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# Imposing early stability to ecological and biological networks through Evolutionary Network Control

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**Abstract**
The stability analysis of the dynamical networks is a well-studied topic, both in ecology and in biology. In this work, I adopt a different perspective: instead of analysing the stability of an arbitrary ecological network, I seek here to impose such stability as soon as possible (or, contrariwise, as late as possible) during network dynamics. Evolutionary Network Control (ENC) is a theoretical and methodological framework aimed to the control of ecological and biological networks by coupling network dynamics and evolutionary modelling. ENC covers several topics of network control, for instance a) the global control from inside and b) from outside, c) the local (step-by-step) control, and the computation of: d) control success, e) feasibility, and f) degree of uncertainty. In this work, I demonstrate that ENC can also be employed to impose early (but, also, late) stability to arbitrary ecological and biological networks, and provide an applicative example based on the nonlinear, widely-used, Lotka-Volterra model.

**Keywords** Evolutionary Network Control; genetic algorithms; Lotka-Volterra system; network stability; predator-prey model.

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## 1 Introduction

Evolutionary Network Control (ENC; Ferrarini, 2013a; Ferrarini, 2013b) has been introduced with the goal to control ecological and biological networks (Ferrarini, 2011a; Ferrarini, 2011b) both from the outside (Ferrarini, 2013a) and from inside (Ferrarini, 2013b) by coupling network dynamics, stochastic simulations and evolutionary modelling (Holland, 1975; Goldberg, 1989).

ENC also estimates the reliability of the achieved solutions (Ferrarini, 2013c). This is an important topic because it's not sure that, while managing a network-like system, we are able to impose to nodes and edges exactly the optimized values required to achieve the desired control. In addition, ENC has solved the issue of seeking the most feasible solution to network control by introducing the concepts of control success and feasibility (Ferrarini, 2013d). ENC also has shown its ability to locally (step-by-step) drive ecological and biological networks so that also intermediate steps (and not only the final state) are under control by using an *ad hoc* intermediate control functions (Ferrarini, 2014).

The study of the stability of dynamical networks is a common topic, both in ecology and in biology. In this paper, I adopt a different perspective: instead of analysing the stability of an arbitrary ecological network, I search to impose such stability as soon as possible (or, contrariwise, as late as possible) during network dynamics, and provide an applicative example based on the nonlinear, widely-used, Lotka-Volterra model (Lotka, 1925; Volterra, 1926).

I show here that Evolutionary Network Control is on top of this task. It's not purpose of this work to discuss the ecological implications of these results, but it's evident that they are not irrelevant.

## 2 Imposing Early Stability to Ecological and Biological Networks: Mathematical Formulation

Given a generic ecological (or biological) dynamical system with *n* interacting actors

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t) \tag{1}$$

where $S_i$ is the amount (e.g., number of individuals, total biomass, density, covered surface etc…) of the generic *i-th* actor, if we also consider inputs (e.g. species reintroductions) and outputs (e.g. hunting) from-to the outside, we must write

$$\frac{d\vec{S}}{dt} = \varphi(\vec{S}, t) + \vec{I}(t) + \vec{O}(t) \tag{2}$$

with initial values

$$\vec{S}_0 = <S_1(0), S_2(0)...S_n(0)> \tag{3}$$

and co-domain limits

$$\begin{cases} S_{1\min} \le S_1(t) \le S_{1\max} \\ ... & \forall t \\ S_{n\min} \le S_n(t) \le S_{n\max} \end{cases} \tag{4}$$

and stability happening at time $T = t_{equilibrium}$ when

$$\begin{cases} \frac{dS_1}{dt} = 0 \\ ... \\ \frac{dS_n}{dt} = 0 \end{cases} \tag{5}$$

Now, in order to impose early stability to the arbitrary network above, ENC acts as follows

$$minimize\ t_{equilibrium} \tag{6}$$

by ruling the network parameters and initial values as in Ferrarini (2013a, 2013b).

As an example, let's consider the widely used Lotka-Volterra predator-prey model (Lotka, 1925; Volterra, 1926). The Lotka-Volterra equations (Lotka, 1925; Volterra, 1926), otherwise known as the predator-prey equations, are a combination of first-order, non-linear, differential equations widely used to describe the dynamics of biological systems with two species interacting (one as a prey and the other as a predator). The Lotka-Volterra model makes five assumptions about the environment and the dynamics of the two interacting species: 1) the prey population finds food at any times; 2) the food supply for the predator depends completely on the size of the prey population; 3) the rate of change of each population is proportional to its size; 4) during the interaction, the environment remains unvarying; 5) predators have unbounded appetency. Since differential equations are used, the solution is deterministic and continuous; this means that the generations of both the predator and prey continually overlap. The nonlinear Lotka-Volterra model with logistic grow of the prey $S_1$ is a particular case of (1), and it reads as follows

$$\begin{cases} \dfrac{dS_1}{dt} = \alpha S_1 (1 - \dfrac{S_1}{\kappa}) - \beta S_1 S_2 \\ \dfrac{dS_2}{dt} = \beta\gamma S_1 S_2 - \delta S_2 \end{cases} \tag{7}$$

with initial values

$$\vec{S}_0 = <S_1(0),\ S_2(0)> \tag{8}$$

and co-domain limits

$$\begin{cases} S_{1\min} \le S_1(t) \le S_{1\max} \\ S_{2\min} \le S_2(t) \le S_{2\max} \end{cases} \quad \forall t \tag{9}$$

In order to get global control of such model, Evolutionary Network Control acts upon the previous Lotka-Volterra model as follows

$$\begin{cases} \dfrac{d\tilde{S}_1}{dt} = \tilde{\alpha} S_1 (1 - \dfrac{S_1}{\kappa}) - \tilde{\beta} S_1 S_2 \\ \dfrac{d\tilde{S}_2}{dt} = \tilde{\beta}\tilde{\gamma} S_1 S_2 - \tilde{\delta} S_2 \\ \tilde{\vec{S}}_0 = <\tilde{S}_1(0),\ \tilde{S}_2(0)> \end{cases} \tag{10}$$

where the tilde symbol means that the ENC is active over such actors by controlling equation parameters and initial values, in order to determine as soon as possible (or, also, as late as possible)

$$\begin{cases} \dfrac{dS_1}{dt} \le \varphi \\ \dfrac{dS_2}{dt} \le \varphi \end{cases} \quad \text{with } \varphi \to 0 \tag{11}$$

The control equations in (10) are able to drive the nonlinear Lotka-Volterra model to the desired final state with an uncertainty degree that can be calculated as proposed in Ferrarini (2013c).

$$\begin{cases} \frac{d\tilde{S}_1}{dt} = \underline{\tilde{\alpha}} S_1 (1 - \frac{S_1}{\underline{\tilde{\kappa}}}) - \underline{\tilde{\beta}} S_1 S_2 \\ \frac{d\tilde{S}_2}{dt} = \underline{\tilde{\beta}}\underline{\tilde{\gamma}} S_1 S_2 - \underline{\tilde{\delta}} S_2 \\ \tilde{\tilde{S}}_0 = <\underline{\tilde{S}}_1(0),\ \underline{\tilde{S}}_2(0)> \end{cases} \tag{12}$$

where the underscores represent 1%, 5% or 10% uncertainties about the optimized parameters. Thus for example:

$$\begin{cases} 0.99 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.01 * \tilde{\alpha} \\ or \\ 0.95 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.05 * \tilde{\alpha} \\ or \\ 0.90 * \tilde{\alpha} \le \underline{\tilde{\alpha}} \le 1.10 * \tilde{\alpha} \end{cases} \tag{13}$$

If we stochastically vary *n* times (e.g. 10,000 times) the parameters that have been optimized via ENC, we can compute how many times such uncertainty makes the optimization procedure useless. Hence, uncertainty about network control can be computed as in Ferrarini (2013c)

$$U_{\%} = 100 * \frac{k}{n} \tag{14}$$

where *k* is the number of stochastic simulations acting upon the optimized parameters that make the optimization procedure useless (i.e. the goal of the optimization procedure is not reached).

## 3 An Applicative Example

Let's consider the Lotka-Volterra system of eq. (7) with the following parameters and constants:

$$\begin{cases} S_1(0) = 100 \\ S_2(0) = 10 \\ \alpha = 4 \\ \beta = 0.05 \\ \gamma = 1 \\ \delta = 4 \\ \kappa = 500 \\ dt = 0.01 \\ \varphi = 0.0001 \end{cases} \tag{15}$$

Fig. 1 shows its dynamical behaviour.

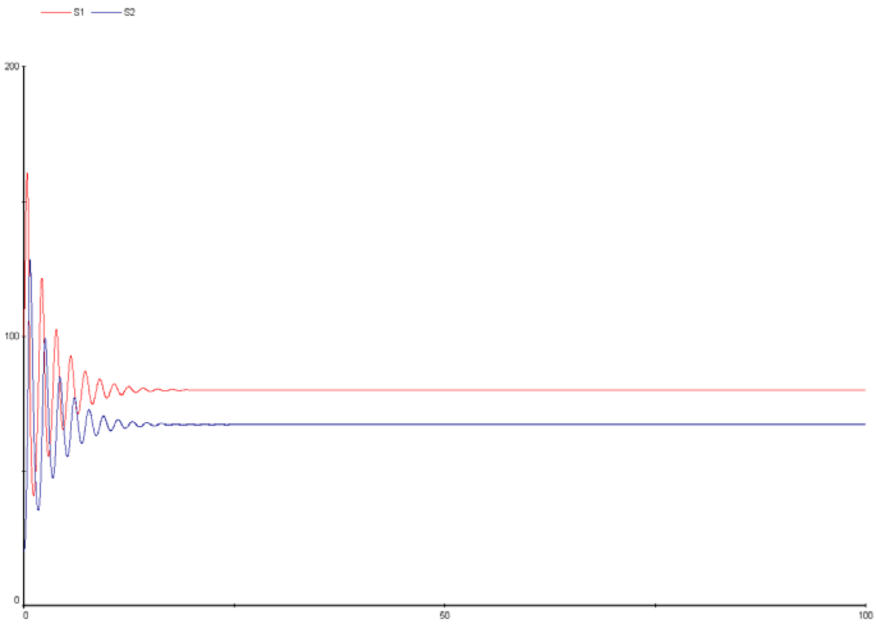


**Fig. 1** Time plot of the nonlinear Lotka-Volterra dynamical system described by eq. (15).

The previous nonlinear system goes at the steady state at $t$= 61.75 with $S_1$= 80.00 and $S_2$= 67.20.

Now let's suppose that we seek to stabilize the system earlier than $t$= 61.75 by optimizing via ENC the parameters in (15) of no more than 10% with respect to their initial values (slight changes to the Lotka-Volterra model). Fig. 2 and Table 2 show the optimal solution detected via ENC. The system becomes stabilized at $t_{equilibrium}$ = 27.15 with final values $S_1$= 108.11 and $S_2$= 74.11. Thus, the initial $t_{equilibrium}$ has been lowered by 40.05, passing from 67.20 to 27.15.

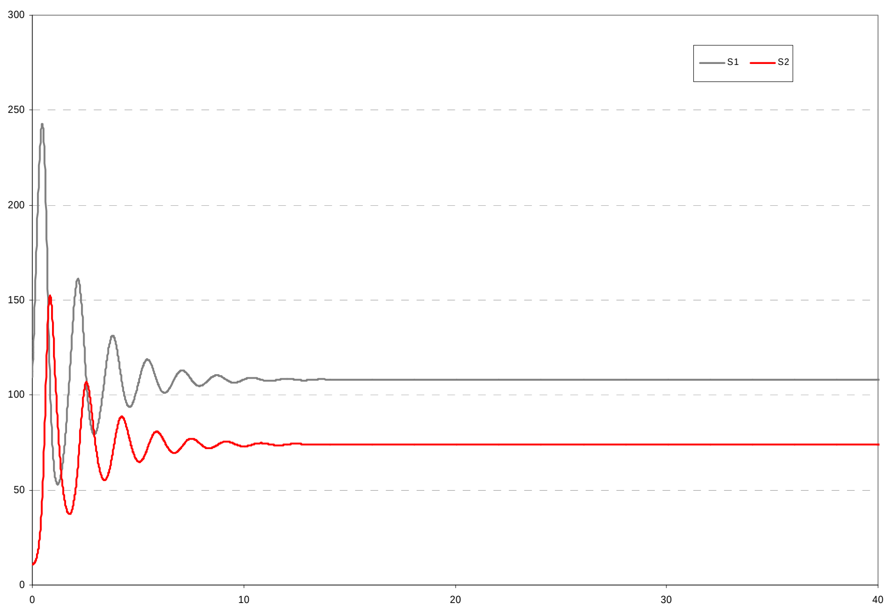


**Fig. 2** The Lotka-Volterra model of Fig. 1 has been stabilized at $t_{equilibrium}$= 27.15 using Evolutionary Network Control. The solution has been found through the software Control-Lab (Ferrarini, 2013e).

**Table 1** The optimized parameters detected via ENC in order to stabilize as early as possible the Lotka-Volterra system of Fig. 1 by changing the initial parameters of no more than 10%.

| | |
|---|---|
| **$S_1(0)$** | 109.4208223 |
| **$S_2(0)$** | 10.95093103 |
| **a** | 4.397088922 |
| **b** | 0.045137919 |
| **k** | 451.0657846 |
| **c** | 0.900567487 |
| **d** | 4.394835934 |

Now let's suppose that we want to stabilize the system earlier than *t*= 61.75 by optimizing via ENC the parameters in (15) of no more than 20% (average changes to the initial Lotka-Volterra model).

Fig. 3 and Table 2 show the solution detected via ENC. The optimized system becomes stabilized at $t_{equilibrium}$ = 16.06 with final values $S_1$= 136.41 and $S_2$= 73.01. Thus the initial $t_{equilibrium}$ has been lowered by 51.14, passing from 67.20 to 16.06.

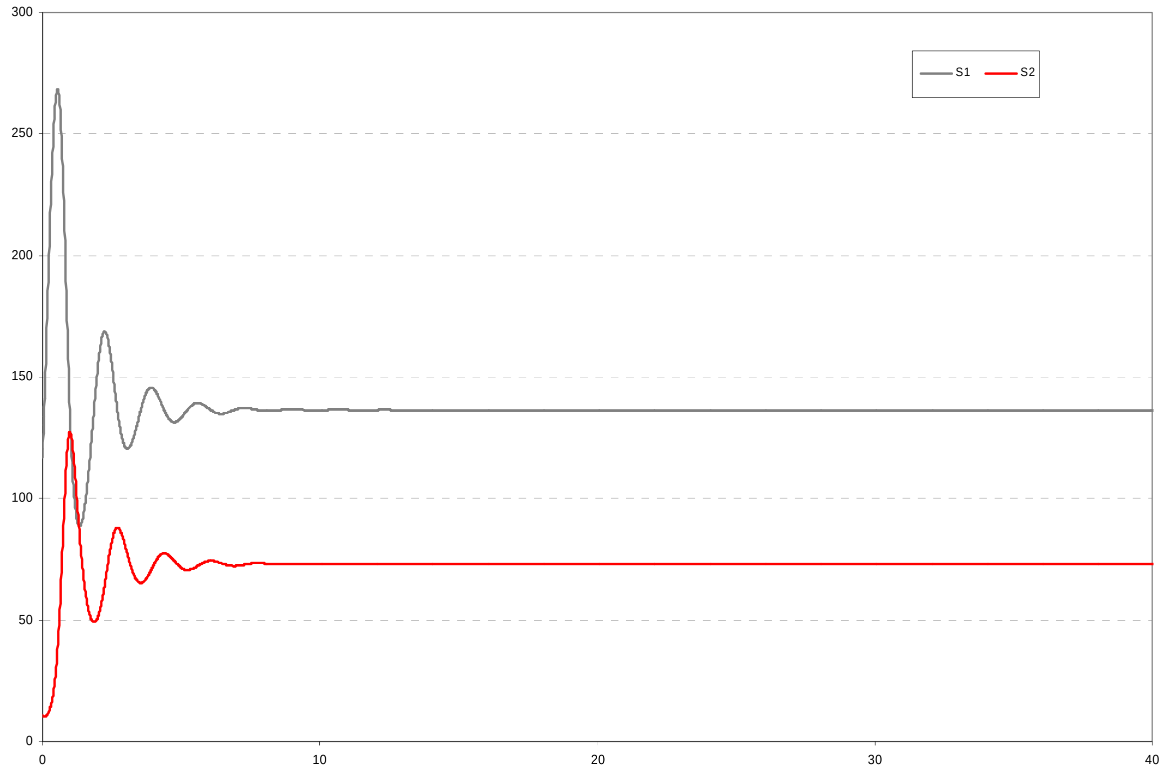


**Fig. 3** The Lotka-Volterra model of Fig. 1 has been stabilized at *t*= 16.06 using Evolutionary Network Control. The solution has been found through the software Control-Lab (Ferrarini, 2013e).

**Table 2** The optimized parameters detected via ENC in order to stabilize as early as possible the Lotka-Volterra system of Fig. 1 by changing the initial parameters of no more than 20%.

| | |
|---|---|
| **$S_1(0)$** | 116.9693275 |
| **$S_2(0)$** | 10.67240538 |
| **a** | 4.580402248 |
| **b** | 0.041595202 |
| **k** | 404.8463846 |
| **c** | 0.833454426 |
| **d** | 4.729172808 |

In addition, ENC can also impose more complex kinds of control, for instance by ruling not only the equilibrium time, but also the final values for $S_1$ and $S_2$ (Ferrarini, 2015). It's clear that ENC can also do opposite, i.e. imposing a late stability to the dynamics of any kind of network.

## 4 Conclusions

The analysis of the stability of dynamical networks is a well-studied topic both in ecology and in biology. In this paper, I have adopted a different perspective: instead of analysing the stability of an arbitrary ecological network, I have searched to impose such stability as soon as possible (or, contrariwise, as late as possible) during network dynamics.

I have showed that Evolutionary Network Control is on top of this task. What's more, ENC can also tame the final values of network's actors, thus assuming a forceful position over the network control. While it's not purpose of this work to discuss the implications of these results in ecology and biology, it's evident that they are not irrelevant.

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