Article

Multi-purpose control of ecological and biological networks

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Abstract
Evolutionary Network Control (ENC) allows the control of any kind of ecological and biological network, with an arbitrary number of nodes and links, acting from inside and/or from outside. To date, ENC has been applied to drive the dynamics of ecological and biological networks so that the target variable can reach the desired equilibrium value. In this work, ENC has been expanded to incorporate the multipurpose control of any kind of ecological and biological network. The rationale here is that, not one, but at least two, or even more than two, variables can be contemporaneously driven towards the desired equilibrium values. In theory, multipurpose ENC can lead an arbitrary number of network actors towards the desired equilibrium values. It is useful whenever ecological and biological networks present several taxonomic resolutions that are worthy to be controlled simultaneously.

Keywords dynamical networks; genetic algorithms; Evolutionary Network Control; edge control; node control; network control; system dynamics.

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1 Introduction

Evolutionary Network Control (ENC) has been introduced to allow the control of any kind of ecological and biological network, with an arbitrary number of nodes and links, acting from inside (Ferrarini, 2013) and/or from outside (Ferrarini, 2013b). ENC opposes the idea, very common in the scientific literature, that controllability of networks should be based on the identification of a subset of nodes to be permanently controlled (Ferrarini, 2011).

ENC can be applied to both discrete-time (i.e., systems of difference equations) and continuous-time (i.e., systems of differential equations) networks. ENC uses an integrated solution (system dynamics - genetic optimization - stochastic simulations) to compute uncertainty about network control (Ferrarini, 2013c) and to compute control success and feasibility (Ferrarini, 2013d). ENC employs intermediate control functions to locally (step-by-step) drive ecological and biological networks (Ferrarini, 2014). ENC can globally subdue nonlinear networks (Ferrarini, 2015), impose early or late stability to any kind of ecological and biological network (Ferrarini, 2015b) and locally control nonlinear networks (Ferrarini, 2016). A decentralized network control has already been conceptualized that is particularly suitable for big data networks (Ferrarini, 2016b).

Table 1 Theory and developed applications of the Evolutionary Network Control (ENC).

Reference	Goal
Ferrarini 2011	Theoretical bases of Evolutionary Network Control
Ferrarini 2013	Endogenous control of linear ecological and biological networks
Ferrarini 2013b	Exogenous control of linear ecological and biological networks
Ferrarini 2013c	Computing the uncertainty associated with network control
Ferrarini 2013d	Computing the degree of success and feasibility of network control
Ferrarini 2014	Local control of linear ecological and biological networks
Ferrarini 2015	Global control of nonlinear ecological and biological networks
Ferrarini 2015b	Imposing early/late stability to linear and nonlinear networks
Ferrarini 2016	Local control of nonlinear ecological and biological networks
Ferrarini 2016b	Decentralized control of ecological and biological networks
This work	Multipurpose control of ecological and biological networks

To date, ENC has been applied to drive the dynamics of ecological and biological networks so that the target variable can reach the desired equilibrium value. In this work, ENC is expanded to incorporate the multipurpose control of any kind of ecological and biological network. The rationale here is that, not one, but at least two, or even more than two, variables can be driven towards the desired equilibrium values. In theory, multipurpose ENC can lead all the network actors towards the desired equilibrium values. An applicative example is given.

2 Multipurpose Evolutionary Network Control: Mathematical Formulation

An ecological (or biological) dynamical system of *n* interacting taxonomic resolutions (species, genera, family, etc.) or aggregated assemblages of taxa (e.g., phytoplankton) is as follows

$$\frac{d\mathbf{S}}{dt} = \gamma(\mathbf{S}(t)) + \mathbf{I}(t) + \mathbf{O}(t) \qquad (1)$$

where S_i is the number of individuals (or the total biomass) of the generic *i-th* taxonomic resolution (species, genera, family, or aggregated assemblages of taxa) with inputs (e.g. species reintroductions) and outputs (e.g. hunting) from outside.

Most real systems' dynamics can be modelled and simulated using a system of canonical linear equations (Luenberger, 1979; Slotine and Li, 1991) which represents a simplification of Eq. (1) as follows

$$\frac{dS_i}{dt} = \underbrace{\alpha_{ii} S_i}_{\text{intra-specific}} + \underbrace{\sum_{j \neq i} \alpha_{ij} S_j}_{\text{inter-specific}} + \underbrace{\sum_k I_k + \sum_k O_k}_{\text{exogenous input-output}} \qquad (2)$$

with initial values

$$\mathbf{S}_0 = \langle S_1(0), S_2(0) \ldots S_n(0) \rangle \qquad (3)$$

and co-domain limits

$$\begin{cases} \mathrm{S}_{1\min} \le S_1(t) \le \mathrm{S}_{1\max} \\ \dots \\ \mathrm{S}_{\mathrm{n}\min} \le S_n(t) \le \mathrm{S}_{\mathrm{n}\max} \end{cases} \tag{4}$$

and where

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \tag{5}$$

is the matrix of the per unit time effect on S_i due to unitary S_j.

The multipurpose control of Eq.2 requires that ENC is on top of the control of the equilibrium values of at least two taxonomic resolutions. For instance, the multipurpose control of 3 variables at equilibrium reads as follows

$$\begin{cases} S_p^{\,Eq} = c_p \\ S_q^{\,Eq} = c_q \\ S_r^{\,Eq} = c_r \end{cases} \tag{6}$$

where $<c_p, c_q, c_r>$ is the vector of equilibrium values *a priori* chosen for the generic variables $<S_p, S_q, S_r>$.

Multipurpose ENC solves the multipurpose control of Eq. 2 using the following control equation

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11*}S_1^{\,*} + \dots + a_{1n*}S_n^{\,*} + I_{1*} + O_{1*} \\ \dots \\ \dfrac{dS_n}{dt} = a_{n1*}S_1^{\,*} + \dots + a_{nn*}S_n^{\,*} + I_{n*} + O_{n*} \\ M_c = \gamma \end{cases} \tag{7}$$

with

$$\gamma \in [0,1] \tag{8}$$

where any component (variable, parameter or coefficient) can be tamed, as denoted by the asterisk, using genetic algorithms (GA; Holland, 1975; Goldberg, 1989) to drive the network towards the desired state.

The variable M_c takes into account if all the taxonomic resolutions chosen for the multiple control have reached the desired values at equilibrium. It can be thought as a binary variable (1 if multipurpose control was reached, 0 otherwise) that switches on (until $M_c = 0$) and off (when $M_c = 1$) the control effort by ENC.

In a wider context, it could be conceived as a continuous variable ranging in the [0-1] interval that computes the normalized Euclidean distance of actual variables’ state with respect to the vector of desired equilibrium values. In case a threshold value, a priori decided by the user, is reached, M_c becomes equal to 1 and the multipurpose control effort stops.

Multipurpose ENC can also use an exogenous network control through an 1-external-controller C_1

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} \\ \ldots \\ \dfrac{dS_n}{dt} = a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} \\ \dfrac{dC_1}{dt} = f_1S_1 + \ldots + f_nS_n \\ M_c = \gamma \end{cases} \tag{9}$$

where asterisks stand for the genetic optimization of exogenous node's edges (i.e., coefficients of interaction with the inner system) and exogenous node's stock, i.e. the modification of such values at the beginning of network dynamics in order to get a certain goal. The controller C_1 can receive feedbacks (i.e. dC_1/dt) from the network, that in turn could be subject to control by taming $<f_1 \ldots f_n>$.

In case one controller is not enough, the model in (9) must be expanded to the following *k*-external-controllers model:

$$\begin{cases} \dfrac{dS_1}{dt} = a_{11}S_1 + \ldots + a_{1n}S_n + I_1 + O_1 + c_{11*}C_{1*} + \ldots + c_{1k*}C_{k*} \\ \ldots \\ \dfrac{dS_n}{dt} = a_{n1}S_1 + \ldots + a_{nn}S_n + I_n + O_n + c_{n1*}C_{1*} + \ldots + c_{nk*}C_{k*} \\ \dfrac{dC_1}{dt} = f_{11}S_1 + \ldots + f_{1n}S_n \\ \ldots \\ \dfrac{dC_k}{dt} = f_{k1}S_1 + \ldots + f_{kn}S_n \\ M_c = \gamma \end{cases} \tag{10}$$

However, many ecological (or biological) dynamical systems can be more properly described using difference (recurrent) equations rather than differential ones. This is true for many systems where dynamics happen on discrete time rather than on continuous one. In this case, Eq. 2 becomes

$$\begin{cases} \left(S_1\right)_{t+1} = a_{11}\left(S_1\right)_t + \ldots + a_{1n}\left(S_n\right)_t + \left(I_1\right)_t + \left(O_1\right)_t \\ \ldots \\ \left(S_n\right)_{t+1} = a_{n1}\left(S_1\right)_t + \ldots + a_{nn}\left(S_n\right)_t + \left(I_n\right)_t + \left(O_n\right)_t \end{cases} \tag{11}$$

and multipurpose ENC solves the control of Eq. 11 using the following

$$\begin{cases} (S_1)_{t+1} = a_{11*}(S_1)_t + \dots + a_{1n*}(S_n)_t + (I_1)_{t*} + (O_1)_{t*} \\ \dots \\ (S_n)_{t+1} = a_{n1*}(S_1)_t + \dots + a_{nn*}(S_n)_t + (I_n)_{t*} + (O_n)_{t*} \\ M_c = \gamma \end{cases} \qquad (12)$$

where asterisks stand for the optimization of edges (i.e., coefficients of interaction among variables) or nodes (i.e., initial stocks), that is the modification of their values at the beginning of the network dynamics in order to get a certain goal.

3 An Applicative Example

Figure 1 depicts an *in silico* simulation of a real ecological network. Greenish nodes represent actors or events that act positively for the goal of the network control, i.e. the increase or preservation of the target species. Reddish nodes represent ecological actors or events with negative impact on the target species. Bluish nodes represent resources needed by the target species. Stocks stand for the actual amounts of individuals or biomass. Updates stand for yearly internal dynamics (i.e., intra-specific gains due to births and/or immigration rates minus losses due to deaths and/or emigration rates). Minimum and maximum values stand for lowest and highest values of stock values. For the sake of simplicity, the maximum possible value for each actor has been set to 100. Hunters and poachers remain constant (i.e. 10) during the simulation. The percent value associated to links represents the percentage of the receiver that is yearly consumed by the transmitter at the beginning of the network simulation. Traps mortality and re-introductions accounts for 19 and 10 individuals per year respectively. Predators can also gain resources from outside, so their internal dynamics (updates) are not limited to the presence of the target species.

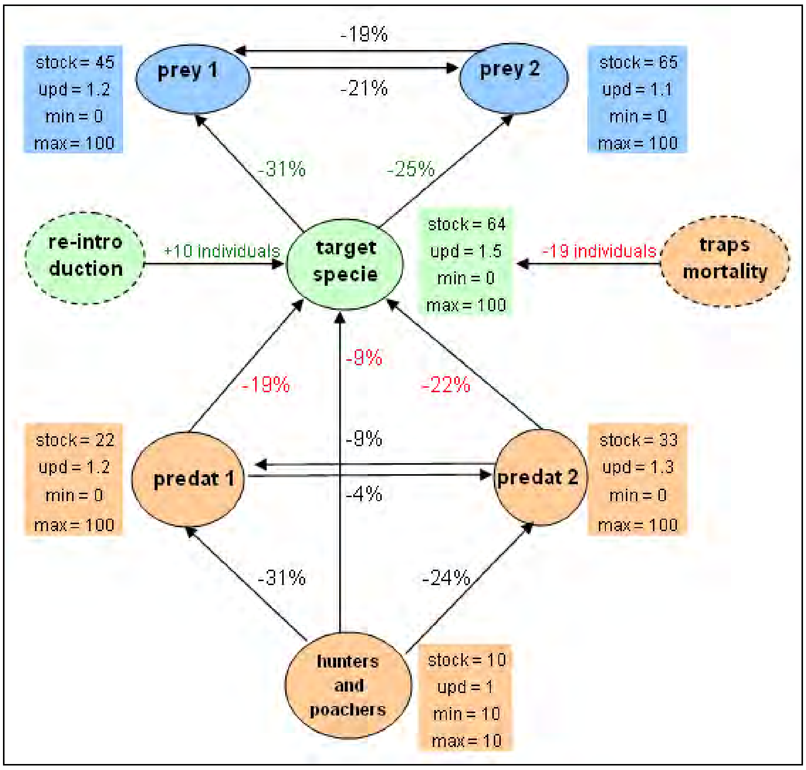
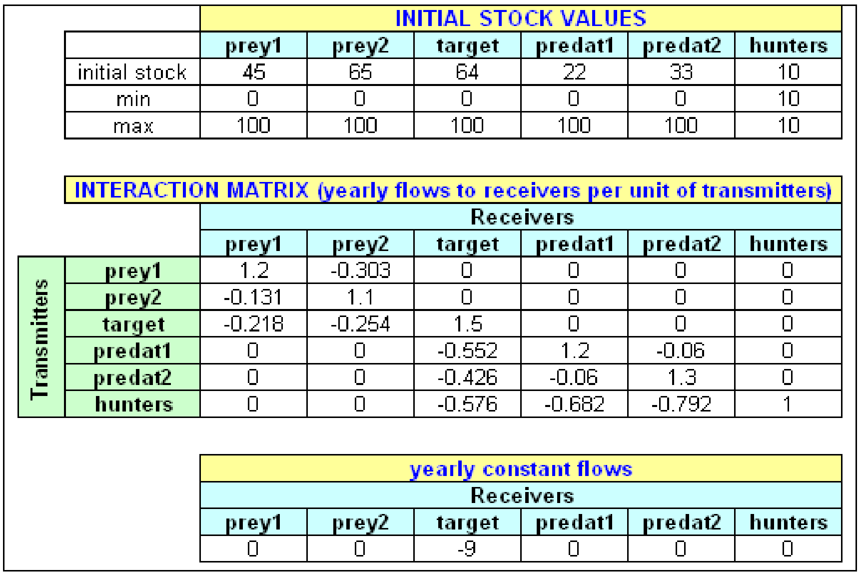


Fig. 1 Initial conditions of the ecological network under study.

Table 1 Parameter declaration corresponding to the ecological network of Fig. 2, calculated using the software Control-Lab 6 (Ferrarini, 2015c).



INITIAL STOCK VALUES

	prey1	prey2	target	predat1	predat2	hunters
initial stock	45	65	64	22	33	10
min	0	0	0	0	0	10
max	100	100	100	100	100	10

INTERACTION MATRIX (yearly flows to receivers per unit of transmitters)

		Receivers					
		prey1	prey2	target	predat1	predat2	hunters
Transmitters	prey1	1.2	-0.303	0	0	0	0
	prey2	-0.131	1.1	0	0	0	0
	target	-0.218	-0.254	1.5	0	0	0
	predat1	0	0	-0.552	1.2	-0.06	0
	predat2	0	0	-0.426	-0.06	1.3	0
	hunters	0	0	-0.576	-0.682	-0.792	1

yearly constant flows

Receivers					
prey1	prey2	target	predat1	predat2	hunters
0	0	-9	0	0	0

The previous ecological network has the following inertial dynamics (Fig. 2), with equilibrium time $E_T = 22$ years and the *target species* (green line) disappearing in the studied ecosystem after 5 years. The final vector at $E_T = 22$ is given by <*prey1*=100, *prey2*=0, *target*=0, *predat1*=0, *predat2*=100>.

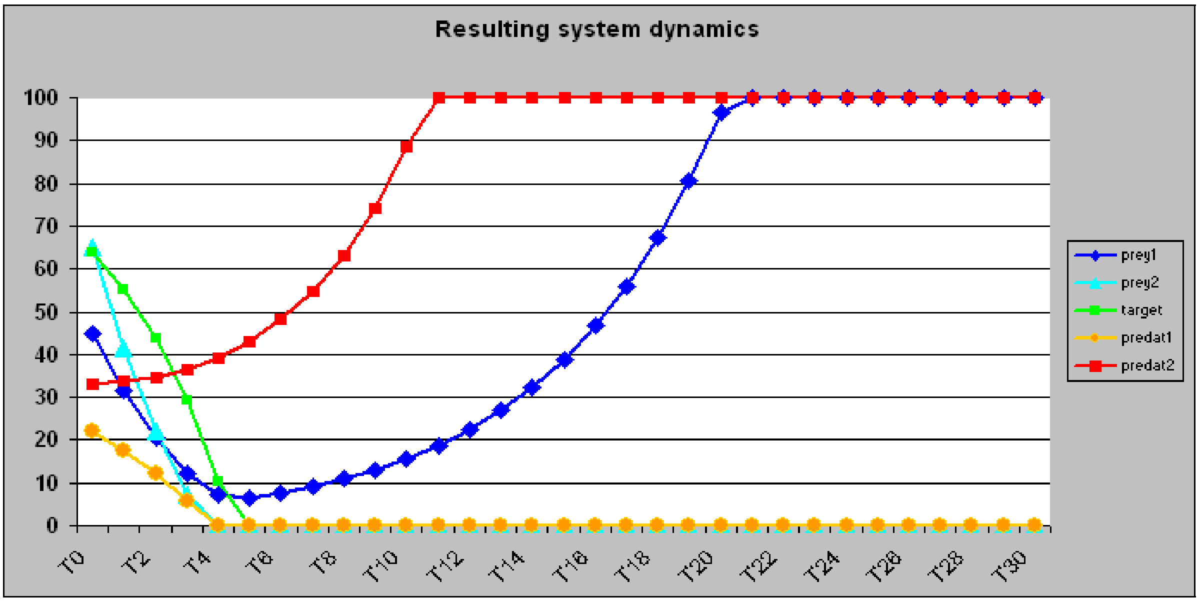


Fig. 2 Inertial dynamics for the network of Fig. 2. X-axis measures time in years. Dynamics have been calculated using the software Control-Lab 6 (Ferrarini, 2015c).

Now let's suppose we want to achieve the following multipurpose control for the network of Fig. 1

- target species equal to 100 at equilibrium

- *prey1* equal to 100 at equilibrium
- *predat2* equal to 100 at equilibrium

by acting upon the target species': a) initial stock, b) intra-specific coefficient, c) inter-specific coefficient upon prey1, d) inter-specific coefficient upon prey2.

Multipurpose ENC has found the solution depicted in Fig. 3 that has been achieved through the following parameters (and leaving all the other network's parameters unchanged):

a) target species' initial stock= 64

b) target species' intra-specific coefficient= 1.93

c) target species' inter-specific coefficient upon *prey1*= -0.06

d) target species' inter-specific coefficient upon *prey2*= -0.52

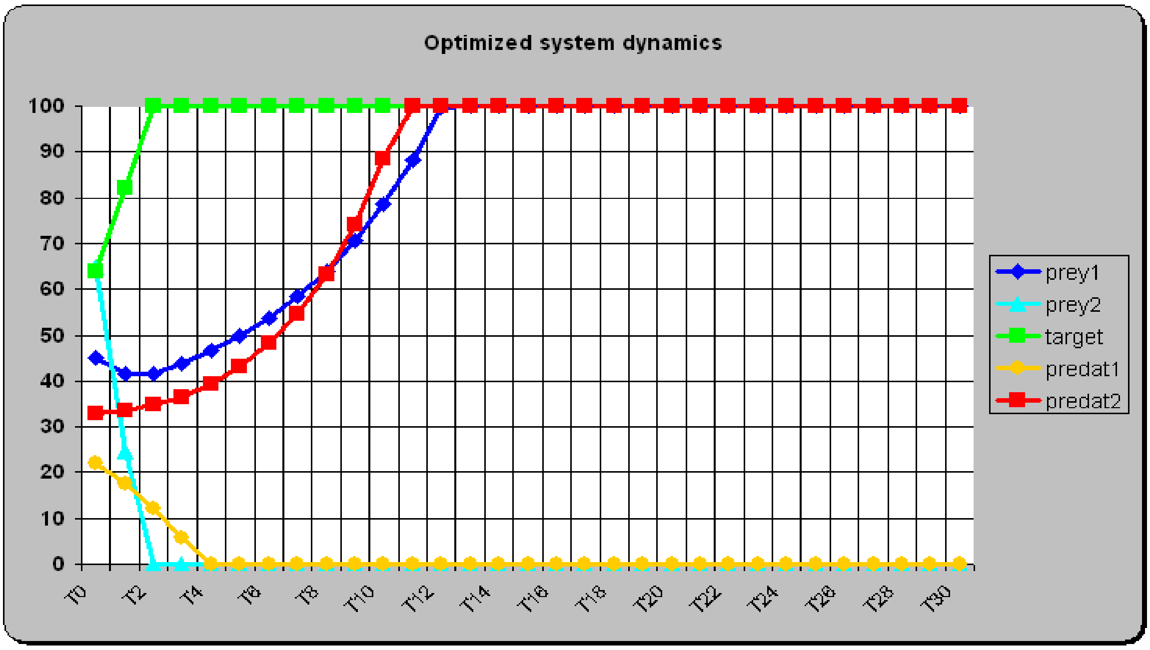


Fig. 3 A solution detected by ENC to the multipurpose control of the ecological network depicted in Fig. 1. X-axis measures time in years. Solutions have been detected using the software Control-Lab 6 (Ferrarini, 2015c).

Now let's suppose we seek a more complex, multipurpose control at equilibrium

- target species equal to 100
- *predat2* equal to 100
- equilibrium time ≤12

by changing network parameters of no more than 10%.

Multipurpose ENC has found the solution shown in Fig. 4 that has been achieved through the network parameters depicted in Table 2.

Of course, any other kind of multipurpose control is feasible. The modelling framework proposed here might also be applied to semi-quantitative networks (Ferrarini 2011b). Multipurpose ENC has been applied using the software Control-Lab 6 (Ferrarini, 2015c) written in Visual Basic (Balena, 2001; Pattison, 1998).

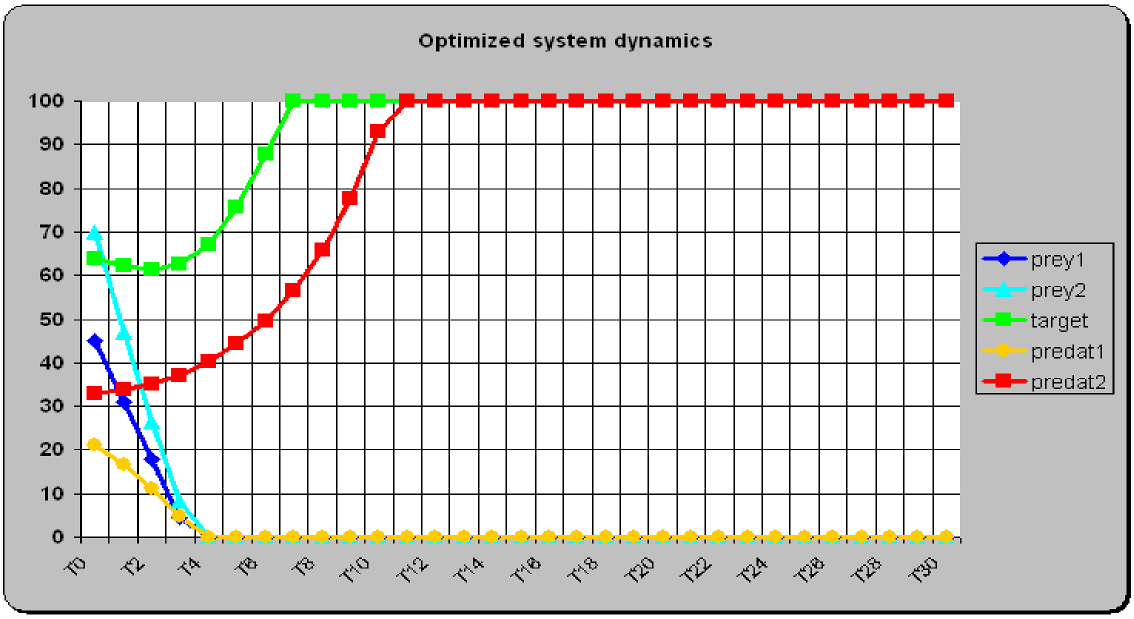
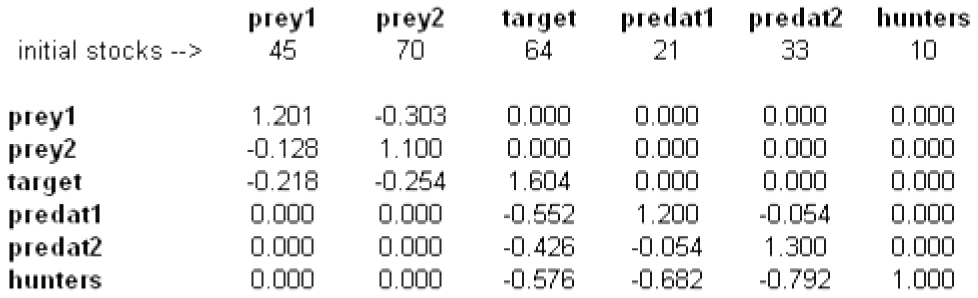


Fig. 4 A solution detected by ENC to the multipurpose control of the ecological network presented in Fig. 1 by changing network parameters of no more than 10%. X-axis measures time in years. Solutions have been detected using the software Control-Lab 6 (Ferrarini, 2015c).

Table 2 Network control achieved by Multipurpose ENC by changing network parameters of no more than 10% in order to drive the ecological network of Fig. 1 towards the dynamics depicted in Fig. 4. Transmitters are on the rows, receivers on the columns. Multipurpose network control has been achieved using the software Control-Lab 6 (Ferrarini, 2015c).



	prey1	prey2	target	predat1	predat2	hunters
initial stocks -->	45	70	64	21	33	10
prey1	1.201	-0.303	0.000	0.000	0.000	0.000
prey2	-0.128	1.100	0.000	0.000	0.000	0.000
target	-0.218	-0.254	1.604	0.000	0.000	0.000
predat1	0.000	0.000	-0.552	1.200	-0.054	0.000
predat2	0.000	0.000	-0.426	-0.054	1.300	0.000
hunters	0.000	0.000	-0.576	-0.682	-0.792	1.000

4 Conclusions

Multipurpose Evolutionary Network Control (ENC) has been introduced here as an expansion of single-target ENC. It allows to drive an arbitrary number of network actors toward the desired equilibrium values. It is helpful whenever ecological and biological networks present several taxonomic resolutions that are worthy to be controlled simultaneously.

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