Article

One-way ANOVA and comfortless questions: Direct computer experiment

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Abstract
For matrix 5x5 (5 gradations of a factor with 5 values of characteristics in every gradation) values of normally distributed stochastic variables were generated with equal averages (for all gradations of a factor) and equal variances. Results of application of Kolmogorov's test to samples of variance ratios (for checking of correspondence of samples to Fisher distribution) are presented in table for 5% significance level. Every element of table was obtained after analysis of 10^7 independent samples. Changing of elements of table was analyzed with Theil criterion. Obtained results demonstrate that there exist some problems with identification of distribution of variance ratio and in analysis of its correspondence to Fisher distribution.

Keywords one-way ANOVA; Kolmogorov's test; distribution of variance ratio; Theil criterion.

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1 Introduction

In our previous publications (Nedorezov, 2012, 2016, 2017) it was noted that a lot of comfortless questions can be addressed to one-way ANOVA. Let's consider a matrix $m \times n$ (n is a number of factor gradations, and m is a number of measurements of characteristic for every gradation) with equal elements: $x_{ij} = a = const$. For this dataset we can conclude that factor has no influence onto value of characteristic: all averages for all gradations of factor are equal to each other. Thus factor is very weak. On the other hand, factor is extremely strong because influence of factor is so hard that for every gradation we can't observe variation of characteristic.

In other words, if $D_y \approx 0$ we may have serious problems in identification of a role of considering factor. But we can decrease a value of D_y artificially by changing a scale of measurements (for example, if we will use tons for larva weights instead of grams…). The second within the framework of traditional one-way ANOVA we have to calculate variance ratio and compare it with respective/table value of Fisher distribution (Hollander and Wolfe, 1973; Lakin, 1990; Kobzar, 2006; Vasiliev and Melnikova, 2009). But we have a background for this comparison if and only if variance ratio has Fisher distribution. Theory of one-way ANOVA is perfect (Hollander and Wolfe, 1973; Scheffe, 1980; Aivazyan et al., 1985). But pointed out property of variance ratio and its correspondence to Fisher distribution we have to and must check with direct computer experiment.

In current publication for matrix 5x5 (5 gradations of factor and 5 values of characteristic per every gradation) we modeled artificial normally distributed datasets with equal averages and variances. Within the framework of traditional one-way ANOVA such factor A must be determined as a weak factor (because averages for all gradations of factor are equal to each other). For every matrix variance ratio was calculated. After that 10^7 independently generated samples (with same characteristics) of variance ratios were tested by Kolmogorov's test onto correspondence to Fisher distribution with respective parameters. There were analyzed a certain number of various cases (with various sample sizes of variance ratios, and various values of variances) with 5% significance level of Kolmogorov's test.

Obtained results allow concluding that distribution of variance ratios can be rather close to Fisher distribution. But some additional effects don't allow us concluding that there is strong correspondence between distributions. Results are presented as numbers of negative results (when Null hypothesis about correspondence of sample distribution to Fisher distribution must be rejected by Kolmogorov's test) after checking of 10^7 independent samples of variance ratios. Moreover, if distribution of variance ratios corresponds to Fisher distribution we have to have decrease (tendency to decreasing) of number of negative results. Application of Theil criterion (with 5% significance level) shows that we can observe (statistically confident) tendency to increasing of number of negative results.

2 Description of Computer Experiment

Let's assume that x_{ij} for all i and j are values of independent stochastic variables with Normal distribution and with equal averages (below we'll consider a case when all averages are equal to one). After providing of experiments (or observations) we get a set of numbers x_{11}, x_{12},…, x_{1m_1} which correspond to first gradation of a factor A. Number m_1 is a sub-sample size. We have also numbers x_{21}, x_{22},…, x_{2m_2} corresponding to second gradation of a factor and so on. Last part of initial sample is x_{n_A1}, x_{n_A2},…, $x_{n_Am_{n_A}}$. Let N be a (total) sample size, $N = m_1 + m_2 + ... + m_{n_A}$. We'll also assume that sample variances for all gradations are equal. As it was pointed out above, in such a situation factor must be identified as weak regulator which has no influence on values of considering characteristic.

For applying of analysis of variance to matrix $\|x_{ij}\|$ we have to calculate following amounts:

$$D_y = \sum_{ij} (x_{ij} - \bar{x})^2 . \tag{1}$$

$$D_x = \sum_{i=1}^{n_A} \frac{m_i}{N} (\bar{x} - \bar{x}_i)^2 . \tag{2}$$

$$D_e = \sum_{i=1}^{n_A} \left[\sum_{j=1}^{m_i} (x_{ij} - \bar{x}_i)^2 \right]. \tag{3}$$

In expression (1) (total deviate) summarizing is provided for all possible values of i and j (below we'll consider a particular case when $i, j = 1,2,3,4,5$; in other words, we'll assume that we have five gradations of a factor, sample size is equal to 25, $N = 25$, and we have 5 elements for every gradation of a factor, $m_1 = ... = m_5 = 5$). Amount $\bar{x}$ is an average for whole sample:

$$\bar{x} = \frac{1}{N}\sum_{ij} x_{ij} .$$

Amount $\bar{x}_i$ is an average for gradation i. For every deviate (1)-(3) there is a certain number of degrees of freedom: we can divide deviates (1)-(3) onto respective number of degrees of freedom, and get sample variances:

$$s_y^2 = \frac{D_y}{N-1},\ s_x^2 = \frac{D_x}{n_A - 1},\ s_e^2 = \frac{D_e}{N - n_A} .$$

Amount s_y^2 is a total sample variance for all initial values; s_x^2 is between group sample variance (factorial variance); and s_e^2 is residual variance. Final solution (about influence of a factor A onto values of characteristics) is based on relation of two sample variances (variance ratio):

$$F_{fact} = \frac{s_x^2}{s_e^2} . \quad (4)$$

Amount (4) we have to compare with table value for Fisher distribution with fixed value of confidence level and numbers of degrees of freedom $n_A - 1$ and $N - n_A$. Note that this comparison has real sense if and only if (4) has Fisher distribution too. For checking of this hypothesis for matrix 5x5 we modeled values x_{ij} with following formula: $x_{ij} = 1 + \sigma\xi_{ij}$. In this expression ξ_{ij} are independent stochastic variables with standard Normal distribution. σ is variance of stochastic amounts x_{ij}. Values of stochastic variables ξ_{ij} were modeled with formula:

$$\xi = \sqrt{\frac{12}{n}}\left(\sum_{k=1}^{n}\alpha_k - \frac{n}{2}\right).$$

In this expression α_k are independent stochastic variables with uniform distribution (Rnd) on $[0,1]$, $n = 12$.

Table 1 (part 1) Results of computer experiments: numbers of negative results (Null hypothesis of correspondence of distribution of sample of variance ratios to Fisher distribution) at application of Kolmogorov's test.

σ	Sample size M								
	8	9	10	11	12	13	14	15	16
0.1	324352	334864	343602	353313	362127	366680	373895	379953	382566
0.2	324399	334897	343506	353328	362205	366785	373943	379932	382554
0.3	324370	334932	343470	353418	362352	366381	373891	380225	382554
0.4	324344	334810	343565	353447	362344	366788	373971	380130	382596
0.5	324376	334939	343483	353266	362236	366597	373950	380070	382533
0.6	324437	335032	343580	353034	362155	366688	373784	380045	382524
0.7	324390	334934	343535	353126	362269	366318	373926	380167	382546
0.8	324388	335029	343252	353158	362248	366555	373829	380107	382545
0.9	324330	334886	343496	353236	362088	366392	373844	380056	382483
1.0	324338	334955	343240	353428	362101	366412	373959	379958	382541
1.1	324372	334851	343586	353260	362123	366773	374033	380045	382551
1.2	324396	334936	343647	353205	362197	366415	373821	379970	382553
1.3	324438	335065	343265	352989	362121	366437	373850	380283	382582
1.4	324396	334850	343500	353478	362268	366511	373946	380119	382584
1.5	324374	334984	343564	353306	362114	366374	373851	380045	382564
1.6	324376	334881	343510	353294	362344	366600	373816	380259	382476
1.7	324453	334852	343543	353132	362139	366578	373725	379987	382480

1.8	324359	334875	343352	353375	362217	366346	373839	380124	382536
1.9	324360	334905	343648	353313	362234	366307	373991	380048	382569
2.0	324404	334974	343507	353421	362141	366767	374024	380077	382531

Table 1 (part 2)

σ	Sample size M								
	17	18	19	20	21	22	23	24	25
0.1	388124	393174	395027	397048	400574	403337	406462	405117	409926
0.2	388200	393244	395080	397151	400617	403135	406656	405116	409789
0.3	388136	393106	395068	397119	400531	403154	406692	405106	409591
0.4	388103	393332	395089	397235	400638	403171	406629	405113	410076
0.5	388149	393132	394985	397009	400506	403482	406691	405059	409762
0.6	388229	393291	395087	397179	400700	403535	406698	405059	409816
0.7	388025	393416	395215	397066	400561	403553	406378	405120	409767
0.8	388306	393290	395076	397040	400687	403600	406617	405068	409828
0.9	388305	393098	395127	397142	400487	403321	406412	405135	409981
1.0	388074	393240	395001	397056	400676	403590	406296	405153	409980
1.1	388201	393227	395056	397078	400666	403106	406642	405058	410008
1.2	388103	393098	395064	397119	400612	403560	406568	405028	409799
1.3	388131	393100	395109	397175	400683	403664	406609	405116	409841
1.4	388357	393137	395193	397141	400517	403643	406689	405052	409788
1.5	388309	393440	395168	397076	400635	403496	406328	405037	410086
1.6	388218	393234	395174	397175	400694	403050	406438	405076	409765
1.7	388213	393074	395168	397105	400660	403624	406715	405114	410012
1.8	388297	393251	395094	397084	400535	403599	406643	405094	410006
1.9	388185	393098	395149	397072	400708	403491	406698	405088	409710
2.0	388163	393333	395083	397056	400669	403551	406685	405061	409843

After modeling of matrix $\|x_{ij}\|$ and calculating of value of variance ratio (4) new sample was organized: this sample contained independent values of variance ratios only. This new sample (with sample size M) was tested by Kolmogorov's test (with 5% significance level; Bolshev and Smirnov, 1983) onto its correspondence with Fisher distribution. For every fixed value of sample size M , $M = 8,9,...,25$, and fixed value of variance σ , $\sigma = 0.1,0.2,...,2.0$, 10^7 independent samples (of variance ratios) were tested onto its correspondence to Fisher distribution. Results of calculations are presented in Table 1.

As we can see in this Table 1 number of negative results (when Kolmogorov's test shows that Null hypothesis about correspondence of sample distribution to Fisher distribution, must be rejected) increases at increase of sample size M . But it is in contradiction with natural imagination about dependence on sample size: if distribution of variance ratios corresponds to Fisher distribution than increase of sample size must lead to decrease of number of negative results. Moreover, it is important to note that increase of sample size M leads to situation when frequency of appearance of negative results becomes bigger than significance level. In particular, for significance level 0.1% we have following values of frequencies q of negative results: for $\sigma = 2$ and $M = 195$ $q = 0.0010914$, for $\sigma = 0.1$ and $M = 195$ $q = 0.0010924$, for $\sigma = 1$ and $M = 140$ $q = 0.0010218$, for $\sigma = 1$ and $M = 185$ $q = 0.0011025$ etc. For analysis of dependence of numbers of negative results on sample size Theil criterion was used (we have to be sure that observed dependence isn't not a pure stochastic effect).

3 Theil Criterion: Analysis of Dependence of Frequencies on Sample Size

For every fixed value σ hypothesis about equivalence of coefficient of linear regression a is equal to zero, $H_0:\ a = 0$ (with alternative hypothesis $H_1:\ a \neq 0$) was checked with Theil criterion with 5%

significance level. Critical level (when sample size is equal to 18; see Table 1) for Theil criterion is equal to 51.745. With critical level we have to compare absolute value of following function:

$$C = \sum_{i<j}^{m} \delta(y_j - y_i),$$

where C is calculated for fixed value of σ, y_i are elements of respective row of table 1, and function

$$\delta(x) = \begin{cases} 1, x > 0, \\ 0, x = 0, \\ -1, x < 0. \end{cases}$$

For all rows of table 1 $C = 151$ (maximum is equal to 153). It means that we have to reject Null hypothesis (about absence of increasing of number of negative results with increasing of sample size) with rather small significance level. Analysis of particular case when $\sigma = 1$ and sample size is equal to 43 showed that $C = 885$. Critical value for Theil criterion is equal to 187.28 (for 5% significance level). Finally, as it was obtained with Theil criterion effect of increasing of number of negative results at increase of sample size (and it is in contradiction with standard imagination about correspondence of any sample to considering distribution: increase of sample size must lead to decrease of number of negative results when criterion allows rejecting of Null hypothesis) isn't pure stochastic effect.

4 Conclusion

Theory of one-way ANOVA is perfect (Hollander and Wolfe, 1973; Scheffe, 1980; Aivazyan et al., 1985). But direct computer experiments don't allow us concluding that we have strong correspondence between Fisher distribution and distribution of variance ratios. It is necessary to check the problem for matrices with other characteristics, to check the correspondence with other statistical criterions etc.

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