Abstract

Robustness refers to a system’s capacity for maintaining some performance when the system’s internal structure is perturbed. In previous study, network robustness, i.e., network structure robustness, includes both resistance capacity (connection robustness) of network structure to perturbation and restoration capacity (restoration robustness) of network structure if it is perturbed. Besides network structure robustness, in present paper I defined two more categories of robustness, network parameter robustness, and comprehensive robustness. Network parameter robustness refers to a network’s capacity, without any structural changes, for maintaining between-node flows (fluxes) / link weights if it is perturbed. Comprehensive robustness refers to the network’s capacity that not only the topological structure of the network, e.g., nodes and links, are not or less changed, but also between-node flows (fluxes), link weights, nodes’ state values are maintained also if the network is perturbed. Comprehensive robustness considers both structure and parameter changes of a network. Furthermore, some new indices for network parameter robustness, and comprehensive robustness were proposed. In addition to specialized indices for network robustness, the inverse of various indices of global sensitivity analysis were suggested as indices for network robustness. Differences between robustness and stability were discussed. Misuse or inaccurate use of robustness / stability in ecology was clarified. In addition, I proposed methods to facilitate network robustness. Parameters / properties of some robust bio-networks were analyzed and summarized.

Keywords network; structure robustness; parameter robustness; comprehensive robustness; stability; sensitivity analysis; biological networks

1 Introduction

1.1 Definition and implication

Perturbation of a system’s structure / parameters occurs occasionally. System perturbation occurs due to (1) external disturbances, and (2) internal factors such as unknown variables or mechanisms, structural catastrophe,
or slow drift of parameters / properties, etc. Robustness refers to a system’s capacity for maintaining some performance (stability, topological structure, functionality, or flux, etc.) if the system’s internal structure (or parameters) is perturbed. It denotes the insensitivity of system’s performance to perturbation of its structure / parameters (Zhang, 2016d). According to the specific types of system’s performances, robustness is divided into stability robustness and performance robustness.

Robustness is the key for the survival of the system in the case of abnormal and dangerous situations. Robust control refers to control theory and methodology that maintains satisfied performance when system model encounters perturbation or other uncertain disturbances (Ferrarini, 2011, 2013, 2014, 2015).

In terms of robustness of biological systems, Alon (2006) defined robustness as that a biological network can almost make its basic functions irrelevant to original biochemical parameters, while non-robustness was defined as fine-tuned, i.e., system properties greatly change when biochemical parameters are perturbed (Alon et al., 1999). Robustness is related to the survival of organisms, which is a response to internal perturbation and which reflects the capacity of an organism’s internal organization (Zhang and Zhang, 2009).

Exploiting the mechanism of robustness is very important for understanding biological networks, through which we can understand how biological networks maintain its own features under various disturbances, such as changes in the environment (lack of nutrition level, chemical induction, temperature), internal faults (DNA damage, genetic failure of metabolic pathways), etc (Zhang and Wang, 2009; Gao and Guo, 2011).

1.2 Robustness and stability
Stability is divided into state stability and structure stability. State stability is a system’s capacity to maintain its operation state after it is disturbed by external factors, while the system’s structure / parameters is / are maintained. State stability includes uniform stability, and asymptotic stability, etc. Structure stability denotes a system’s capacity to maintain its structure / parameters after it / they is / are disturbed by external factors.

Robustness is a mapping from system’s structure / parameters to system’s performance, driven by the system’s perturbation, while stability is driven by external disturbance.

Both robustness and stability are determined by a system’s structure / parameters.

2 Network Robustness
Complex networks are characterized by some or all of the properties of self-organization, self-similarity, attractors, small-world, and scale-free. Some of these properties, however, are occasionally disturbed or even destroyed.

Network structure robustness is the capacity of the network for maintaining its functionality when network nodes or links are damaged (by random failures, malicious attacks, etc.) (Newman et al., 2006; Barabasi and Albert, 2000; Zhu and Liu, 2012). It includes both resistance capacity (connection robustness) of network structure to external damage and restoration capacity (restoration robustness) of network structure if it is damaged (Du et al., 2010). Research on network robustness have been widely reported (Kwon and Cho, 2007; Ash and Newt, 2007; Gao et al., 2006; Wang et al., 2006a, 2006b). Kwon and Cho (2007) studied the relationship between feedback structure and network structure robustness, and found that scale-free network model may evolve more feedback structures than random graph model and its network structure robustness enhanced considerably. Ash and Newt (2007) optimized network structure using evolutionary algorithm and found that clustering, modularity and length of long paths all have important effects on the network structure robustness. Gao et al. (2006) demonstrated that for most food webs the attacks based on betweenness centrality are more effective than that based on node degree. Wang et al. (2006a, b) argued that network structure robustness to random failures can be improved by optimizing the efficiency of the network (average inverse
length of paths). So far, network structure robustness is usually represented by network connectivity (connection robustness). Restoration robustness is seldom studied.

Besides network structure robustness, here I define two more categories of robustness, network parameter robustness, and comprehensive robustness. Network parameter robustness refers to a network’s capacity, without any structural changes, for maintaining between-node flows (fluxes) / link weights if it is perturbed. Comprehensive robustness refers to a network’s capacity that not only topological structure of the network, e.g., nodes and links, are not or less changed, but also between-node flows (fluxes), link weights, nodes’ state values are maintained also if the network is perturbed. Comprehensive robustness considers both structure and parameter changes of a network.

In addition to conventional indices for network robustness, I propose to use the inverse of some indices of global sensitivity analysis such as Sobol index (Sobol, 1993), and Extended Fourier Amplitude Sensitivity Test, etc (Tarantola et al., 2002; Xu et al., 2004; Zhang, 2012c, 2016d) as indices of network robustness.

As a basic formula, for instance, we may define network robustness as

\[ R = 1/(dN/dp/N) \]

or

\[ R = 1/(dN/dp) \]

where \( N \): network measure, \( dN \): final variation of network measure due to perturbation; \( p \): strength of perturbation, which is usually expressed by variation of network measure itself, e.g., number of removed nodes / links, or amount of reduced flux, etc. The greater \( R \) value means the stronger robustness.

There are many network measures for uses, i.e., total number of nodes or links, network fluxes, etc.

### 2.1 Structure robustness

#### 2.1.1 Connection robustness

Connection robustness refers to the capacity of a network to maintain connectivity among remaining nodes if some nodes are attacked and destroyed. Suppose \( N_r \) nodes with maximal degree, along with their links are simultaneously removed from a network \( X \). Connection robustness is thus (Dodds et al., 2003)

\[ R = c/(n-n_r) \]

where \( n \): total number of nodes in the original network, \( n_r \): number of nodes removed, and \( c \): number of nodes in the maximal connected subgraph (i.e., components; Zhang, 2012a) after \( n_r \) nodes have been removed.

#### 2.1.2 Restoration robustness

The full information of a node with unknown or incomplete information can be achieved by consulting the information of its adjacent nodes, as used in detection of key terrorist in the terrorist organization (Bohannon, 2009). Restoration robustness refers to the capacity of a network to restore missed nodes / links if they are destroyed.

Restoration robustness in terms of nodes (\( D \)) and links (\( L \)) are

\[ D = 1-(n_r-n)/n \]
\[ L = 1-(l_r-l)/l \]
respectively, where $n$: total number of nodes in the original network, $l$: total number of links in the original network, $n_s$: number of nodes restored, $n_r$: number of nodes removed, $l_s$: number of links restored, and $l_r$: number of links removed.

### 2.2 Parameter robustness

Suppose the topological structure of a network, e.g., nodes and links, are not changed if the network is perturbed. Parameter robustness refers to a network’s capacity, without any structural changes, for maintaining between-node flows (fluxes) / link weights if it is perturbed. Here I propose to use the following index, revised from adjacency matrix index (Zhang, 2012c), to represent parameter robustness. Following the definition of Zhang (2012c), suppose flows (fluxes) / link weights matrix of a network with $n$ nodes is $w=(w_{ij})_{n\times n}$, where $w_{ij}$ is the flow (flux) or the weight of the link between nodes $v_i$ and $v_j$; $w_{ij}=0$ if there is not a link between nodes $v_i$ and $v_j$; $i, j=1,2,\ldots, n$. The index is

$$S=\sum_{i,j} |w_{ij0}-w_{ijt}|$$

where $w_{ij0}, w_{ijt}$: flow (flux) or link weight between nodes $v_i$ and $v_j$ after and before a network is perturbed. The less $S$ value means the stronger robustness.

### 2.3 Comprehensive robustness

Following the definition of Zhang (2012c), suppose adjacency matrix of a network with $n$ nodes is $d=(d_{ij})_{n\times n}$, If $d_{ij}=d_{ji}=0$, then there is not link between nodes $v_i$ and $v_j$; if $d_{ij}=d_{ji}=1$, then there is a link between nodes $v_i$ and $v_j$. Suppose $w_{ij}$ is the flow (flux) or the weight of the link between nodes $v_i$ and $v_j$, $i, j=1,2,\ldots, n$. The index for comprehensive robustness is

$$S=\sum_{i,j} |w_{ij0}-d_{ij0}-w_{ijt}d_{ijt}|$$

In addition to the measures on network robustness, other robustness measures can be defined according to various performance indices of a network, e.g., network type, variation coefficient, network entropy, etc (Zhang and Zhan, 2011). For instance, robustness of network entropy is defined as a network’s capacity to maintain network entropy if the network’s structure (or parameters) is perturbed.

For specific networks, e.g., ecosystems, network performance indices may be additionally defined as total productivity, number of functional groups, etc.

### 3 Theoretical Analysis of Structure Robustness of Typical Networks

In the random network of $n$ nodes, two nodes are connected at a certain probability $p$ (Zhang, 2012a). In the regular network, nodes are connected following certain rules. For instance, the nearest neighbors coupling network (Wang et al., 2006a, 2006b), namely for a given $k$ ($k$ is an even number), $n$ nodes in the network are linked to generate a ring, in which each node is only connected with its $k/2$ neighborhood nodes. Scale-free network is the most popular network (Barabasi and Albert, 1999; Zhang, 2012a). Small-world network was proposed by Watts and Strogatz (1998). Methods for network generation / evolution are from Barabasi and Albert (1999) (scale-free network), Watts and Strogatz (1998) (small-world network), Wang et al. (2006a, 2006b) (regular network), and Zhang (2012a) (random network).

#### 3.1 Connection robustness

For the random network, increasing connection probability $p$ enhances connection robustness. Network connectivity will not be destroyed if $p\geq 0.3$ (the threshold density for connection robustness).

As increase of the nodes removed from scale-free network, the decrease of network connection capacity
produces “emergence” phenomena. Connection robustness enhances as the increase of network density. In general, the connection robustness of regular network and random network is stronger than scale-free network. In terms of connection robustness, small-world network is the interim between regular network and random network. Connection robustness of all networks produces “emergence” as change of number of nodes removed and network density.

3.2 Restoration robustness

3.2.1 Node restoration

Suppose that node $i$ is adjacent to node $j$. If node $i$ is removed from node $j$, we may try to restore node $i$ and the link to node $j$ according to the information of node $j$.

For the random network, if network density is less than 0.3, the restored nodes decline as increase of the nodes removed. Removed nodes can be restored if network density is not less than 0.3 (threshold density for node restoration robustness). Similar to threshold density for connection robustness, the threshold density for node restoration robustness declines as the increase of network size.

The scale-free network can be thoroughly restored if only a few of nodes are removed. After the removed nodes have reached a threshold, the number of restored nodes will decrease quickly. However, node restoration robustness increases as network density.

In the small-world network, for a fixed re-connection probability $p$, the decline of node restoration rate shows “emergence” as increase of nodes removed. The increase of node degree or re-connection probability $p$ will enhance node restoration robustness.

Overall, node restoration robustness of all networks produces “emergence” as change of number of nodes removed and network density.

3.2.2 Link restoration

For the random network, link restoration robustness increases as network density. However, link restoration robustness will be almost irrelevant to network density after network density has reached a threshold.

Link restoration robustness of scale-free network decreases linearly as the number of nodes removed, and has not significant relationship with network density.

For the small-world network, with a fixed re-connection probability $p$, the decline of link restoration rate shows “emergence” as increase of nodes removed. The increase of node degree or re-connection probability $p$ will enhance link restoration robustness.

In summary, both link and node restoration robustness of the random network are the best among four types of networks, and the regular network is the worst. Small-world network is the interim between the random network and the regular network. Node restoration robustness of scale-free network is better but its link restoration robustness is as worse as the regular network.

In addition to network evolution methods and resultant networks above, more complex network evolution model has demonstrated that network density (connectance; Zhang, 2011, 2012a, 2012b) increases as time (Zhang, 2016a), which means that the network evolves to the stronger robustness according to the conclusions above.

As theoretical models, all of network evolution methods and resultant networks above do not limit the number of links connected to a node, i.e., node degree is limitless in these methods and networks.

4 Facilitation of Network Structure Robustness

Different from theoretical networks above, practical networks sometimes demonstrate different mechanisms (e.g., food webs with random degree distributions were highly fragile to removals of species, see Montoya and
Sole, 2003) for robustness-maintaining due to the limitation of node degree (e.g., many species have only one or two links in food webs), etc. Based on both theoretical analysis and practical observations, here I summarize the following methods to enhance structure robustness of a given network.

1. Maintaining a certain network density / connectance / connectivity (Dunne et al., 2002; Allesina et al., 2005; Zhang, 2011, 2012a). It has been reported that a food web with the higher connectance has more numerous reassembly pathways and can thus restore faster from perturbation (MacArthur, 1955; Law and Blackford, 1992; Zhang, 2012a). However, some models suggested that food webs with the lower connectance restored faster after a disturbance (May, 1973; Pimm, 1991; Chen and Cohen, 2001; Cohen et al., 1990; Zhang, 2012a). Therefore, for a network with fixed number of nodes, maintaining (increasing or reducing) a certain number of links may improve network structure robustness.

2. Deploying network circuits (Alon, 2006; Zhang, 2012a, 2016f). Deploying some circuits in network means the existence of some feedback controls, which help to improve network robustness. Feedback control is the basis of robustness of bio-systems and bio-processes, e.g., the chemotaxis and heat shock response of *Escherichia coli*, biological rhythms, and cell cycles, etc (Oleksiuk et al., 2011).

3. Constructing hierarchical sub-networks / modules / connected components (Zhang, 2012a, 2016e). A large network is always organized from various small mosaics (modules). Organizing mosaics to a large network will probably influence the robustness of entire network (May, 1973). Pinnegar et al. (2005) used a detailed Ecopath with Ecosim model to examine the impacts of food web aggregation and the removal of weak linkages. They found that aggregation of a 41-compartment food web to 27 and 16 compartment systems greatly affected system properties (e.g. connectance, system omnivory and ascendancy) and influenced dynamic stability. Highly aggregated webs restored more quickly following disturbances compared to the original disaggregated model.

   Existence of hierarchical sub-networks / modules / connected components may prevent failures diffuse across over a network, simplify the evolution and update of nodes and links, and thus help to enhance network structure robustness. In addition, utilization of several hierarchical sub-networks / modules / connected components may avoid malfunction of the major components in a network. A biological cell is a typical example. In a cell, mitochondria, ribosomes, chloroplasts, etc., are sub-networks / modules / connected components.

4. Incorporating redundancy. Besides useful nodes and links, adding redundant (i.e., temporarily not useful, or candidate) nodes, links and circuits in a network will help to enhance structure robustness. Redundant nodes / links / circuits are expected to play key roles if some original nodes, links and circuits are destroyed. In biological networks, repeated genes, homeotic genes (McAdams and Arkin, 1999), redundant metabolites, multi-pathway signaling, and similar metabolic circuits (Edwards et al., 2001), etc., are all examples of network redundancy.

Some methods above can be used to facilitate parameter robustness and comprehensive robustness. Actually comprehensive robustness is more reasonable than structure robustness, because network structure is usually determined by the strengths of between-node interactions (i.e., link weights, between-node fluxes, etc.). As an example, past studies have demonstrated that under the condition of constant structure robustness, to increase nodes and links in a network must be at the cost of weakening the added links (weak interactions, i.e., weak weights, weak fluxes, etc.; McCann, 1988; Paine, 1992; Zhang, 2011, 2012a).

Computational simulation, e.g., the network evolution model (Zhang, 2015) can be used to exploit the relationship between network structure and network robustness.
5 Biological Networks and Robustness

5.1 Misuse or inaccurate use of stability in biological studies

So far, most research used stability, etc., to describe the robustness of various biological systems. This includes such topics as “relationship between biodiversity and stability”, “stability of ecosystems”, “stability of metabolic networks”, etc. It is unsurprising because the terminology “robustness” was defined as late as about 30 years ago, while “stability” had been used by ecologists for more than 40 years (May, 1973). Obviously, “stability” in these topics was substantially robustness in most situations. Definition and implication of robustness and stability, as discussed above, are soundly distinctive. We concern both external disturbance and internal perturbation in biodiversity and ecosystems. According to the substantial implication of so-called stability in these topics, I argue to use robustness to replace misused or inaccurately used stability in these situations in exception of the fewer cases for stability-specific topics. In most of the situations, the exact and correct designate should be “relationship between biodiversity and robustness”, “robustness of ecosystems”, “robustness of metabolic networks”, etc.

5.2 Robust structure and parameters of some biological networks (systems)

I assume that naturally existing and sustainable networks (food webs, biochemical networks, etc.) are robust. I summarize some network structures and parameters from references in Table 1, in which $S$ is the number of species, $L$ is the number of actual links, and $C$ is connectance. $\lambda$ is the parameter of power-law distribution of node degrees, $p(x)\propto x^{-\lambda}$. It can be concluded that overall power-law $\lambda$ is around 1.5 (1.5±0.4), the mean node degree is around 2~3 (Zhang and Li, 2016).

<table>
<thead>
<tr>
<th>Structures</th>
<th>Parameters</th>
<th>Values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L=CS^2$</td>
<td>$C$</td>
<td>0.14</td>
<td>Martinez, 1991, 1992</td>
</tr>
<tr>
<td>$S=C^a$</td>
<td>$a$</td>
<td>-0.5</td>
<td>Montoya and Sole, 2003</td>
</tr>
<tr>
<td>$L=as^b$</td>
<td>$a$, $b$</td>
<td>$a=1.3$, $b=1.1$</td>
<td>Cohen and Briand, 1984</td>
</tr>
<tr>
<td>$L=as^b$</td>
<td>$b$</td>
<td>1.5</td>
<td>Sugihara et al., 1989; Schoenly et al., 1991; Havens, 1992; Martinez, 1994</td>
</tr>
<tr>
<td>$L=as^b$</td>
<td>$a$, $b$</td>
<td>$a=2$, $b=1$</td>
<td>Cohen et al., 1990; Martinez, 1992</td>
</tr>
<tr>
<td>$C=L/S^2$</td>
<td>$C$</td>
<td>0.1~0.15</td>
<td>Martinez, 1992; Warren, 1994</td>
</tr>
</tbody>
</table>

Table 1 Network structure and parameters of food webs, metabolic pathways, etc.
\[ p(x) = x^{-\lambda} \]

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Degree Distribution</th>
<th>Mean Degree</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transcription network</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Signaling network</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metabolic network</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goemann et al., 2011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arthropod family networks</td>
<td>1.64</td>
<td>1.43</td>
<td>1.11</td>
</tr>
<tr>
<td>Zhang, 2011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arthropod species networks</td>
<td>2.16</td>
<td>4.07</td>
<td>3.14</td>
</tr>
<tr>
<td>Zhang, 2011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arthropod species networks</td>
<td>2.84</td>
<td>2.19</td>
<td></td>
</tr>
<tr>
<td>Arthropod family networks</td>
<td>2.1</td>
<td>2.9</td>
<td></td>
</tr>
<tr>
<td>Huang and Zhang, 2012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tumor pathway</td>
<td>2.35</td>
<td>2.18</td>
<td>3.09</td>
</tr>
<tr>
<td>Goemann et al., 2011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food webs</td>
<td>~2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohen et al., 1990; Martinez, 1992</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Immunization network</td>
<td>4–15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shams and Khansari, 2014</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal pathway</td>
<td>4.68</td>
<td>10.58</td>
<td></td>
</tr>
<tr>
<td>Rahman et al., 2013</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: \( S \) is the number of species, \( L \) is the number of actual links, and \( C \) is connectance. \( \lambda \) is the parameter of power-law distribution of node degrees, \( p(x) = x^{-\lambda} \).

**Acknowledgment**

I am thankful to the support of High-Quality Textbook Network Biology Project for Engineering of Teaching Quality and Teaching Reform of Undergraduate Universities of Guangdong Province (2015.6-2018.6), from Department of Education of Guangdong Province, Discovery and Crucial Node Analysis of Important Biological and Social Networks (2015.6-2020.6), from Yangling Institute of Modern Agricultural Standardization, and Project on Undergraduate Teaching Reform (2015.7-2017.7), from Sun Yat-sen University, China.

**References**

Alon U. 2006. An Introduction to Systems Biology: Design Principles of Biological Circuits. CRC Press, USA
Monographs, 61: 367-392
Wang XF, Li X, Chen GR. 2006b. The Theory of Complex Network and Its Application. Tsinghua University Press, Beijing, China
Zhang WJ. 2012b. How to construct the statistic network? An association network of herbaceous plants
constructed from field sampling. Network Biology, 2(2): 57-68
Zhang WJ. 2016b. Screening node attributes that significantly influence node centrality in the network. Selforganizology, 3(3): 75-86