Article

Non-traditional approach to fitting of time series of larch bud moth dynamics: Application of Moran – Ricker model with time lags

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Abstract
In current publication analyses of time series of larch bud moth (*Zeiraphera diniana* Gn.) dynamics are considered. For fitting of time series Moran - Ricker model with time lags was used. Estimations of model parameters were provided with non-traditional approach: for every considered case feasible sets of points in space of model parameters were determined where selected statistical criterions give required results for deviations between theoretical (model) and empirical datasets. In all considered situations obtained results were compared with other results obtained with least squared method. It was obtained that Moran – Ricker model without time lag and with time lag in one year is not suitable for fitting of time series. Best correspondences on quantitative and qualitative levels between model trajectories and empirical dataset were found for cases with time lag in 3 years.

Keywords larch bud moth dynamics; time series; fitting; Moran - Ricker model with time lag.

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1 Introduction

For estimation of ecological model parameters various methods and approaches can be used – from expert estimations up to Bayesian approach (Bard, 1974; Borovkov, 1984; Draper and Smith, 1981; Wood, 2001 a, b and others). For one and the same model which is used for fitting of one and the same time series we can get various estimations for model parameters: expert estimations, estimations obtained with least square method, obtained with method of maximum likelihood etc. Naturally, in some situations question about *better estimations* does not arise, and in some situations question about *correspondence of model and time series* does not arise too.

Let us assume that parameters of following discrete-time model must be estimated:

$$x_{k+1} = F(x_k, \overline{\alpha}). \quad (1)$$

In (1) x_k is population size at moment k, $k = 0,1,2...$; $\overline{\alpha}$ is vector of model parameters; F is non-negative function (for non-negative values x_k and admissible values of parameters). Let us additionally to assume that at initial time moment $k = 0$ population size is equal to x_0 and it is unknown amount which must be estimated too.

Let $\{x_k^*\}$, $k = 0,1,...,N$, be empirical time series of population size changing in time; $N+1$ is sample size. Using this sample $\{x_k^*\}$ we have to estimate model parameters $\overline{\alpha}$ and initial population size x_0.

Use of least square method (LSM) (Bard, 1974; Borovkov, 1984) is based on assumption that best estimations of model parameters can be found with minimizing of sum of squared deviations between theoretical (model) and empirical datasets. If time series is approximated by model trajectory (global fitting; Wood, 2001 a, b) loss-function can be presented in the following form:

$$Q(\overline{\alpha}, x_0) = \sum_{k=0}^{N} \left(x_k(\overline{\alpha}, x_0) - x_k^* \right)^2. \quad (2)$$

In (2) $\{x_k(\overline{\alpha}, x_0)\}$ is model (1) trajectory obtained for fixed values of $\overline{\alpha}$ and x_0. Let's also assume that for certain point $(\overline{\alpha}^{**}, x_0^{**})$ there is a global minimum in (2):

$$Q(\overline{\alpha}^{**}, x_0^{**}) = \min_{\overline{\alpha}, x_0} \left(\sum_{k=0}^{N} \left(x_k(\overline{\alpha}, x_0) - x_k^* \right)^2 \right). \quad (3)$$

Following a traditional approach (Bard, 1974; Borovkov, 1984; Draper and Smith, 1981) after determination of estimations $(\overline{\alpha}^{**}, x_0^{**})$ analysis of set of deviations $\{e_k\}$ between theoretical and empirical datasets must be provided:

$$e_k = x_k(\overline{\alpha}^{**}, x_0^{**}) - x_k^*. \quad (4)$$

Model (1) is recognized to be suitable for fitting of considering time series if following conditions are truthful: deviations $\{e_k\}$ are values of independent stochastic variables with Normal distribution and with zero average. Following these assumptions Kolmogorov – Smirnov, Lilliefors, Shapiro – Wilk or other tests are used for checking of Normality of deviations (Bolshev and Smirnov, 1983; Lilliefors, 1967; Shapiro et al., 1968). For checking of independence of stochastic variables Durbin – Watson and/or Swed – Eisenhart tests are used (Draper and Smith, 1981; Hollander and Wolfe, 1973; Likes and Laga, 1985).

If in the sequence of residuals (4) serial correlation is observed it gives a background for conclusion that considering model isn't suitable for fitting and needs in modification. It means also that some of important factors or processes were not taken into account within the framework of model. Similar conclusion about model and its applicability to fitting can be made in situation when hypothesis about Normality of deviations must be rejected (for selected significance level). In other words, final conclusion about suitability of model for approximation of considering time series is based on analysis of properties of unique point $(\overline{\alpha}^{**}, x_0^{**})$ in the space of model parameters.

In our opinion, this is one of basic problems of LSM: a'priori it is impossible to exclude from consideration a situation when nearest to $(\overline{\alpha}^{**}, x_0^{**})$ points have required properties. One more problem of LSM is absence of a background for assumption about Normality of deviations. This is very strong condition and it must be reduced to two other conditions – symmetry of distribution with respect to origin and monotonic behaviour of branches of density function (Nedorezov, 2012, 2015a, b).

Next problem of LSM is following: there are no criterions for selection of loss-function. This function may have form (2) (with or without system of weights), it can be a sum of absolute values of deviations etc. This freedom in selection of loss-function is based on absence of real correlation with biological problem, and it is based on desire for obtaining of one point only but not set of points. Note that even in a case when model (1) is a law of population dynamics with natural values of parameters, there are no reasons for assumption that it must give minimum of any abstract loss-function.

Before finding of any special point in a space of parameters basic criterions for sets of deviations between theoretical and empirical datasets must be determined. On the next step it will allow determination of a *feasible set* – this is a set in the space of model parameters where all statistical criterions demonstrate required results. After that it is possible to use any loss-functions within the limits of feasible set. It is possible but not obligatory step – structure of feasible set depends on significance levels, and changing of these levels it will be possible to find points with extreme properties. These extreme points give best approximation for time series from the standpoint of selected criterions (but not from standpoint of loss-function).

In current publication this alternative approach is under consideration. Moran – Ricker model with time lags is applied for fitting of larch bud moth (LBM) time series (*Zeiraphera diniana* Gn.; Baltensweiler, 1964, 1978). For every considering particular case (which are determined by length of time lags) obtained results compare with results obtained with LSM and biological imaginations about LBM dynamics.

In our previous publication (Nedorezov and Sadykova, 2015) we applied Moran – Ricker model with time lags for fitting of larch bud moth time series. But not all basic dynamic regimes were presented in pointed out publication. In current publication we analyze wider spectrum of dynamic regimes which may have relation to larch bud moth dynamics.

2 Basic Requirements to Model and Set of Deviations

Mathematical model can be recognized as suitable for approximation of time series if following requirements are truthful:

1. Deviations between theoretical and empirical datasets must have a symmetric distribution with respect to origin. Branches of density function must be monotonic curves – it must increase in negative part of straight line, and it must decrease in right part.

Let $\{e_k^+\}$ be a set of positive deviations, and $\{-e_k^-\}$ is a set of negative deviations (4) with sign minus. Distribution has symmetry with respect to origin if and only if samples $\{e_k^+\}$ and $\{-e_k^-\}$ have one and the same distribution function. It allows using criterions for homogeneity – Kolmogorov – Smirnov test, Lehmann – Rosenblatt test, and Mann – Whitney test ((Bolshev and Smirov, 1983; Hollander and Wolfe, 1973; Likes and Laga, 1985). For testing of property of monotonic behavior of branches of density function Spearman rank correlation coefficient was used.

2. Hypotheses about existence of serial correlation in sequences of residuals must be rejected. For testing of this property Swed – Eisenhart test (Draper and Smith, 1981) and test "jump up –jump down" (Likes and Laga, 1985) were used below.

3. Behavior of model trajectory must correspond to behavior of time series. If for every increasing of values in time series model demonstrates decreasing of values there are no backgrounds for conclusion about suitability of model for fitting of considering sample. Thus, quota q of cases when time series and model trajectory demonstrate different changes must be rather small. In other words, Null hypothesis $H_0: \ q = 0.5$ with alternative hypothesis $H_1: \ q > 0.5$ must be rejected.

Pointed out set of statistical criterions was used as for determination of points of feasible sets as for finding points with *strongest properties*. For example, if hypothesis about symmetry of distribution cannot be rejected with 5% significance level it does not mean that Null hypothesis must be accepted (p-value can be rather small and close to 0.05). But if this hypothesis cannot be rejected with 95% significance level it means that we *have to accept it*.

Changing of values of significance levels allows finding small number of points in a space of model parameters with extreme properties. These points can be used as estimations for parameters. This is the first basic idea of considering approach. The second idea is following: assuming that every point of feasible set can be used as estimation of model parameters stochastic point with uniform distribution within the limits of feasible set will allow obtaining a *distribution of possible dynamic regimes*. This distribution can also be used as background for conclusion about population dynamics.

3 Larch Bud Moth Population Dynamics

3.1 Used time series and models

Regular observations of the changing of larch bud moth (*Zeiraphera diniana* Gn.) population densities in time in Swiss Alps (Upper Engadine valley) had been started in 1949 (Auer, 1977; Baltensweiler, 1964, 1978). In current publication time series on larch bud moth dynamics were used which can be free downloaded in Internet (NERC Centre for Population Biology, Imperial College (1999) The Global Population Dynamics Database, N 1407). Unit of measurement is number of larva per kilogram of branches. As it was pointed out in GPDD, data were collected on 1800 m above sea level that corresponds to optimal zone of species living (Isaev et al., 1984, 2001). Sample contains 38 values (first point corresponds to 1949).

Below we'll follow one of basic concepts about larch bud moth dynamics: periodicity of population fluctuations can be explained by influence of time lag in a reaction of self-regulation mechanisms (Isaev et al., 1984, 2001; Berryman and Stark, 1985; Berryman, 1981, 2002; Sadykova and Nedorezov, 2013; Nedorezov and Utyupin, 2011). Moran – Ricker model is one of well-known and well-studied models, and it has very rich set of dynamic regimes (Moran, 1950; Ricker, 1954):

$$x_{k+1} = Ax_k \exp\left\{-\sum_{j=0}^{m} a_j x_{k-j}\right\}, \ m = 0,1,2,3. \qquad (5)$$

In (5) all parameters are non-negative, $\forall k \quad A, a_k = const \geq 0$. Initial values of population sizes must be positive, $x_k^0 > 0$, $k = 0,1,2,3$. Note, that it was proved that model (5) with $m = 1$ can give good approximation for some time series of larch bud moth dynamics (McCallum, 2000).

3.2 Moran – Ricker model with $m = 0$

Within the framework of traditional approach following estimations were obtained (Nedorezov, Sadykova, 2015): $Q_{\min} = 465435.7$, $x_0 = 53.44$, $A = 517.0$, $a_0 = 0.1168$. $Q_{\min}$ is minimum value of functional form (2)-(3). Obtaining values of stochastic points with uniform distribution in $\Delta = [0,100] \times [0,1000] \times [0,1]$ we get a possibility to analyze structure of feasible set Ω. In Fig. 1 there is a projection of feasible set onto plane (A, a_0) (for all criterions one and the same 5% significance level was used). As one can see in fig. 1 LSM-estimation is far from a domain of maximum of point's concentration: $18.5 \le A \le 70$, $a_0 \le 0.05$ approximately. Moreover, LSM-estimation doesn't belong to Ω.

For LSM-estimations analysis of properties of deviations $\{e_k\}$ (4) allowed obtaining following results: hypothesis about Normality of deviations must be rejected with 0.1% significance level (Shapiro – Wilk test). Probability that distribution of deviations is symmetry is very small: $p = 0.002507$ for Wald – Wolfowitz test and $p < 0.025$ for Kolmogorov – Smirnov test. Thus, within the framework of traditional approach we have to conclude that Moran – Ricker model (5) with $m = 0$ is not suitable for fitting of considering time series.

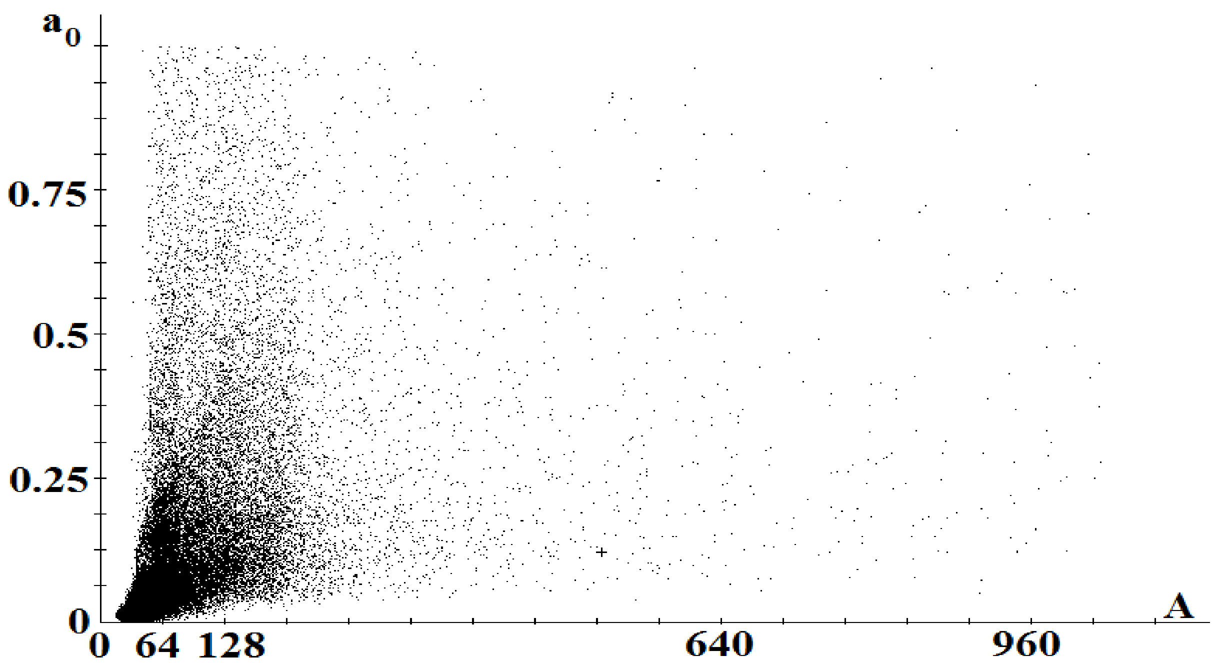


Fig. 1 Projection of feasible set for Moran – Ricker model without time lag onto plane (A, a_0). Crest corresponds to point of minimum of functional form (2)-(3).

Remark. Marked in Fig. 1 points correspond to situations when for fixed values of parameters A and a_0 it is possible to point out initial value x_0^0 which gives a model trajectory and set of deviations satisfying to all statistical criterions described above. If point doesn't mark it means that for fixed values of parameters A and a_0, and all possible values of initial population size one of criterions cannot be applied (for example, when we have small number of negative or positive deviations) or when one of criterions give negative result (absence of symmetry, existence of serial correlation etc.).

Point $x_0^0 = 50.6028$, $A = 29.0773$, $a_0 = 0.0263$ has following properties. Hypothesis about symmetry cannot be rejected with 99.9996% significance level (Kolmogorov – Smirnov test), with 99.7% significance level (Lehmann – Rosenblatt test), with 96.8% significance level (Mann – Whitney test). Hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with o.1% significance level (critical value for t-test is equal to 0.513 for sample size 38 and 0.1% significance level; real value for t-test for considering point 0.63913). Thus, hypothesis about symmetry and monotonic behavior of branches of density function *must be accepted*.

Hypothesis about absence of serial correlation cannot be rejected with 25.4% significance level (Swed – Eisenhart test) and with 20% significance level (test "jump up –jump down"; critical levels are equal to 21 and 29, and value of test is equal to 25).

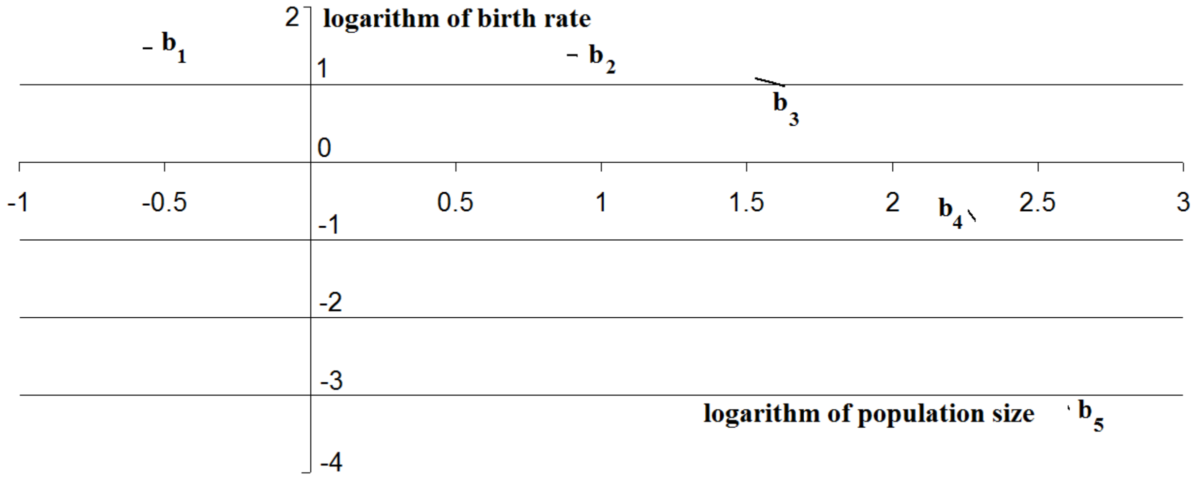


Fig. 2 Asymptotic stable dynamic regime for Moran – Ricker model with $m = 0$. Sets $b_1, \ldots, b_5$ are elements of fuzzy 5-cycle.

Values of autocorrelation function calculated for 20000 values of trajectory after $2 \cdot 10^6$ free steps of model belong to close interval $[-0.48115, 0.999827]$. Every fifth value of autocorrelation function is greater than 0.000715. Thus, asymptotic stable dynamic regime can be classified as (close to) fuzzy 5-cycle. Subsets of this 5-cycle $b_1, \ldots, b_5$ are presented on plane $(\log_{10} x, \log_{10} y)$ in Fig. 2 where x is population density, and y is a birth rate, $y_k = x_{k+1} / x_k$.

Let ξ be a stochastic variable with uniform distribution on feasible set which is equal to length of (asymptotic) cycle which is realized for respective point of the space of model parameters. In such a case probability $P\{\xi = k\}$ is equal to quota of respective dynamic regimes among all dynamic regimes which can be observed for points from feasible set. Distribution of stochastic variable ξ can be as element of background of final conclusion about population dynamic regime. For considering situation it was obtained that probability of event $\{\xi > 1000\}$ is equal to 0.935 approximately. Some other probabilities have following values: $P\{\xi = 4\} \approx 0.0184$, $P\{\xi = 5\} \approx 0.0102$, $P\{\xi = 6\} \approx 0.015$, $P\{\xi = 8\} \approx 0.0029$ (other probabilities are much smaller). Probability of event $\{29 < \xi \le 1000\}$ is equal to zero.

It allows concluding that regimes with strongest properties, most probable regimes, and regimes with LSM-estimations don't correspond to biological imaginations about larch bud moth fluctuations (Isaev et al., 1984, 2001; Auer, 1977; Baltensweiler, 1964, 1978).

3.3 Moran – Ricker model with $m = 1$

Within the framework of traditional approach following results were obtained (Nedorezov and Sadykova, 2015): $Q_{\min} = 154148.4$, $x_0^0 = 4.148 \cdot 10^{-14}$, $x_1^0 = 0.1274$, $A = 8.3$, $a_0 = 2.205 \cdot 10^{-3}$, $a_1 = 0.02214$. For these parameters asymptotic stable regime is 18-cycle with rather close values of cyclic coordinates (Fig. 3). It looks like double 9-cycle, and in this occasion it corresponds to biological imagination about larch bud moth dynamics (Auer, 1977; Baltensweiler, 1964, 1978). But analysis of deviations shows that hypothesis about Normality must be rejected with 1% significance level; serial correlation is also observed in sequence of residuals (Swed – Eisenhart test).

Within the boundaries of set $\Delta = [0,100] \times [0,100] \times [0,1000] \times [0,1] \times [0,1]$ 50000 points of feasible set Ω were found (with the help of stochastic variable with uniform distribution). Probability to find a point in Δ belonging to Ω is equal to $5.3356 \cdot 10^{-5}$ approximately. Final picture is close to presented in fig. 1 (it is obvious that set Ω for Moran – Ricker model with $m = 0$ is a subset for new one). New domains of point's concentration on the plane (A, a_0) were not identified.

The following point $x_0^0 = 44.705$, $x_1^0 = 53.333$, $A = 719.01$, $a_0 = 0.0318$, $a_1 = 0.0017$ has best characteristics with respect to symmetry. Value of Kolmogorov – Smirnov test is equal to 0.4425 (hypothesis about symmetry cannot be rejected with 98.7411% significance level); value of Lehmann – Rosenblatt test is equal to 0.03305 (93.315%); value of Mann – Whitney test is equal to 0.0908 (92.04%). It means that Null hypothesis about symmetry of distribution of residuals *must be accepted*.

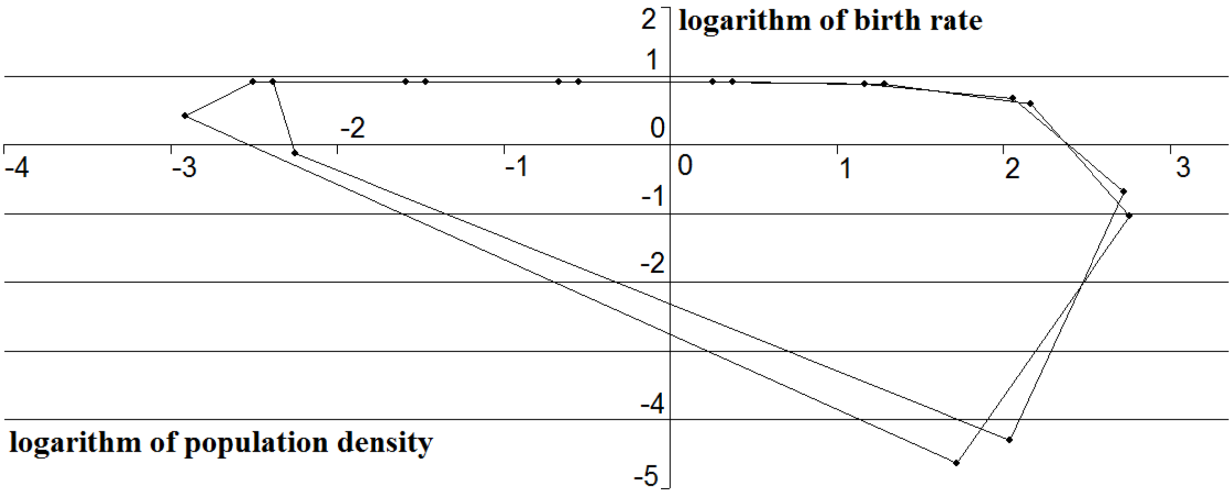


Fig. 3 Asymptotic stable dynamic regime (18-cycle) for Moran – Ricker model with $m=1$ for LSM-estimations.

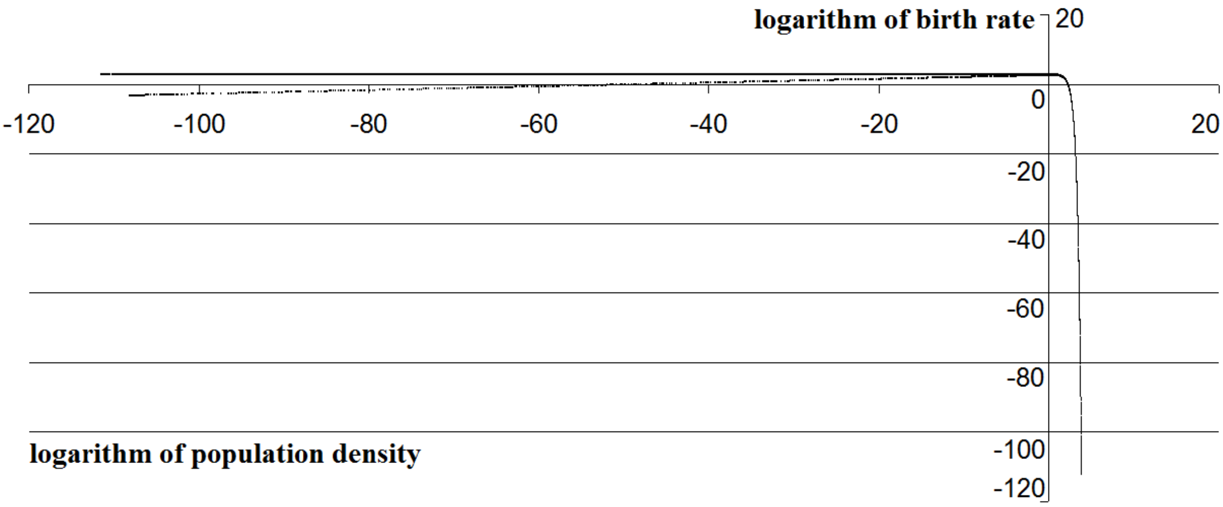


Fig. 4 Asymptotic stable dynamic regime for Moran – Ricker model with $m=1$.

Like in previous case hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with 0.1% significance level (t-test is equal to 0.74483>0.513). Hypothesis about absence of serial correlation cannot be rejected with 67.44% significance level (Swed – Eisenhart test); value of test "jump up –jump down" is equal to 21: hypothesis about absence of serial correlation cannot be rejected with 5% significance level and must be rejected with 20% level.

Values of autocorrelation function calculated for 200000 values of model trajectory and after 200000 free steps of model belong to close interval $[-0.033, 0.076]$. Dynamics has irregular nature: monotonic increasing, fast decreasing of population size (during one step of model) with further monotonic increasing. In Fig. 4 set of points belonging to asymptotic stable dynamic regime is presented on the plane $(\log_{10} x, \log_{10} y)$. Similar behavior of autocorrelation function and similar structure of set of points on the plane $(\log_{10} x, \log_{10} y)$ were observed in cases when probabilities of rejection of hypotheses about absence of serial correlation were minimized.

Analysis of points of feasible set shows that cycles of various lengths can correspond to considering time series: 1-cycle (regime of asymptotic stabilization or rather small fluctuations near stable level with variance which is less than 10^{-40}), 2,…, 7, 11, 15, 16, 19, 27, 30, 56. 8-cycles and 9-cycles were not found. Cycles with lengths 16 and 27 don't look like double or triple 8-cycle or 9-cycle: in these cases dynamic regimes with very long phases of population increasing were observed.

For considering case $P\{\xi = 1\} \approx 0.0164$, $P\{\xi = 2\} \approx 0.0131$. Other probabilities are much less these values. Biggest probability is equal to 0.9546 for event $\{\xi > 1000\}$. Thus, in both considered cases ($m = 0$ and $m = 1$) dynamic regimes don't correspond to existing imaginations about population dynamics.

3.4 Moran – Ricker model with $m = 2$

Let's consider situation when time lag is equal to 2. Minimum of squared deviations (3) $Q_{\min} = 92099.1$ was observed for the following values of model parameters: $x_0^0 = 5.638 \cdot 10^{-15}$, $x_1^0 = 2.653 \cdot 10^{-16}$, $x_2^0 = 0.07208$, $A = 20.076$, $a_0 = 8.155 \cdot 10^{-3}$, $a_1 = 1.6994 \cdot 10^{-3}$, $a_2 = 0.02097$. Analysis of deviations showed that for Kolmogorov–Smirnov test $p < 0.05$; for Lilliefors test $p < 0.01$; for Shapiro – Wilk test $p < 10^{-5}$ (Bolshev and Smirnov, 1983; Shapiro et al., 1968; Lilliefors, 1967). Thus, model (5) with LSM-estimations cannot be used for fitting of empirical time series.

Within the boundaries of set Δ: $0 < x_k^0 \le 0.1$, $0 < A \le 40$, $0 \le a_k \le 0.03$, $k = 0,1,2$, – 50000 points of feasible set Ω were found. Probability to find in Δ a point belonging to Ω is equal to 0.000915 approximately.

Point $x_0^0 = 0.06564$, $x_1^0 = 0.02832$, $x_2^0 = 0.0211$, $A = 27.607$, $a_0 = 0.01207$, $a_1 = 0.000138$, $a_2 = 0.01257$ of the feasible set has following properties: hypothesis about symmetry of deviations cannot be rejected with 99.9991% significance level (Kolmogorov – Smirnov test), with 99.7% (Lehmann – Rosenblatt test), and with 98.81% (Mann – Whitney test). Hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with 0.1% significance level (t-test is equal to 0.7947). Hypothesis about absence of serial correlation cannot be rejected with 10.37% significance level (Swed – Eisenhart test); value of test "jump up – jump down" is equal to 22: hypothesis about absence of serial correlation cannot be rejected with 20% significance level. This point with extreme properties doesn't correspond to biological imaginations about larch bud moth dynamics: asymptotic stable dynamic regime is strong 7-cycle.

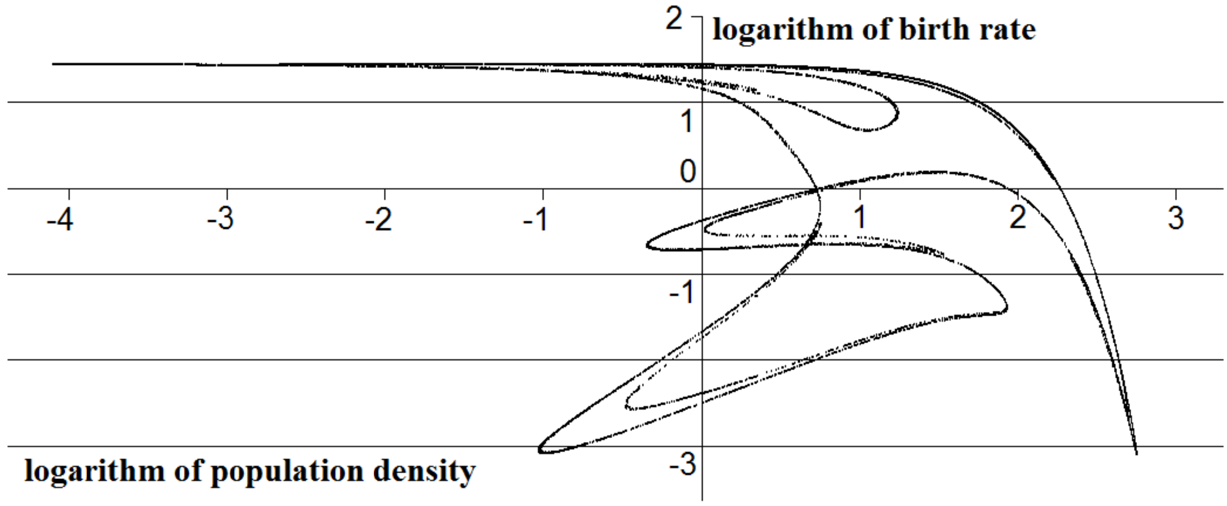
Another point with extreme properties is following: $x_0^0 = 0.0098$, $x_1^0 = 0.0902$, $x_2^0 = 0.0585$, $A = 27.758$, $a_0 = 0.0174$, $a_1 = 0.00655$, $a_2 = 0.0181$. Hypothesis about symmetry of deviations cannot be rejected with 99.9996% significance level (Kolmogorov – Smirnov test), with 99.7% (Lehmann – Rosenblatt test), and with 89.31% (Mann – Whitney test). Hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with 0.1% significance level (t-test is equal to 0.8947). Hypothesis about absence of serial correlation cannot be rejected with 28.37% significance level (Swed – Eisenhart test); value of test “jump up – jump down” is equal to 20: hypothesis about absence of serial correlation cannot be rejected with 5% significance level.

This point with extreme properties doesn’t also correspond to biological imaginations about larch bud moth dynamics: asymptotic stable dynamic regime is non-periodic. Values of autocorrelation function belong to close interval $[-0.2284, 0.4088]$. Asymptotic stable regime constructed for 20000 points (after 10^6 free steps of model) on the plane $(\log_{10} x, \log_{10} y)$ is presented in Fig. 5a. Approximation of considering time series by initial part of model trajectory is presented in Fig. 5b. Visual analysis of these pictures shows that we have not a background for conclusion that in this case Moran – Ricker model gives good fitting. In particular, number of extreme points of trajectory (Fig. 5b) is much bigger than number of extreme points of time series. Asymptotic trajectory of model is rather difficult, and we cannot say it has relation to larch bud moth dynamics or not.

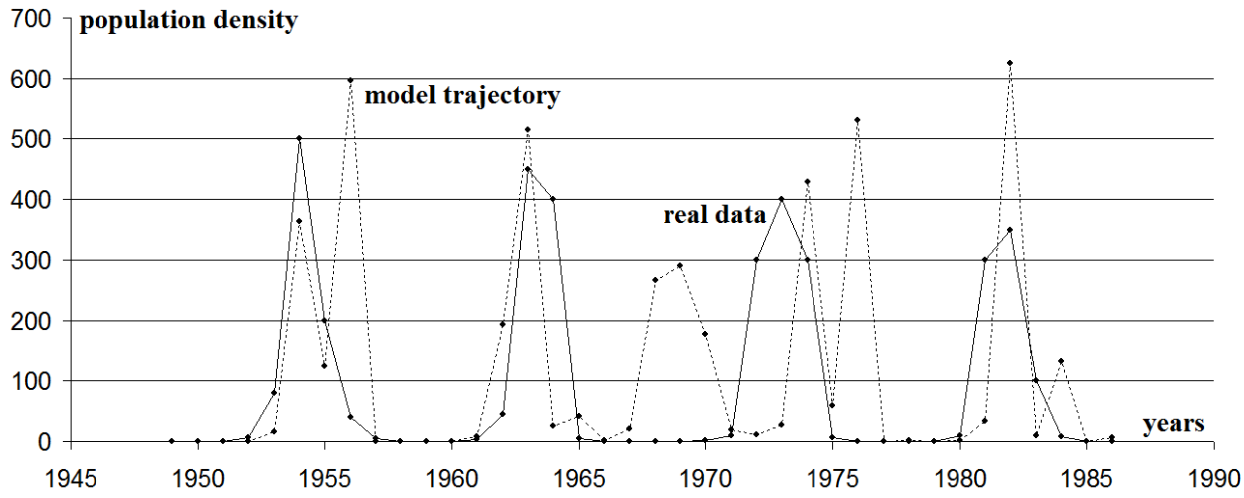
Analysis of points of feasible set shows that cycles of various lengths belong to this set. Like in previous case biggest probability is equal to 0.9 for event $\{\xi > 1000\}$. For 1-cycle probability is equal to 0.01634; for 9-cycle $P\{\xi = 9\} \approx 0.01202$, and for 8-cycle $P\{\xi = 8\} \approx 0.00892$. In a group of regimes which are close to 9-cycle, it is possible to point out strong 9-cycles, double 9-cycles (cycles of the length 18), and fuzzy 9-cycles with close coordinates (every ninth step values of autocorrelation function is bigger than 0.999). It allows concluding that within the boundaries of feasible set it is possible to find dynamic regimes which are close to biological imaginations about larch bud moth dynamics. But quota of these regimes is rather small.

3.5 Moran – Ricker model with $m = 3$

Let us consider the situation with $m = 3$. Minimum $Q_{\min} = 92018.9$ of squared deviations (3) was observed for the following values of model parameters: $x_0^0 = 4.97 \cdot 10^{-14}$, $x_1^0 = 1.61 \cdot 10^{-16}$, $x_2^0 = 4.2 \cdot 10^{-16}$, $x_3^0 = 1.4837$, $A = 19.8439$, $a_0 = 0.0081$, $a_1 = 0.0018$, $a_2 = 0.0186$, $a_3 = 0.0021$. Analysis of deviations showed that for Kolmogorov–Smirnov test $p < 0.05$; for Lilliefors test $p < 0.01$; for Shapiro– Wilk test $p < 10^{-5}$ (Bolshev and Smirnov, 1983; Shapiro et al., 1968; Lilliefors, 1967). Thus, Moran – Ricker model with LSM-estimations of model parameters cannot be applied for fitting of considering time series. Asymptotic stable dynamic regime is not cycling with length of cycle in 1000 years or less.



(a)



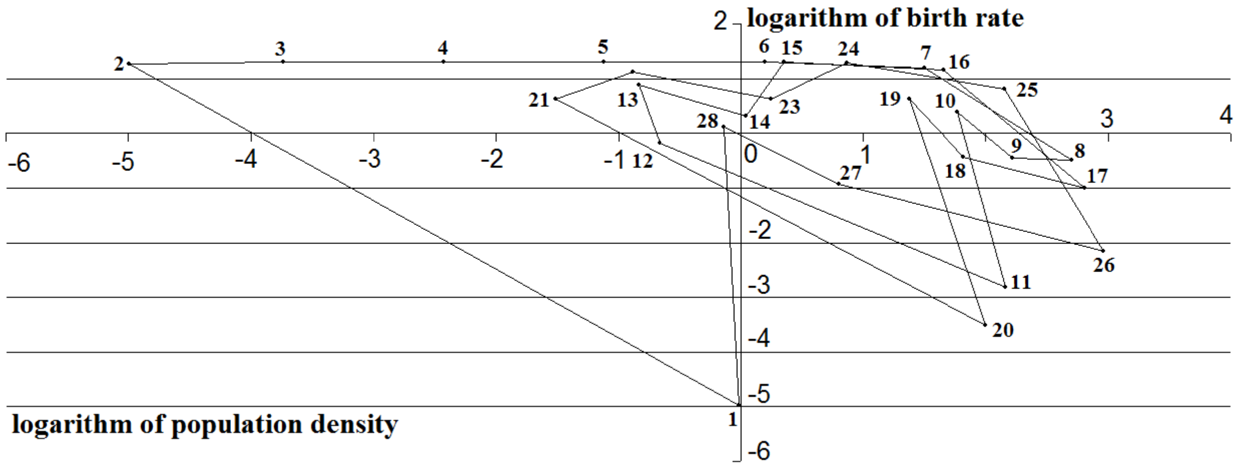
(b)

Fig. 5 Asymptotic stable dynamic regime for Moran – Ricker model with $m = 2$ on the plane $(\log_{10} x, \log_{10} y)$ (a) and approximation of time series (solid line) by initial part of model trajectory (broken line) (b).

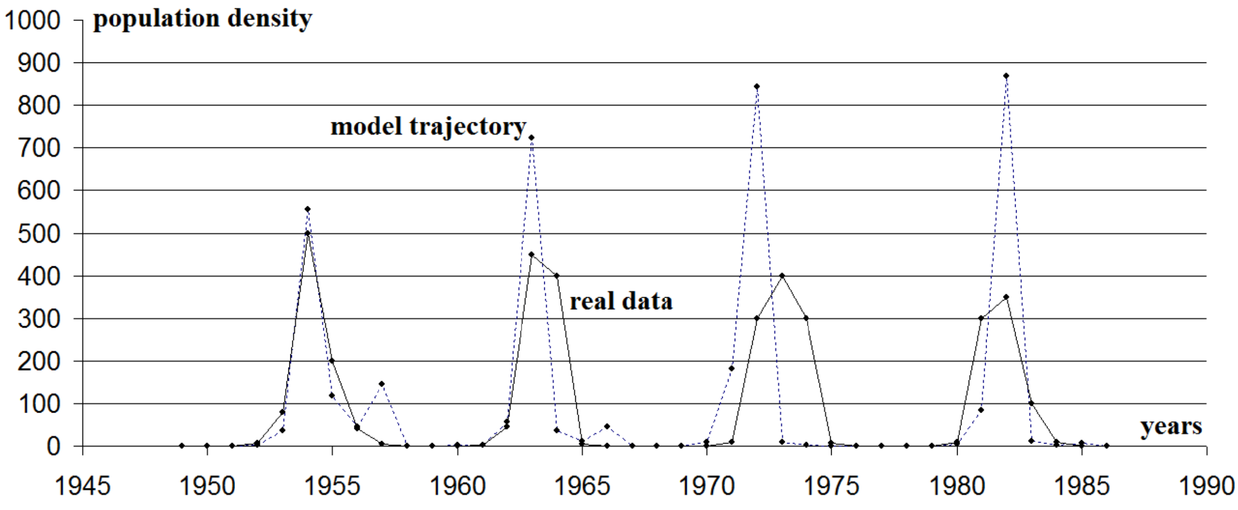
Within the boundaries of set Δ : $0 < x_k^0 \le 0.1$, $k = 0,1,2$, $0 < x_3^0 \le 2$, $0 < A \le 40$, $0 \le a_k \le 0.02$, $k = 0,1,2,3$, – 50000 points of feasible set Ω were found. Probability to find in Δ a point belonging to Ω is equal to 0.002023 approximately.

Point $x_0^0 = 0.0969$, $x_1^0 = 0.0914$, $x_2^0 = 0.0802$, $x_3^0 = 1.7984$, $A = 20.5404$, $a_0 = 0.00788$, $a_1 = 0.00541$, $a_2 = 0.00052$, $a_3 = 0.01584$ of the feasible set has following properties: hypothesis about symmetry of deviations cannot be rejected with 99.9979% significance level (Kolmogorov – Smirnov test), with 97.432% (Lehmann – Rosenblatt test), and with 97.66% (Mann – Whitney test). Hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with 0.1% significance level (t-test is equal to 0.9295). Hypothesis about absence of serial correlation cannot be rejected with 31.63% significance level (Swed – Eisenhart test); value of test “jump up – jump down” is equal to 21: hypothesis about absence of serial correlation cannot be rejected with 5% significance level and must be rejected with 20% level.

Asymptotic dynamic regime with pointed out parameters is strong 28-cycle (Fig. 6a). It consists of three outbreak trajectories of various lengths. Moreover, considering regime has following important features: birth rate decreases in domain where birth rate is bigger than one (parts of trajectory 13-15 and 22-24), and this rate increases in domain where it’s less than one (parts of trajectory 9-11, 18-20, and 26-1). Such kind of behavior of model trajectory was theoretically explained in our publications (within the framework of model of Kolmogorov type; Isaev et al., 1984, 2001) and can be observed for various trajectories of outbreak species of forest insects. Thus, we have good background for conclusion that larch bud moth dynamics corresponds to 28-years cycle.



(a)



(b)

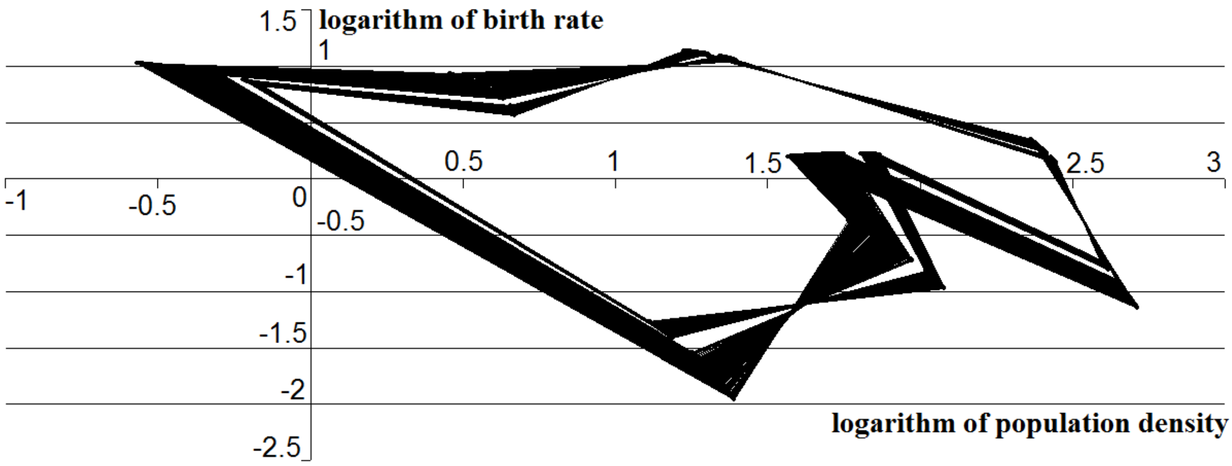
Fig. 6 Asymptotic stable dynamic regime (strong 28-cycle) for Moran – Ricker model with $m = 3$ on the plane $(\log_{10} x, \log_{10} y)$ (a), and approximation of time series (solid line) by initial part of model trajectory (broken line) (b).

Similar properties (Fig. 7) are observed for fuzzy 16-cycle which is realized for following extreme point

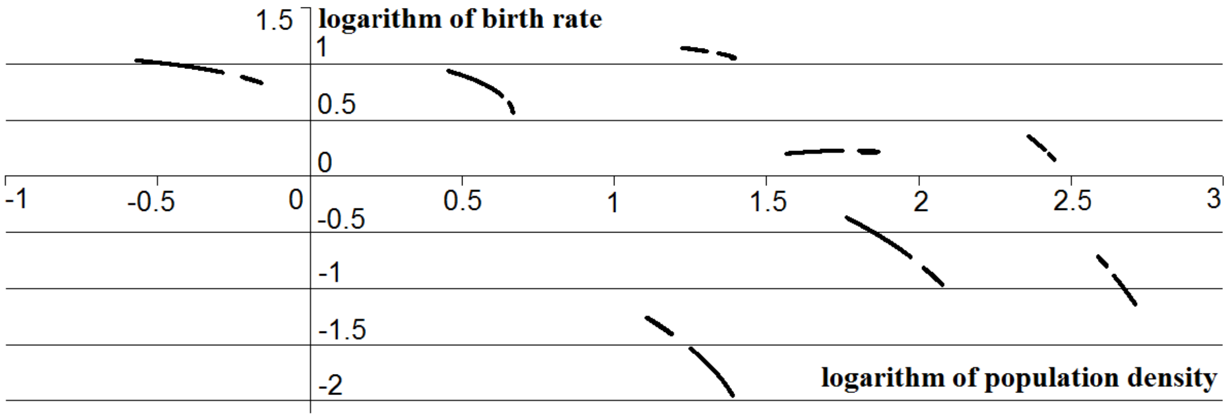
$x_0^0 = 0.0231$, $x_1^0 = 0.0768$, $x_2^0 = 0.0315$, $x_3^0 = 0.2691$, $A = 19.364$, $a_0 = 0.00904$, $a_1 = 0.00375$, $a_2 = 4.48 \cdot 10^{-5}$, $a_3 = 0.0136$ of the feasible set. Like for previous dynamic regime birth rate decreases in domain where birth rate is bigger than one (Fig. 7a), and this rate increases in domain where it's less than one. Autocorrelation function is bigger than 0.995 every 16th value of argument, and bigger 0.95 every 8th value of argument.

Hypothesis about symmetry of deviations cannot be rejected with 99.9999% significance level (Kolmogorov – Smirnov test), with 99.7% (Lehmann – Rosenblatt test), and with 89.48% (Mann – Whitney test). Hypothesis about equivalence of Spearman rank correlation coefficient to 0 must be rejected with 0.1% significance level (t-test is equal to 0.8982). Hypothesis about absence of serial correlation cannot be rejected with 13.68% significance level (Swed – Eisenhart test); value of test "jump up – jump down" is equal to 22: hypothesis about absence of serial correlation cannot be rejected with 20% significance level.

Analysis of points of feasible set Ω shows that cycles of various lengths belong to this set. Like in previous cases biggest probability is observed for the event $\{\xi > 1000\}$ and is equal to 0.7759. But this probability is much less than the same one of previous cases. Big probability is observed for 9-cycle $P\{\xi = 9\} \approx 0.08148$, and it is bigger than all other probabilities for sub-events of $\{\xi \le 1000\}$. Several other probabilities are following: $P\{\xi = 8\} \approx 0.02248$, $P\{\xi = 10\} \approx 0.0229$, $P\{\xi = 18\} \approx 0.02566$. In a group of regimes which are close to 9-cycle, it is possible to point out strong 8- , 9-, and 10-cycles, fuzzy cycles of same lengths etc. It allows concluding that within the boundaries of feasible set it is possible to find other dynamic regimes which are close to biological imaginations about larch bud moth dynamics.



(a)



(b)

Fig. 7 Asymptotic stable dynamic regime (fuzzy 16-cycle) for Moran – Ricker model with $m=3$ on the plane $(\log_{10} x, \log_{10} y)$ (a); (b) – the same regime but without lines between nearest points of cycle.

Thus, use of Moran – Ricker model with time lag in three years, $m=3$, allowed obtaining dynamic regimes which are close to considering time series on quantitative and qualitative levels. For cases when $m<3$, regimes with same characteristics were not found (it is possible to point out regimes which are close on quantitative level but not on qualitative level).

4 Conclusion

One of very important stages in analysis of correspondence between empirical time series and model trajectories is determination of properties of set of deviations. If distribution of deviations is not symmetric with respect to origin or if serial correlation is observed in a sequence of residuals it gives a background for conclusion that model is not suitable for fitting of considered time series.

In current publication for fitting of larch bud moth time series by trajectories of Moran – Ricker model with time lags the following procedure was used. First of all, with the help of Monte Carlo methods in a space of model parameters points of *feasible sets* were determined. Feasible set was defined as a set of points in the space of model parameters which correspond to deviations satisfying to certain statistical criterions. Collection of statistical criterions includes Kolmogorov – Smirnov, Mann – Whitney, and Lehmann – Rosenblatt tests for checking of symmetry of distribution of deviations. It also includes Swed – Eisenhart test and test "jump up – jump down" for checking of absence/existence of serial correlation in sequences of residuals, and some other statistical tests.

For every considered case (for different values of time lag in Moran – Ricker model) about 50000 points of every feasible set were found, and for every set points with extreme properties were determined. Finding of points with extreme properties is based on following idea: these points give best fitting of considering time series *from the stand point of selected statistical criterions*. For example, if hypothesis about symmetry of distribution cannot be rejected with 5% significance level it doesn't mean that we *have to accept* Null hypothesis (because p-value can be rather small, for example, $p \approx 0.06$). Stronger result is following: Null hypothesis cannot be rejected with 20% significance level. But strongest result corresponds to situation when Null hypothesis cannot be rejected with 99% significance level: in such a situation we *have to accept Null hypothesis*.

Analysis of points with extreme properties allowed obtaining that best results are observed for situation when time lag is in three years. In this situation correspondence of theoretical and empirical datasets is observed on quantitative (all statistical criterions are satisfied with extreme values of significance levels) and qualitative (behavior of model trajectory corresponds to behavior of empirical time series) levels. Provided analysis allows concluding that asymptotic stable dynamic regime of larch bud moth is strong 28-cycle (cycle in 28 years with three maximum points) or fuzzy 16-cycle (cycle in 16 years).

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