends on node's degree, communication ability of neighbors and node's edges weight. Communication centrality of a node is defined based on multiplying the *h-degree* of a neighbor in the weight of the edge connecting the node to that neighbor.

Zhai et al. defined the communication centrality of node x as "the largest integer k such that the node x has at least k neighbor nodes satisfying the product of each node's *h-degree* and the weight of the edge linked with node x is no fewer than k" (Zhai et al., 2013). If the *h-degree* of node's neighbors are marked as $d_h(n_1)$, $d_h(n_2)$,..., $d_h(n_M)$ and the node's edge weight are w_1, w_2, …, w_M then their product sequence is:

$$w_1 d_h(n_1), w_2 d_h(n_2), \dots, w_M d_h(n_M)$$

If we suppose:

$$w_1 d_h(n_1) \geq \; w_2 d_h(n_2) \geq \; \dots \geq w_M d_h(n_M)$$

Then,

$$C_{Com}(i) = \max\{k\colon w_k d_h(n_k) \geq k\} \quad (8)$$

2.3 Spectral centralities

Spectral centrality measures are those that their definition and calculation relies on the eigenvalues and the eigenvectors of adjacency and Laplacian matrices of the graph. The most well-known two are:

2.3.1 Laplacian centrality

Laplacian centrality is based on the Laplacian matrix. This matrix contains practical information about dynamics and geometry of the network (Pauls and Remondini, 2012). Laplacian matrix is defined as:

$$L(G) = X(G) - W(G)$$

where, $X(G)$ is the graph's degree matrix and $W(G)$ is the graph's weighted adjacency matrix. If we nominate eigenvalues of $L(G)$as λ_1, λ_2,..., λ_n the Laplacian energy of graph G is:

$$E_L(G) = \sum_{i=1}^{n} \lambda_i^2$$

A node's *Laplacian* centrality is the difference between Laplacian energy of network with and without that node (Qi et al., 2012):

$$E_L(v_i, G) = \frac{E_L(G) - E_L(G_i)}{E_L(G)} \quad (9)$$

2.3.2 PageRank centrality

This centrality measure represents a node's relative importance in the network (Sarma et al., 2011). The node's importance depends on the importance of its neighbors. *PageRank* centrality is based on links analysis:

$$C_{PR}^{w}(i) = (I - d.\overline{W})^{-1} + (1 - d)v_i \quad (10)$$

where d is known as damping factor that is usually set to 0.85.$\overline{W}$is the normalized weighted adjacency matrix and v is the preference network.

It is worth noting that *Subgraph Centrality* can be identified as spectral centrality.

2.4 Hybrid centralities

One of the major problems in pure centralities such as *degree*, *betweenness* and *closeness* centralities is that they are not a convenient measure to estimating node availability. In other words, the degree centrality of a node with n neighbors with edge weight of 1 is equal to the degree centrality of a node with just 1 neighbor with edge weight of$n$. However, by removing an edge from each node the former is available and the latter is not. Hence, hybrid centrality measures are proposed for better understanding of importance and influence of nodes which are applicable to both weighted and binary networks.

2.4.1 Degree-Degree centrality

This measure highlights the nodes which have more connections to more important nodes. In other words, the node degree as well as the degree of node's neighbors are considered in calculation of this centrality measure.

$$DD^{w}(i) = \sum_{j=1}^{n} (w(i,j).C_D(j)) \quad (11)$$

where, $C_D(j)$ is the degree centrality of node j and $w(i,j)$ is the weight of edge between nodes $i$and j.

2.4.2 Closeness-Degree centrality

This measure not only represents the node's influence in control of data flow, but also represents the node's performance in communication with other nodes in the network.

$$DC^{w}(i) = \sum_{j=1}^{n} (w(i,j).C_C(j)) \quad (12)$$

where, $C_C(j)$ is the closeness centrality of node j and $w(i,j)$ is the weight of edge between nodes $i$and j.

2.4.3 Betweenness-Degree centrality

This centrality represents popularity of a node as well as its influence in control of data flows in the network.

$$DB^w(i) = \sum_{j=1}^{n}(w(i,j).C_B(j)) \tag{13}$$

where, $C_B(j)$ is the betweenness centrality of node j and $w(i,j)$ is the weight of edge between nodes $i$and j.

3 The Proposed Centrality Measures

To detect the most effective nodes in the network and study vaccination of these nodes in the immunization network, we propose four new hybrid centrality measures regarding the pattern proposed in (Abbasi et al., 2011):

3.1 PageRank-Degree centrality

As aforementioned, PageRank centrality measure represents the importance of nodes based on their adjacent nodes. We Introduce PageRank-Degree centrality as a combination of PageRank measure with degree centrality. The following formula represents this new centrality measure:

$$C_{DP}(i) = \sum_{j=1}^{n} w(i,j).((1-d) + d\sum_{v\in B(j)}\frac{C_{PR}(v)}{N_v}) \tag{14}$$

where, $w(i,j)$ is the weight of the edge between nodes i and j, $C_{PR}(v)$ is the PageRank centrality of node v, d is damping factor with value of 0.85, $B(j)$ is the set of nodes which have edge with node j and N_v is number of outlinks of node v.

3.2 Radiality-Degree centrality

Radiality centrality exhibits the degree of a node's access to other nodes in the network through its neighbors. Thus, if a node has more neighbors with high Radiality centrality, its Radiality centrality is high as well. We define Radiality-Degree centrality measure as:

$$C_{DR}(i) = \sum_{j=1}^{n} w(i,j).\sum_{u\in V}\frac{\Delta_G + 1 - d^w(j,u)}{n-1} \tag{15}$$

where, $w(i,j)$ is the weight of edge between nodes $i$and j, Δ_G is the diameter of graph G, $V$is the set of nodes of graph G, n is the number of nodes in the graph and $d^w(j,u)$ is the geodesic path between nodes$j$ and u.

3.3 Subgraph-Degree centrality

We define subgraph-degree centrality as:

$$C_{DS}(i) = \sum_{j=1}^{n} w(i,j).\sum_{k=0}^{\infty}\frac{\mu_k(j)}{k!} \tag{16}$$

where, $w(i,j)$ is the weight of edge between nodes $i$and j, $\mu_k(j)$ is the number closed walks of cost k starting and ending on node j.

3.4 Katz-Degree centrality

Hybrid Katz-degree centrality is defined as follows:

$$C_{DK}(i) = \sum_{j=1}^{n} w(i,j).\alpha\sum_j A_{ju}x_{ju} + \beta \tag{17}$$

where, $w(i,j)$ is the weight of edge between nodes $i$and j. α, β are positive constants where, α is attenuation factor and $0 \le \alpha \le 1$.

4 Evaluation Method

The purpose of this study is to vaccinate the nodes with highest effect on disease distribution and remove them from the network. Effective nodes will be selected by centrality measures which were discussed in the previous section. By vaccinating these nodes the links in the network will be decreased therefore the network is assorted into smaller elements. The framework of this model is the same as in (Shams and Khansari, 2014). We use two criteria to estimate immunization algorithms: largest connected component (LCC) and epidemic threshold.

4.1 Increment of epidemic threshold of network

Epidemic threshold of network means the minimum number of people who have to be infected to reach an epidemic level. Higher epidemic threshold indicates lower probability to reach epidemic (Chakrabarti et al., 2008; Kitchovitch and Lio, 2011; Masuda, 2009; Peng et al., 2010; Shams and Khansari, 2014). The epidemic threshold of a network is the inverse of largest eigenvalue of network adjacency matrix (Chakrabarti et al., 2008; Kitchovitch and Lio, 2011; Shams and Khansari, 2014). We study the influence of removal of a specified node in reducing the largest eigenvalue of the network adjacency matrix. The purpose of this study is to assess the capability of considered centrality measures to detect and immunize the most effective nodes. As described in (Shams and Khansari, 2014), we calculate $E_1 = \frac{\lambda_1'}{\lambda_1}$ where λ_1 is the largest eigenvalue of the original network and λ_1' is the largest eigenvalue of the vaccinated network.

4.2 Decrement of Largest-Connected-Component size

A Connected-Component is a subgraph in which there is at least one path between every two nodes. Thus, the largest epidemic size of a network is its largest connected component (LCC) size (Chen et al., 2008; Gallos et al., 2007; Schneider et al., 2012; Shams and Khansari, 2014). Hence, we calculate $E_2 = \frac{LCC'}{LCC}$ where LCC is the largest connected component of the original network and LCC' is the largest connected component of the immunized network.

5 Experimental Results

In this section we investigate the performance of the predefined and proposed centrality measures in detecting and immunizing influential nodes in the network. We calculate the centrality measures in three artificial networks and in one real network and investigate their performance. To generate the artificial networks and to calculate the different centrality measures we use R.3.1.1 software.

5.1 Datasets

To generate the artificial networks we use three most famous models: Scale-free, Erdős-Rényi model (ER) and Small-World. Scale-free network is generated based on (Albert and Barabási, 2002) with 500 nodes and parameter $m = 3$ which leads to 1500 edges. ER network is generated with 500 nodes and 1500 edges which is greater than the threshold of connectedness of a random graph (Erdős and Rényi, 1961). Small-World network is produced with 500 nodes and 3 initial neighbors in each side which result in 1500 edges and rewiring probability is $p = 0.1$ (Watts and Strogatz, 1998). Edge weight is a random number between 1 and 20. For each network and for each centrality measure we generate five datasets and calculate the average of desired parameters. Then we use them in evaluation of immunization performance.

The real network includes Facebook-like (FBL) (Opsahl and Panzarasa, 2009) network which is frequently used in immunization literature and consists of 1899 nodes and 13838 edges. The weight of the edges are uniformly distributed between 1 and 20.

5.2 Diagnose and vaccination

Two methods are proposed to diagnose and vaccinate individuals based on their centrality in networks: the first is initial method in which centrality measure is calculated in initial network and most central nodes will be immunized regarding number of available vaccination resources. In the second method, adaptive method, in each iteration the most central node will be immunized and centrality measure will be recalculated (Schneider et al., 2012; Shams and Khansari, 2014). We determine and vaccinate individuals based on the initial method. Our artificial networks consist of 500 nodes and in each iteration we immunize (remove) 10 nodes. The utilized real network consist of 1890 nodes and in each iteration 40 individuals are vaccinated.

5.3 LCC in network models

Fig. 1 illustrates the efficiency of targeted immunization using LCC size in case of scale-free (BA), Small-world (SW) and Random (ER) networks, regarding different centrality measures. We plotted the proportion of LCC size to network size versus number of vaccinated (removed) nodes.Obviously, for BA model which is presented by ● symbol, by vaccinating 50 as well as 100 individuals *Communication* centrality outperforms others. *Radiality-Degree*, *PageRank* centralities represent the next best performance with almost same level. On the other hand, *Betweenness-Degree* centrality is the worst strategy to immunize the weighted BA network. *Subgraph-Degree* and *Eccentricity* are the next worst metrics. Our experiments show that in all centrality measures except *Betweenness-Degree*, the worst expected epidemic size (LCC) decreases significantly in the first 20 iterations (immunization of 40% of society). By using the *Communication* centrality, the proportion of worst expected epidemic size (LCC) to network size is 0.04 when %20 individuals of network are immunized which has the best result.

Considering different centrality measures to determine and immunize most effective nodes in weighted ER networks is illustrated by ■ symbol. The worst expected epidemic size reduces considerably by using *Subgraph* centrality. *Communication* and *Radiality-Degree* centralities have the next best performance. On the contrary, *Eccentricity* and *Degree-Degree* Centralities exhibit the worst results.

Proportion of LCC size to network size versus vaccinated nodes regarding different centrality measures for Wattz-Strogatz networks are represented by ▲ symbol. The *Betweenness* centrality outperforms others in reducing LCC size. By using *Betweenness* centrality to detect and immunize the most important nodes, the worst expected epidemic size will be reduced to one fifth of network size by vaccinating 40% of society. Table 1 summarizes our experiments in network models.*Closeness-Degree* and *Radiality-Degree* centralities are the next best measures to immunize small-world networks. Take, for example, in order to diminish the size of LCC to one fifth of network, we have to immunize 40%, 44% and 48% of the society by using *Betweenness*, *Closeness-Degree* and *Radiality-Degree* respectively. *Katz-Degree*, *Katz* and *Eccentricity* centralities show the worst results.

5.4 Epidemic threshold in network models

Effectiveness of different centrality measures to reduce the largest eigenvalue of network adjacency matrix versus number of vaccinated individuals are illustrated in Fig. 2 in the case of scale-free (BA), Small-world (SW) and Random (ER) networks.

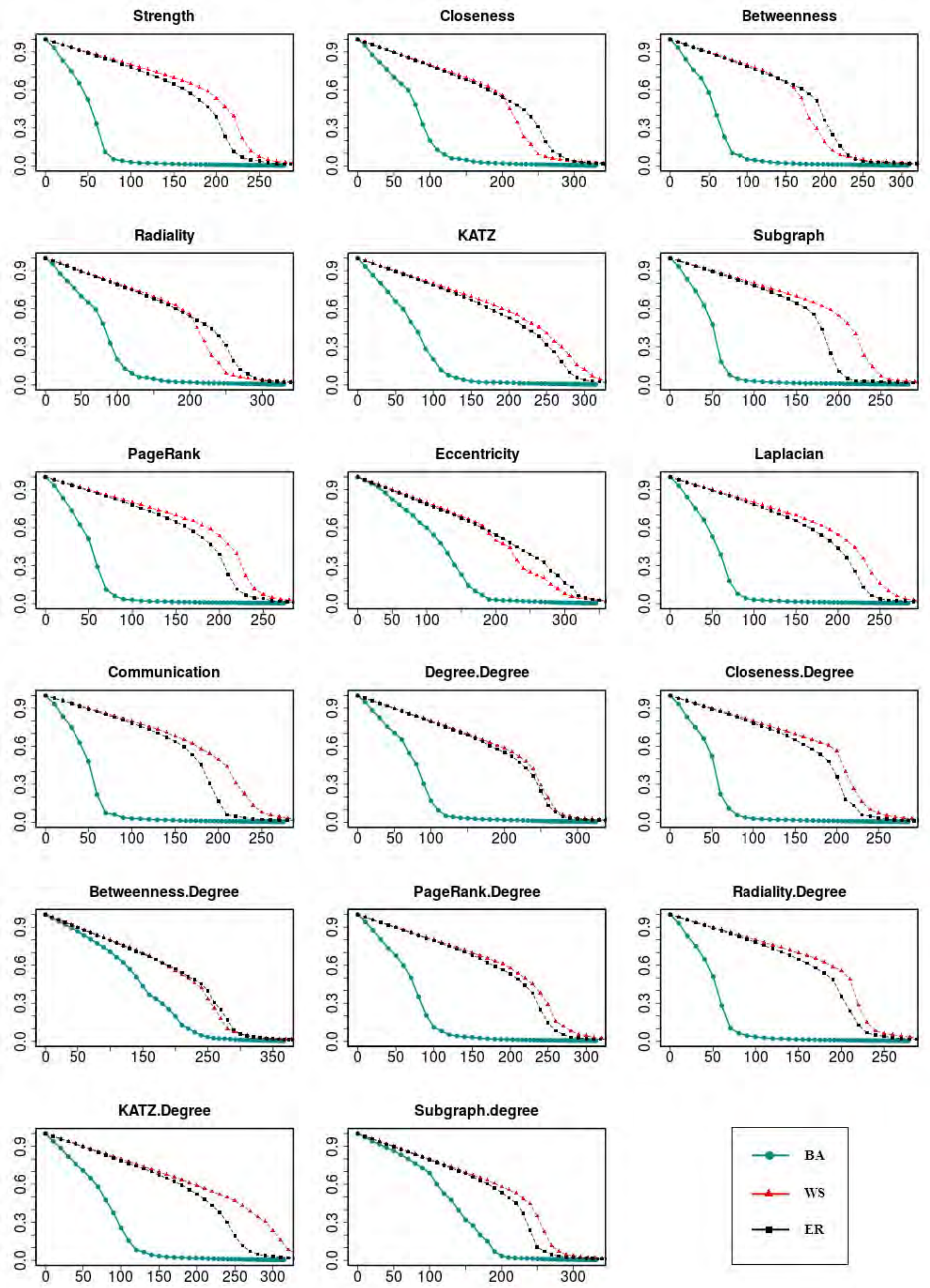

Fig. 1 Proportion of LCC size to inital network size versus number of vaccinated nodes in BA, ER and WS networks.

Table 1 Centrality measures which have the best and worst performance in decreasing LCC size in network models.

Network Model	**Outperform**	**Underperform**
Barabási-Albert	Communication Radiality-Degree PageRank	Betweenness-Degree Subgraph-Degree Eccentricity
Erdős-Rényi	Subgraph Communication Radiality-Degree	Eccentricity Degree-Degree
Watts-Strogatz	Betweeness Closeness-Degree Radiality-Degree	Katz-Degree Katz Eccentricity

Proportion of largest eigenvalue of adjacency matrix of immunized network to largest eigenvalue of original weighted BA network (λ_1) is illustrated by ● symbol. *Radiality-Degree*, *PageRank-Degree* and *Strength* centralities represent the best performance respectively. On the other hand, *Eccentricity*, *Subgraph-Degree* and *Betweenness-Degree* exhibit the worst.

Reducing the largest eigenvalue of immunized network versus immunized nodes is depicted by ■ symbol. Obviously, *Closeness-Degree*, *Degree-Degree* and *PageRank-Degree* centralities have the most performance while *Eccentricity* and *Subgraph* centralities underperform others.

Table 2 Centrality measures which have the best as well as the worst performance in increasing epidemic threshold in network models.

Network Model	**Outperform**	**Underperform**
Barabási-Albert	Radiality-Degree PageRank-Degree Strength	Eccentricity Subgraph-Degree Betweenness-Degree
Erdős-Rényi	Closeness-Degree Degree-Degree PageRank-Degree	Eccentricity Subgraph
Watts-Strogatz	Strength Degree-Degree PageRank-Degree	Betweenness Radiality Eccentricity

The same process has been done for Small-World (Wattz-Strogatz) networks which is illustrated by ▲ symbol. Our experiments show that by vaccination of 47% of society, the proportion of largest eigenvalue of network to largest eigenvalue of initial network would be 0.3 by using *Strength* centrality which is the best result. *Degree-Degree* and *PageRank-Degree* centralities exhibit the next best performance. *Betweenness*, *Radiality* and *Eccentricity* show the worst results respectively. Table 2 summarizes our experiments in increasing epidemic threshold in different network models.

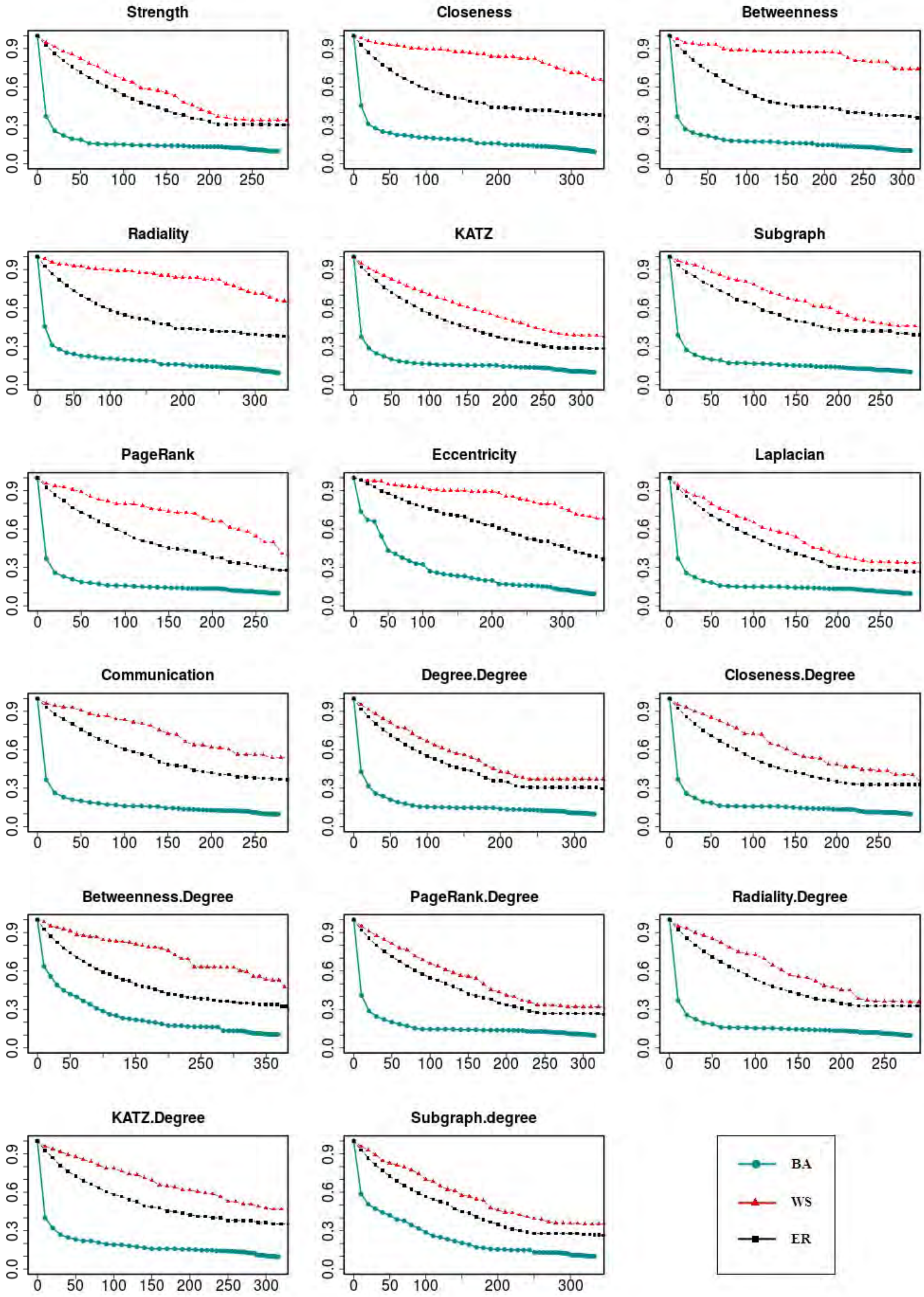

Fig. 2 Proportion of largest eigenvalue of immunized network to largest eigenvalue of initial network versus number of vaccinated nodes in BA, ER and WS networks

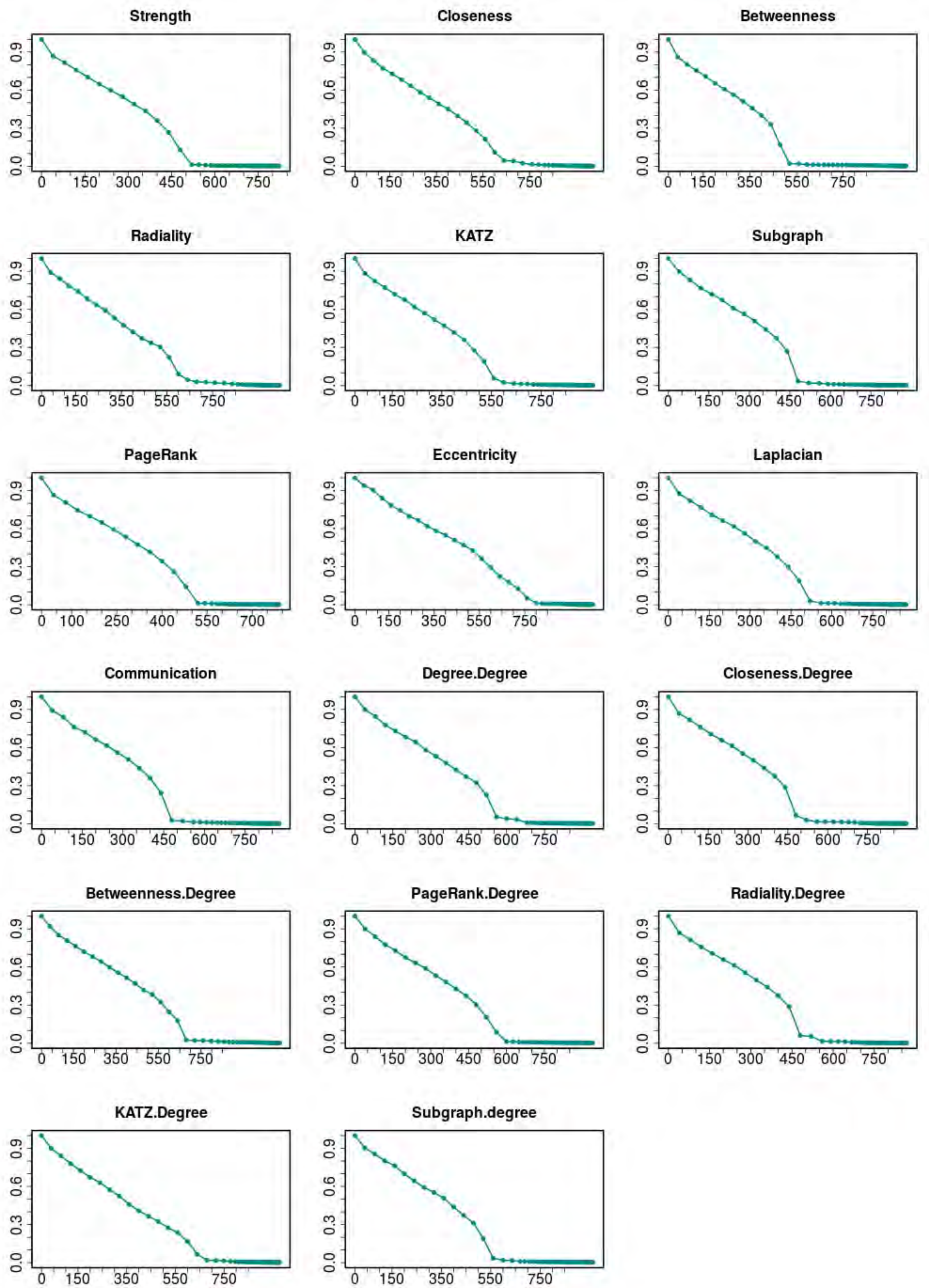

Fig. 3 Proportion of LCC size to initial network size versus number of vaccinated nodes in real network (FBL).

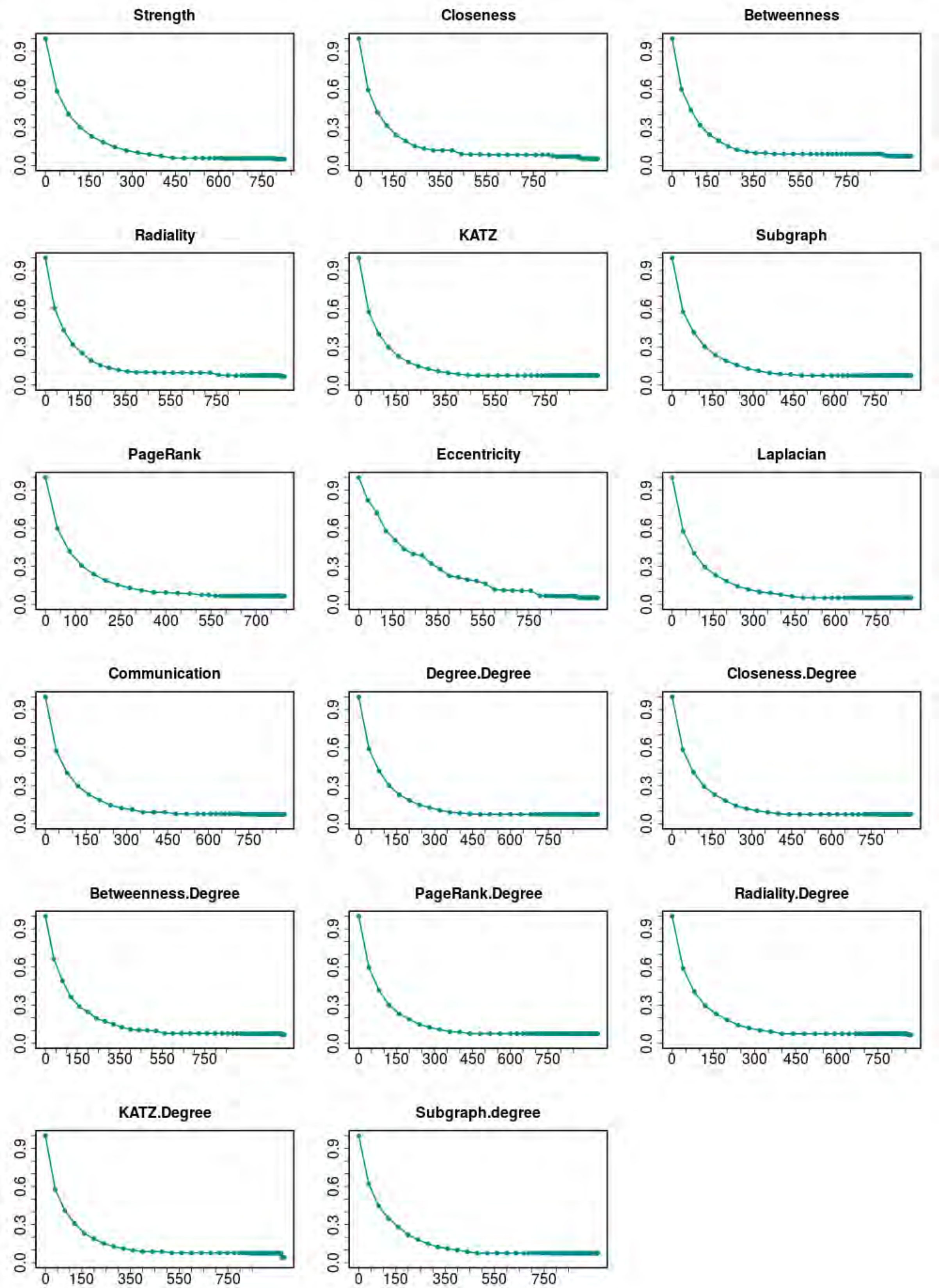

Fig. 4 Proportion of largest eigenvalue of immunized network to largest eigenvalue of initial network versus number of vaccinated nodes in real network (FBL).

5.5 Real network

As aforementioned we use the Facebook-like (FBL) (Opsahl and Panzarasa, 2009) as the real network. The major drawback of this dataset is that it has a cybernetic essence and does not reflect individuals physical contacts. However, due to lack of information in contact networks we used this dataset. Our experiments show that *Radiality-Degree* centrality is the best measure to detect most influenced nodes. *Communication* and *PageRank* centralities have the next best fulfillment as illustrated in Fig. 3.

To reduce the worst epidemic size to one fifth of the network size, we have to vaccinate %23, %23.7 and 24.3% of most important nodes by using *Radiality-Degree*, *Communication* and *PageRank* centralities, respectively.

Fig. 4 illustrates the impact of targeted vaccination on reducing the largest eigenvalue of epidemic threshold of proposed real network regarding different centrality measures. Our experiments show that *Closeness-Degree* centrality has the best performance in increasing the network epidemic threshold. *Degree-Degree* and *Katz-Degree* centralities show the next best results. By contrast, *Eccentricity* and *Betweenness-Degree* centralities exhibit the worst results respectively.

6 Conclusions

Due to accuracy of weighted networks in representing physical contacts in society, we employed them in this work. Hence, we took advantage of 13 predefined centrality measures for weighted networks to determine prominent nodes as central nodes. Furthermore, we proposed 4 new hybrid centrality measures which weight of links are considered in their computations. These 17 centrality measures were considered in targeted immunization realm. We made a comparison between these centrality measures for eminent network models: BA, ER and WS, as well as a real network (FBL) based on two metrics: *epidemic threshold* and *largest connected component* of network.In case of increasing size of largest connected component, *Radiality-Degree centrality* represented satisfying results as well as *PageRank-Degree* used to be applicable in increasing epidemic threshold in all three models: BA, ER and WS. On the other hand, *Subgraph-Degree centrality* exhibited poor performance among 4 proposed centrality measures. Moreover, *Katz-Degree* demonstrated unacceptable outcomes in almost all networks although exhibited valuable results in increasing epidemic threshold in real network (FBL).

Concisely, among pure centralities, Communication and Strength were applicable in decreasing LCC and increasing epidemic threshold respectively. Moreover, *Eccentricity* presented entirely unacceptable results. Among predefined hybrid centrality measures, (*Degree-Degree*, *Betweenness-Degree* and *Closeness-Degree*) we concluded that *Betweenness-Degree* centrality is impractical in decreasing LCC. Results to the other two measures were not so clear. Last but not least, for weighted BA network, *Radiality-Degree* and *Betweenness-Degree* centralities represented best and worst performance respectively. No such unique conclusion was inferred for the other two network models, i.e. Erdős-Rényi and Wattz-Strogatz.

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