Article

How to find cut nodes and bridges in the network? A Matlab program and application in tumor pathways

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Abstract
A connected graph *X* is a block, if and only if for any three vertices *u*, *v* and *w* in *X*, there exists a path from *u* to *w* and the path does not contain *v*. In present article I present full Matlab codes of the algorithm for finding cut nodes and bridges in the network.

Keywords network; cut nodes; bridges; Matlab.

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1 Introduction

A connected graph *X* (in a connected graph, each pair of vertices is connected) is a block, if and only if for any three vertices *u*, *v* and *w* in *X*, there exists a path from *u* to *w* and the path does not contain *v*. In present article I present full Matlab codes of the algorithm for finding cut nodes and bridges in the network.

2 Algorithm

The algorithm to calculate cut nodes (vertices) and bridges of a network (graph) is based on the following therorem (Chan et al., 1982; Tarjan, 1972; Zhang, 2012)

Theorem: A connected graph *X* is a block, if and only if for any three vertices *u*, *v* and *w* in *X*, there exists a path from *u* to *w* and the path does not contain *v*.

The theorem states that there is not any bottleneck in a block. The node (vertex) *v* is a bottleneck if any path from *u* to *w* must go through *v*. In this case *v* is a cut node (vertex). Lose of a cut node will lead to disconnection of connected blocks. Cut nodes are thus crucial nodes of a network.

According to the theorem, the cut nodes (vertices), blocks and bridges of a network (graph) can be obtained by calculating fundamental circuit set of the network (graph).

The following are Matlab codes of the function, cutVertex.m, for calculating cut nodes (vertices). The bridges and blocks are easily found by discriminating cut nodes from all nodes in the network.

```
%Matlab function to obtain cut nodes (vertices) in a network/graph.
function cutset=cutVertex(d)
%d: adjacency matrix of the network; Adjacency matrix is d=(dij)n*n,where n is the number of nodes in the network. dij=1 if vi and vj are adjacent, and dij=0, if vi and vj are not adjacent; i, j=1,2,…, n.
%cutset: string of cutvertex set.
n=size(d,1);
r=sum(d);
e=max(r);
num=zeros(1,n);
t1=zeros(1,n);
t2=zeros(1,n);
b1=zeros(1,n*e);
b2=zeros(1,n*e);
lw=zeros(1,n);
cut=zeros(1,n);
[tree,et,eb,t1,t2,b1,b2,num]=DFS(d);
for i=1:n;
cut(i)=0;
lw(i)=num(i);
end
for i=1:eb
v1=b1(i);
v2=b2(i);
if (lw(v1)>=num(v2)) lw(v1)=num(v2); end
end
for i=1:et
v1=t1(et-i+1);
v2=t2(et-i+1);
if (lw(v2)<=lw(v1)) lw(v1)=lw(v2); end
end
s=0;
for i=1:et
v1=t1(i);
v2=t2(i);
if (v1==1) s=s+1; end
if ((lw(v2)>=num(v1)) & (v1~=1)) cut(v1)=v1; end
end
if (s>=2) cut(1)=1; end
cutset='Cutvertex set:\n{';
for i=1:n
if (cut(i)~=0)
cutset=strcat(cutset,num2str(cut(i)));
if (i~=n) cutset=strcat(cutset,','); end
end
end
```

```
cutset=strcat(cutset,'}\n');
```

The function, DFS.m, used in the program above, can be found in Zhang (2016c).

3 Application

Use the following adjacency matrix (40 by 40) in the typical example of Zhang (2016a), the calculated cut vertex set is achieved as: {6,7,8,9,10,13,14,15,18,19,20,21,22,23,24,31,32,33,34,35}. Obviously, there are four bridges, {6,7,8,9,10},{13,14,15},{18,19,20,21,22,23,24}, and {31,32,33,34,35}.

0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0

Use DFS algorithm and the adjacency matrices of tumor networks (Huang and Zhang, 2012; Li and Zhang, 2013; Zhang, 2016), the set of cut nodes for p53 tumpr pathway is {5,32,38,40,42,47,49,50,52}, for Ras tumpr pathway is {2,3,4,5,10,12,16,19,22,23}, and for MARK is {6,13,15,21,23,25,31,33,36,58}. These cut nodes are crucial nodes in the tumor pathways.

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