Article

A cellular automaton for population diffusion in the homogeneous rectangular area

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Abstract
In this paper, a cellular automaton for population diffusion was introduced. A group of discrete partial differential equations was used to simulate population diffusion in the homogeneous rectangular area. The population dynamics was described by Malthus model, Logistic model, and oscillation model. The cellular automaton can be used to analyze the effects of initial distribution of organisms on diffusion process and distribution pattern, to estimate the diffusion speed and possible diffusion directions, and to determine the major regions occupied by organisms.

Keywords cellular automaton; homogeneous environment; population diffusion.

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1 Introduction

Cell automata are a kind of agent-based modeling (Zhang, 2014b). They have been widely used in emergent modeling and are proved to be a powerful tool (Qi et al., 2004; Zhang, 2012). Population diffusion is a self-organizing process, which possesses the basic properties that a self-organizing system holds (Zhang, 2013, 2014a). In this paper, a cellular automaton for population diffusion in the homogeneous rectangular area was introduced, aiming to provide a tool for exploiting the mechanism of population diffusion in the field.

2 Cellular Automaton

The cellular automaton was used to describe population diffusion in the homogeneous rectangular area (Qi, 2003; Qi et al., 2004).

Suppose the area has $c \times f$ cells. Given a cell, individuals in the cell diffuse to adjacent four cells; or, individuals in the adjacent four cells diffuse to this cell. Population diffuses until all cells have the same number of individuals. At this time the number of individuals for immigration and migration are the same in each cell. The time step for diffusion is ∂t and the diffusion coefficient is a. The diffusion rules are thus

$$\partial u(1,1,t)/ \partial t=a*(u(1,2,t)+u(2,1, t)-2*u(1,1,t))$$
$$\partial u(1,c,t)/ \partial t=a*(u(1,c-1,t)+u(2,c,t)-2*u(1,c,t))$$
$$\partial u(1,j,t)/ \partial t=a*(u(1,j-1,t)+u(2,j,t)+u(1,j+1,t)-3*u(1,j,t))$$
$$\partial u(f,1,t)/ \partial t=a*(u(f-1,1,t)+u(f,2,t)-2*u(f,1,t))$$
$$\partial u(f,c,t)/ \partial t=a*(u(f-1,c,t)+u(f,c-1,t)-2*u(f,c,t))$$
$$\partial u(f,j,t)/ \partial t=a*(u(f,j-1,t)+u(f-1,j,t)+u(f,j+1,t)-3*u(f,j,t))$$
$$\partial u(i,1,t)/ \partial t=a*(u(i-1,1,t)+u(i,2,t)+u(i+1,1,t)-3*u(i,1,t))$$
$$\partial u(i,c,t)/ \partial t=a*(u(i-1,c,t)+u(i,c-1,t)+u(i+1,c,t)-3*u(i,c,t))$$
$$\partial u(i,j,t) /\partial t=a*(u(i-1,j,t)+u(i,j-1,t)+u(i+1,j,t)+u(i,j+1,t)-4*u(i,j,t))$$
$$i=1,2,\ldots,f;\ j=1,2,\ldots,c$$

if $u(i,j,t)<0$, then let $u(i,j,t)=0$.

Regardless of diffusion process, the natural population dynamics in a cell can be described by Malthus model, Logistic model, or oscillation model

$$\partial u(i,j,t) /\partial t=ru$$
$$\partial u(i,j,t) /\partial t=ru(K-u)/K$$
$$\partial u(i,j,t) /\partial t=-2\pi/T*A*\sin(2\pi t/T+\varphi)$$

where, $u=u(x,y,t)$: population size of the cell (x,y) at time t; r: intrinsic rate, $-\infty<r<\infty$; K: environmental capacity of a cell; φ: phase, $0\leq\varphi\leq 2\pi$; $A=(u-b)/\cos(\varphi)$, and b is the mean number of individuals per cell; T: oscillation cycle, $0<T<\infty$.

3 Results and Discussion

Suppose the diffusion coefficient a=0.5, and the environmental capacity per cell K=30. Initial distribution of population in the area (10×8 cells) is

3	18	21	3	1	0	0	0
1	1	0	2	1	0	0	0
0	1	2	6	1	1	0	0
1	0	0	1	0	1	0	1
0	0	1	1	0	12	24	0
2	1	2	2	0	1	0	0
0	0	0	0	1	0	1	0
0	0	1	0	0	1	0	0
0	0	0	2	0	1	0	0
0	0	0	0	0	0	0	0

Assume the population grows in Logistic pattern. Let intrinsic rate r=0.1, 0.2, 0.3, 0.4 and 0.5, respectively. Simulated dynamics of total population is shown in Fig. 1.

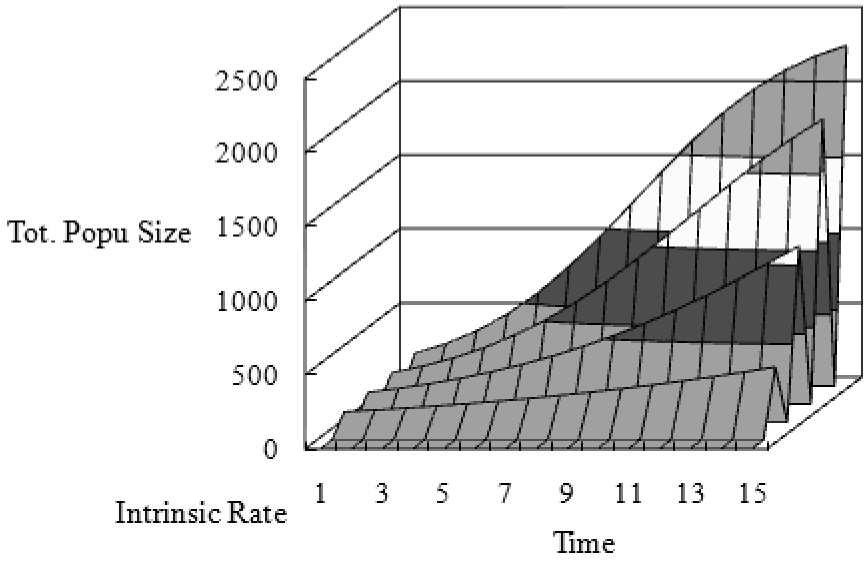


Fig. 1 Comparison of total population using different intrinsic rates in the model.

Overall the population grows in the Logistic pattern. A greater intrinsic rate will result in the larger population size. On the other hand, population size of each cell will not exceed the environmental capacity, *K*=30. Due to the effect of diffusion, the population dynamics in a given cell is not certainly a Logistic growth, as indicated in Fig. 2.

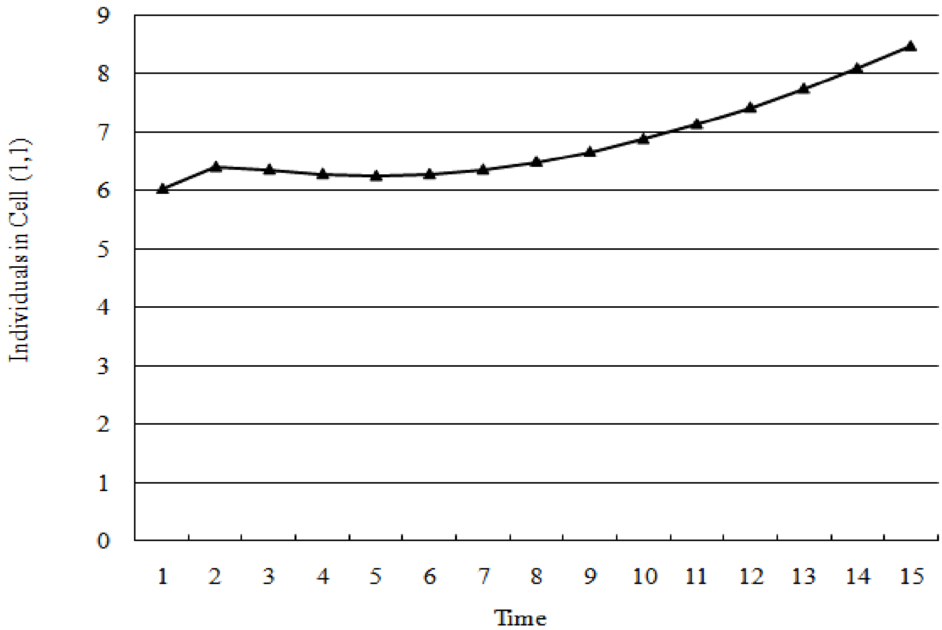
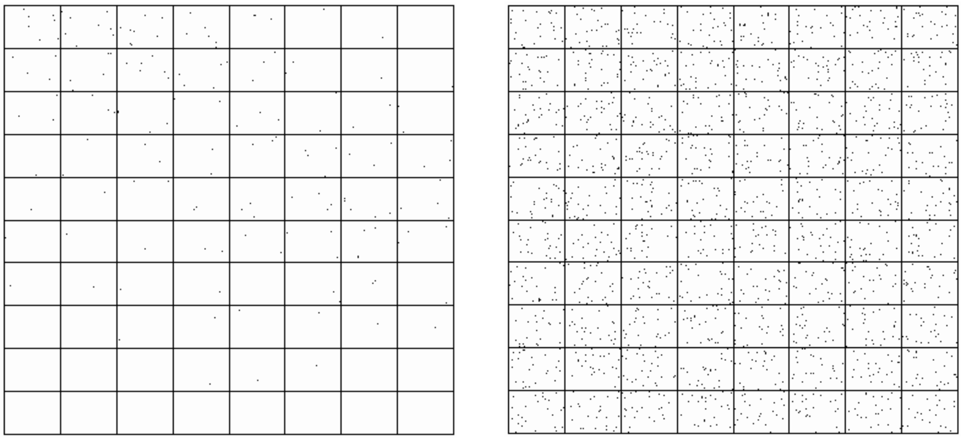


Fig. 2 Population dynamics in the cell (1, 1), with intrinsic rate r=0.1.

Fig. 3 shows the comparison of distribution of individuals. Although the initial distribution is highly heterogeneous, it will tend to be uniform after a period of diffusion due to the limitation of given uniform environmental capacity and the little intrinsic rate compared to diffusion coefficient. In this case, the latter is more significant.



(a) (b)

Fig. 3 Distribution of individuals during Logistic growth, intrinsic rate r=0.2. (a) t=2; (b) t=25.

Population will reach a steady state, with the same number of individuals (30 in the situation of Logistic growth with environmental capacity, K=30) in each cell and, population will change if a cell is disturbed, or the environmental capacity is changed, until the steady state is reached. It is a self-organizing process.

For a population dynamics with oscillation pattern (generally found in the situation of year by year changes of biological population), let oscillation cycle T=5, phase φ=0, mean number of individuals per cell b=10. The initial distribution of population is the same as the mentioned above. For a given cell, the oscillation amplitude decreases with the increase of diffusion coefficient (Fig. 4; Table 1).

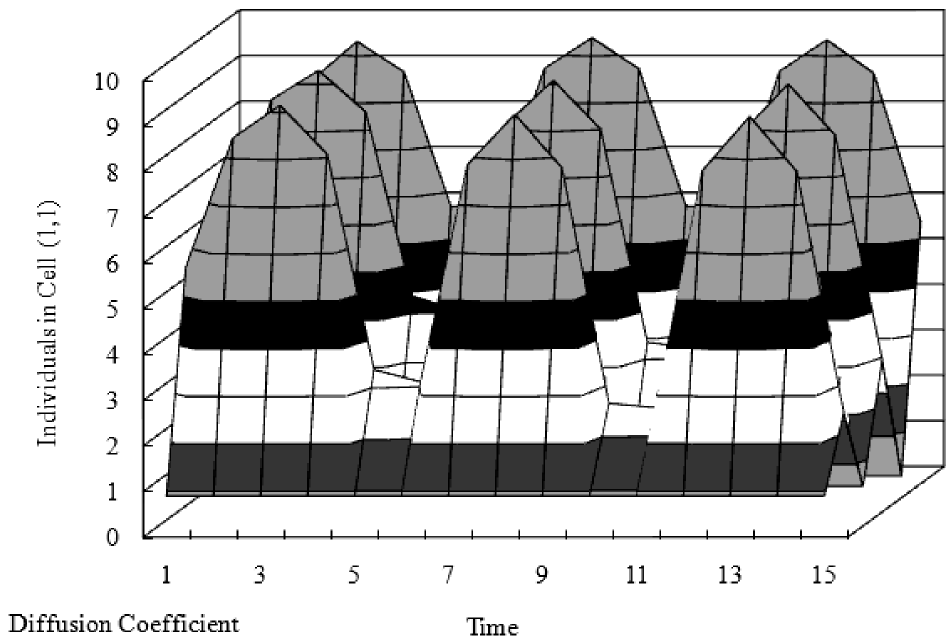


Fig. 4 Population dynamics in the cell (1, 1), for oscillating dynamics.

Table 1 Population dynamics in the cell (1, 1), for oscillating dynamics with different diffusion coefficients (*a*).

a	*u*	*a*	*u*	*a*	*u*
0.9	5.6113	0.5	5.616	0.1	4.0637
0.9	8.438	0.5	8.6589	0.1	8.424
0.9	9.1551	0.5	9.2958	0.1	9.3119
0.9	8.0858	0.5	8.3945	0.1	8.6432
0.9	3.3623	0.5	4.3504	0.1	5.6938
0.9	3.0944	0.5	4.0322	0.1	5.7591
0.9	7.8625	0.5	8.1275	0.1	8.7288
0.9	8.9579	0.5	9.0763	0.1	9.3905
0.9	7.7845	0.5	8.0165	0.1	8.7122
0.9	2.6219	0.5	3.3382	0.1	5.7077
0.9	2.5293	0.5	3.205	0.1	5.6292
0.9	7.7313	0.5	7.9233	0.1	8.6609
0.9	8.9093	0.5	8.996	0.1	9.3492
0.9	7.7056	0.5	7.8773	0.1	8.6144
0.9	2.4221	0.5	2.9579	0.1	5.3648

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