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# Bit by bit control of nonlinear ecological and biological networks using Evolutionary Network Control

**Alessandro Ferrarini**
Department of Evolutionary and Functional Biology, University of Parma, Via G. Saragat 4, I-43100 Parma, Italy
E-mail: sgtpm@libero.it, alessandro.ferrarini@unipr.it, a.ferrarini1972@libero.it

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**Abstract**

Evolutionary Network Control (ENC) has been first introduced in 2013 to effectively subdue network-like systems. ENC opposes the idea, very common in the scientific literature, that controllability of networks should be based on the identification of the set of driver nodes that can guide the system's dynamics, in other words on the choice of a subset of nodes that should be selected to be permanently controlled. ENC has proven to be effective in the global control (i.e. the focus is on mastery of the final state of network dynamics) of linear and nonlinear networks, and in the local (i.e. the focus is on the step-by-step ascendancy of network dynamics) control of linear networks. In this work, ENC is applied to the local control of nonlinear networks. Using the Lotka-Volterra model as a case study, I show here that ENC is capable of locally driving nonlinear networks as well, so that also intermediate steps (not only the final state) are under our strict control. ENC can be readily applied to any kind of ecological, biological, economic and network-like system.

**Keywords** Evolutionary Network Control; genetic algorithms; intermediate control function; local dynamics; Lotka-Volterra model; network systems; nonlinear networks; predator-prey model.

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## 1 Introduction

Evolutionary Network Control (ENC) has been recently developed to control any kind of ecological and biological networks from inside (Ferrarini, 2013) and from outside (Ferrarini, 2013b), by coupling network dynamics and evolutionary modelling (Holland, 1975). The endogenous control requires that the network is optimized at the beginning of its dynamics, by acting upon nodes, edges or both, so that it will inertially go to the desired state. The exogenous control requires that one or more exogenous controllers act upon the network at each time step.

ENC can be applied to both discrete-time (i.e., systems of difference equations) and continuous-time (i.e., systems of differential equations) networks. ENC opposes the common idea in the scientific literature that controllability of networks should be based on the identification of the set of driver nodes that can guide the system's dynamics, in other words on the choice of a subset of nodes that should be selected to be permanently controlled (Ferrarini, 2011). ENC makes use of an integrated solution (network dynamics - genetic optimization - stochastic simulations) to compute reliability of network control (Ferrarini, 2013c), and introduced the concepts of control success and feasibility (Ferrarini, 2013d). ENC makes use of intermediate control functions to locally (step-by-step) drive ecological and biological networks, so that also intermediate steps (not only the final state) are under its control (Ferrarini, 2014). ENC can also globally subdue nonlinear networks (Ferrarini, 2015), and impose early or late stability to any kind of ecological and biological network (Ferrarini, 2015b).

**Table 1** Evolutionary network control and its developed variants.

| Reference | Purpose |
|---|---|
| Ferrarini 2011 | Theoretical bases of Evolutionary Network Control |
| Ferrarini 2013 | Endogenous control of linear ecological and biological networks |
| Ferrarini 2013b | Exogenous control of linear ecological and biological networks |
| Ferrarini 2013c | Computing the uncertainty associated with network control |
| Ferrarini 2013d | Computing the degree of success and feasibility of network control |
| Ferrarini 2014 | Local control of linear ecological and biological networks |
| Ferrarini 2015 | Global control of nonlinear ecological and biological networks |
| Ferrarini 2015b | Imposing early/late stability to linear and nonlinear networks |
| This work | Local control of nonlinear ecological and biological networks |

In this work, I show how ENC can locally control any kind of nonlinear networks, and I provide an applicative example based on the nonlinear, widely-used, Lotka-Volterra model (Lotka, 1925; Volterra, 1926). Of course, any other kind of ecological, biological, economic and network-like system is prone to be controlled by ENC.

## 2 Mathematical Formulation

### 2.1 Opening definitions

A generic ecological (or biological) dynamical system with *n* interacting actors is given as follows

$$\frac{d\mathbf{S}}{dt} = \varphi(\mathbf{S}(t)) \tag{1}$$

where $S_i$ is the amount (e.g., number of individuals, total biomass, density, covered surface etc…) of the generic *i-th* actor (i.e., species, population, taxonomic group etc.) . If we also consider time-dependent inputs (e.g., species reintroductions) and outputs (e.g., hunting) from-to outside, we must write

$$\frac{d\mathbf{S}}{dt} = \varphi(\mathbf{S}(t)) + \mathbf{I}(t) + \mathbf{O}(t) \tag{2}$$

with initial values

$$\mathbf{S}_0 = <\mathrm{S}_1(0), \mathrm{S}_2(0)...\mathrm{S}_n(0)> \tag{3}$$

and co-domain limits

$$\begin{cases} \mathrm{S}_{1\min} \le S_1(t) \le \mathrm{S}_{1\max} \\ \ldots \\ \mathrm{S}_{\mathrm{n}\min} \le S_n(t) \le \mathrm{S}_{\mathrm{n}\max} \end{cases} \quad \forall t \tag{4}$$

The nonlinear Lotka-Volterra model with logistic grow of the prey $S_1$ is a particular case of (1) and it reads as follows

$$\begin{cases} \dfrac{dS_1}{dt} = \alpha S_1(1-\dfrac{S_1}{\kappa}) - \beta S_1 S_2 \\ \dfrac{dS_2}{dt} = \beta\gamma S_1 S_2 - \delta S_2 \end{cases} \tag{5}$$

with initial values

$$\mathbf{S}_0 = \langle S_1(0),\ S_2(0)\rangle \tag{6}$$

and co-domain limits

$$\begin{cases} \mathrm{S}_{1\min} \le S_1(t) \le \mathrm{S}_{1\max} \\ \mathrm{S}_{2\min} \le S_2(t) \le \mathrm{S}_{2\max} \end{cases} \quad \forall t \tag{7}$$

and equilibrium at

$$\begin{cases} \dfrac{dS_1}{dt} \le \varphi \\ \dfrac{dS_2}{dt} \le \varphi \end{cases} \quad \text{with } \varphi \to 0 \tag{8}$$

**2.2 Local control of nonlinear networks through the intermediate control function (*ICF*)**

ENC has been developed so that it can also control intermediate dynamics of linear and nonlinear networks (Ferrarini, 2014) and not only their final state (Ferrarini, 2013; Ferrarini, 2013b).

To this purpose, ENC introduces the concept of intermediate control function (*ICF*; Ferrarini, 2014) and then controls each single step of network dynamics by subduing *ICF*. Using the *ICF*, ENC can effectively reach the following goals.

2.2.1 Freezing any network actor to a desired state

For instance, we want the prey $S_1(t)$ (or the predator $S_2(t)$) to stay as close as possible to a certain value during its dynamics. In this case, *ICF* must be formulated as follows

$$ICF = \int_{t=0}^{E} \left| k - S_i(t) \right| dt \tag{9}$$

where $t$ is time (independent variable), $E$ indicates the time at which the system goes to equilibrium, $k$ is the value at which we want $S_i(t)$ to be tied to, vertical lines indicates the module of the difference.

ENC reaches its goal by optimizing the ecological/biological network in order to achieve the minimization of *ICF*. This goal can be achieved using genetic algorithms (Holland, 1975) for the endogenous optimization

(Ferrarini, 2013) of network edges (i.e., interaction among species) or nodes (i.e., species stocks), i.e. the modification of their values at the beginning of the network dynamics in order to minimize *ICF*.

Alternatively an exogenous control can be applied on exogenous node's edges (i.e., coefficients of interaction with the inner system) and exogenous node's stock (Ferrarini, 2013b).

As a result, ENC can constrain $S_i(t)$ as close as possible to $k$ along its dynamics.

2.2.2 Binding two or more network actors together

For instance, we want to tie the prey $S_1(t)$ to the predator $S_2(t)$. In this case, ENC uses the following *ICF* to be minimized:

$$ICF = \int_{t=0}^{E} \left| S_1(t) - S_2(t) \right| dt \tag{10}$$

As a results, *S1* and *S2* are bound together (i.e. *S1≈S2*) along the whole network dynamics.

2.2.3 Freezing any actor to a desired function

We want here to tie the generic actor $S_i$ to a function of the other actor $S_j$. For instance, we could force the predator $S_2$ to be always twice the prey $S_1$ during network dynamics.

In this case, ENC uses the following equation to be minimized

$$ICF = \int_{t=0}^{E} \left| f(S_j(t)) - S_i(t) \right| dt \tag{11}$$

As a results, $S_i$ and $f(S_j)$ are tied together (i.e. $S_i \approx f(S_j)$) along the whole network dynamics.

2.2.4 Controlling each single change of any actor

We impose here that the generic actor $S_i$ changes from step to step of a certain value $u$

$$ICF = \int_{t=0}^{E} \left| \frac{dS_i(t)}{dt} - u \right| dt \tag{12}$$

By minimizing *ICF*, we impose that $S_i$ changes of exactly $u$ at each time step.

2.2.5 Limiting the control to a certain time interval

In this case the previous equations must be changed to operate mathematical integration from time $T_1$ to time $T_2$ (and not from 0 to $E$), where $T_1$ and $T_2$ are generic points along the timeline.

## 3 An Applicative Example

Let's consider the Lotka-Volterra system of Eq. (5) with the following parameters and constants:

$$\begin{aligned} &S_1(0)=85 \\ &S_2(0)=20 \\ &\alpha=3 \\ &\beta=0.02 \\ &\gamma=1.2 \\ &\delta=2 \\ &\kappa=500 \\ &dt=0.01 \end{aligned} \tag{13}$$

Fig. 1 shows its dynamical behavior. Fig. 2 depicts its phase plot.

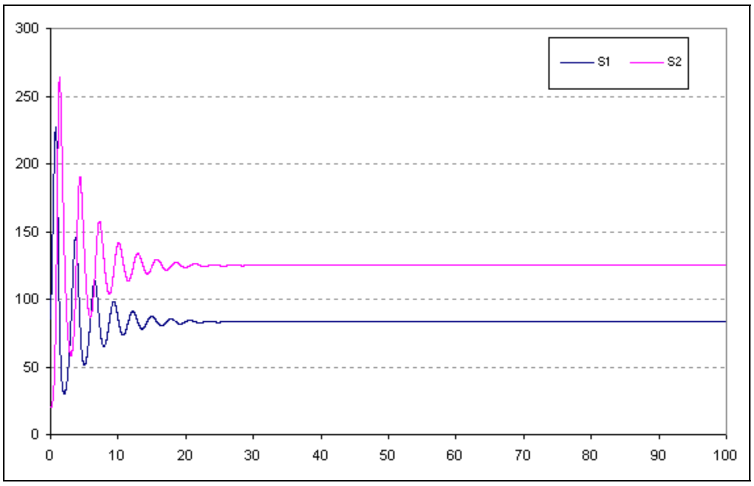


**Fig. 1** Time plot of the nonlinear Lotka-Volterra dynamical system described in (13).

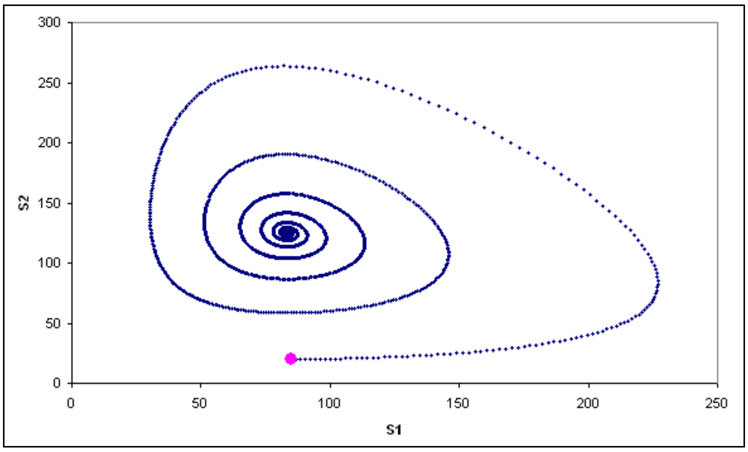


**Fig. 2** Phase plot of the nonlinear Lotka-Volterra dynamical system described in (13).

The previous nonlinear system goes at equilibrium at $E$= 68.74 with $S_1$= 83.33 and $S_2$= 125.00. The average absolute distance between $S_1$ and $S_2$ in the time span [0 - 68.74] is equal to 44.86. Now let's suppose we want to halve this distance, i.e. to impose that it becomes equal to 22.43. By optimizing Eq. 10, ENC found the solution depicted in Fig. 3 with $\alpha$= 2.4612, $\beta$= 0.0190, $\gamma$= 1.1886 while the other parameters were kept constant.

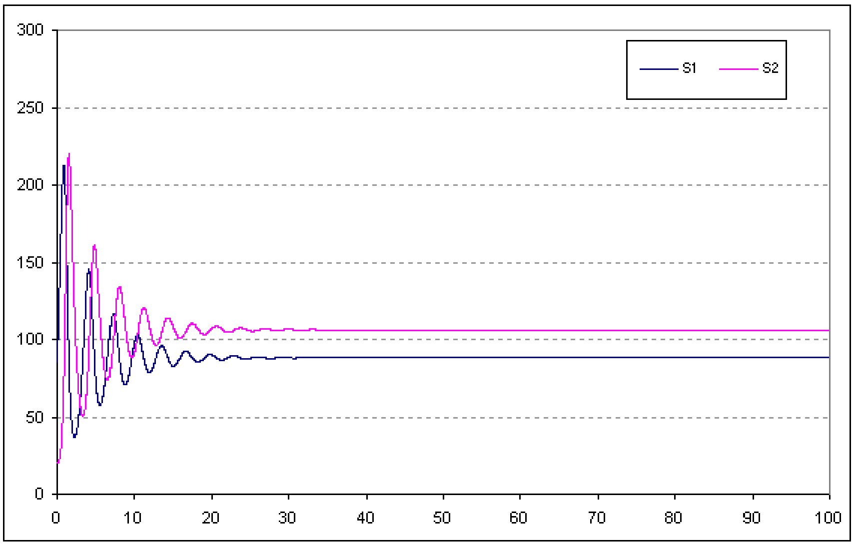


**Fig. 3** ENC has found a solution to halve the average absolute distance between $S_1$ and $S_2$ (i.e., from 44.86 to 22.43) by acting upon $\alpha$, $\beta$ and $\gamma$ while the other parameters were kept constant.

Now let's suppose we want to get the same result but only working on the carrying capacity $\kappa$. ENC found the solution that requires $\kappa$ to be equal to 149.808. The system stabilizes at $E$= 17.55 and the average absolute distance between $S_1$ and $S_2$ results to be 22.44 (Fig. 4).

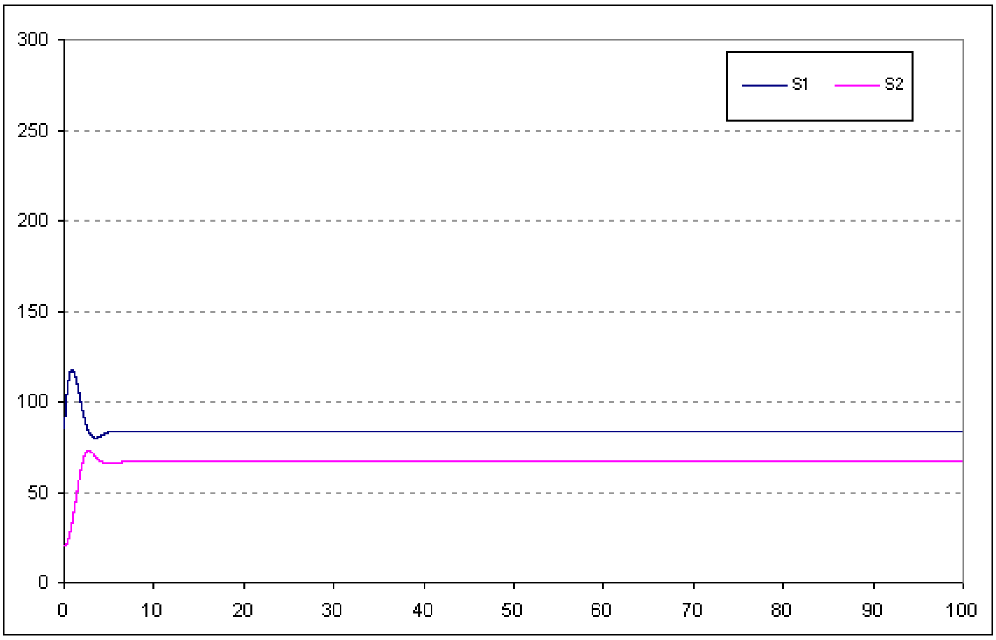


**Fig. 4** ENC has found a solution to halve the average absolute distance between $S_1$ and $S_2$ (i.e., from 44.86 to 22.44) by acting upon the carrying capacity $\kappa$ while the other parameters were kept constant.

With the parameters in (13), during the time span [0 – 68.74] the average absolute distance of two successive steps of $S_1$ was equal to 0.11. Now let's suppose we want to halve it by only working on the initial stocks of $S_1$ and $S_2$. By optimizing Eq. 12, ENC has found the solution depicted in Fig. 5, which stabilizes at $E$= 65.55 with initial stocks $S_1(0)$= 85 and $S_2(0)$= 62.67.

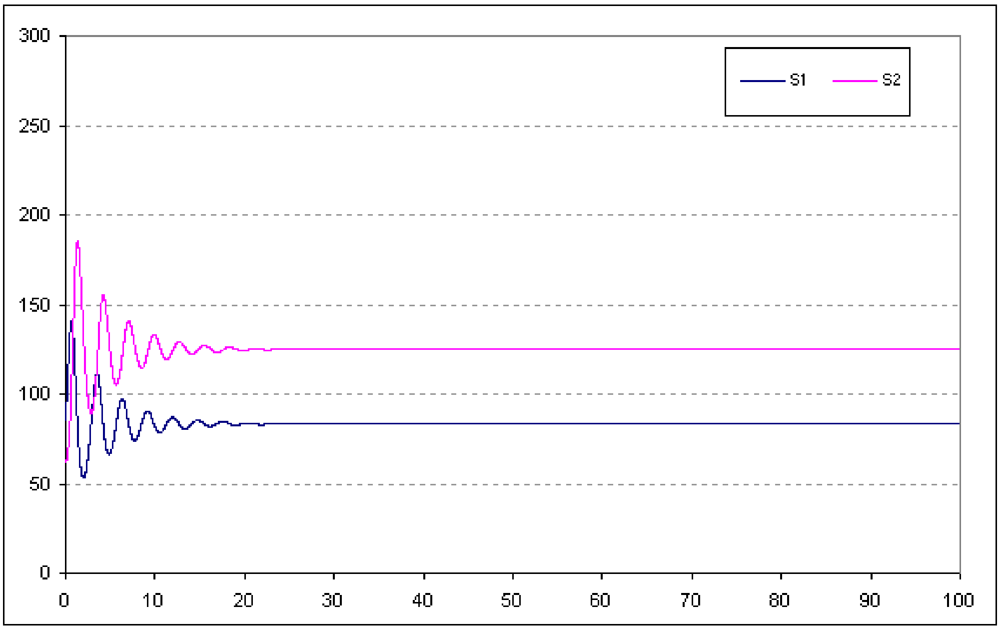


**Fig. 5** ENC has found a solution to halve the average absolute distance between two successive steps of $S_1$ (i.e., from 0.11 to 0.055) by acting upon the initial stocks of $S_1$ and $S_2$ while the other parameters were kept constant.

Of course, by optimizing equations from (9) to (12) ENC is able to realize any other kind of control on nonlinear networks, as already showed for linear networks as well (Ferrarini, 2014). In addition, global and local controls can be coupled in ENC in order to achieve a complete control of network's dynamics. The framework proposed here might also be applied to semi-quantitative networks (Ferrarini, 2011b).

ENC has been applied using the software Control-Lab 5 (Ferrarini, 2015c) written in Visual Basic (Balena, 2001; Pattison, 1998).

## 4 Conclusions

The control of ecological and biological networks is a pivotal and trendy topic. In this work, the theoretical and methodological framework named Evolutionary Network Control (ENC) has showed to be able to locally control any kind of nonlinear network, while in previous works it showed to be on top of globally and locally subduing linear networks, and of globally taming nonlinear networks.

The potential applications of ENC in ecology and biology are virtually unlimited, for instance: a) neutralize damages to ecological and biological networks, b) safeguard rare and endangered species, c) manage ecological systems at the least possible cost, d) counteract the impacts of climate change, and e) balance the negative pressure due to human activities. ENC has been developed exactly with these purposes in mind.

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