Article

Finding minimum cost flow in the network: A Matlab program and application

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Abstract
In this study, I present full Matlab codes of minimum cost flow algorithm and demonstrate an example.

Keywords network; minimum cost flow; Matlab.

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1 Introduction

For a network, suppose the source node is v_0 and sink node is v_n.The weight c_{ij} of directed link e_{ij} is its flow capacity, and f_{ij} is the flow of the link, $0\leq f_{ij}\leq c_{ij}$. The in-flow sum and out-flow sum of a node v_j are equal to each other. We need to calculate the flow $\sum f_{0j}$ while minimize the total cost (Chan et al., 1982; Zhang, 2012). In this study, I present full Matlab codes of minimum cost flow algorithm and demonstrate an example.

2 Algorithm

Given a network X=(V, E, C) with n nodes, v_i, i=1,2,…,n. In the maximum flow problem (Zhang, 2017), assign each link a cost a_{ij}. With v_0 as the source node and v_n as the sink node, calculate the flow $\sum f_{0j}$, and minimize the total cost (Chan et al., 1982; Zhang, 2012)

$$\min \sum_{i,j} a_{ij} f_{ij}$$

Take the initial available flow f as the zero flow. The algorithm to solve minimum cost flow problem is (Chan et al., 1982; Zhang, 2012)

(1) Generate a weighted directed graph X_f =(V, E_f , F), for any $e_{ij}\in E$, E_f and F are defined as

$e_{ij} \in E_f$, $F(e_{ij})=b_{ij}$ if $f_{ij}=0$
$e_{ji} \in E_f$, $F(e_{ji})=b_{ij}$ if $f_{ij}=C_{ij}$
$e_{ij} \in E_f$, $F(e_{ij})=b_{ij}$, $e_{ji} \in E_f$, $F(e_{ji})=-b_{ij}$ if $0<f_{ij}<C_{ij}$

(2) Find the shortest path h from source node v_s to sink node v_t in the weighted directed graph $X_f=(V, E_f, F)$ (Zhang, 2016). If there exists a shortest path h, return (3), otherwise f is the maximum flow with minimum cost, terminate the algorithm.

(3) Enhance flow. The same procedures as finding maximum flow (Zhang, 2017), let

$d_{ij}=c_{ij}-f_{ij}$ if $e_{ij} \in h^+$
$d_{ij}=f_{ij}$ if $e_{ij} \in h^-$
$d=\min \{d_{ij} | e_{ij} \in h\}$

and define the flow $f=\{f_{ij}\}$ as

$f_{ij} = f_{ij}+d$ if $e_{ij} \in h^+$
$f_{ij} = f_{ij}-d$ if $e_{ij} \in h^-$
$f_{ij} = f_{ij}$ otherwise

If M_f is not less than the desired flow, then reduce d, such that M_f is equal to the desired flow, by doing so, f is the minimum cost flow, and terminate the algorithm, otherwise return (1).

The following are Matlab codes, minCost.m, for the minimum cost flow algorithm. The Matlab algorithm needs the user to load an excel file that stores the Two Array Listing data of the form (d_{1i}, d_{2i}, c_i, b_i), where d_{1i}, d_{2i}, c_i, and b_i are start node and end node of the link i, flow capacity of the link i, and cost per unit flow of link i, respectively, i=1,2,…,e.

```
%Data are stored in Two Array Listing.
da=input('Input the excel file name of Two Array Listing data (e.g., adj.xls, etc. Data is d=(d1i, d2i, cci, bbi),where d1i, d2i, cci, bbi are start node and end node of the link i, flow capacity of the link i, and cost of unit flow of link i, respectively, i=1,2,¡-,e): ','s');
da=xlsread(da);
e=size(da,1);
d1=da(:,1); d2=da(:,2); cc=da(:,3); bb=da(:,4);
v=max(max([d1 d2]));
c=zeros(v);
a=c; b=c;
p=zeros(1,v); s=p;
for i=1:v
for j=1:v
for k=1:e
if ((d1(k)==i) & (d2(k)==j))
c(i,j)=cc(k);
b(i,j)=bb(k);
break;
end; end; end; end
```

```
mf=0;
mf0=inf;
f=zeros(v);
while (v>0)
for i=1:v
for j=1:v
if (j~=i) a(i,j)=inf; end
end; end
for i=1:v
for j=1:v
if ((c(i,j)>0) & (f(i,j)==0)) a(i,j)=b(i,j);
elseif ((c(i,j)>0) & (f(i,j)==c(i,j))) a(j,i)=-b(i,j);
elseif (c(i,j)>0)
a(i,j)=b(i,j);
a(j,i)=-b(i,j);
end
end; end
for i=2:v
p(i)=inf;
s(i)=i;
end
for k=1:v
d=1;
for i=2:v
for j=1:v
if (p(i)>(p(j)+a(j,i)))
p(i)=p(j)+a(j,i);
s(i)=j;
d=0;
end; end; end
if (d~=0) break; end
end
if (p(v)>=inf) break; end
dv=inf;
m=v;
while (v>0)
dvt=-inf;
if (a(s(m),m)>0) dvt=c(s(m),m)-f(s(m),m);
elseif (a(s(m),m)<0) dvt=f(m,s(m)); end
if (dv>dvt) dv=dvt; end
if (s(m)==1) break; end
m=s(m);
end
d=0;
if ((mf+dv)>=mf0)
```

```
dv=mf0-mf;
d=1;
end
m=v;
while (v>0)
if (a(s(m),m)>0) f(s(m),m)=f(s(m),m)+dv;
elseif (a(s(m),m)<0) f(m,s(m))=f(m,s(m))-dv; end
if (s(m)==1) break; end
m=s(m);
end
if (d~=0) break; end
mf=sum(f(1,:));
end
mmf=sum(sum(b.*f));
fprintf(['Maximum flow with minimum cost:' '\n']);
f
fprintf(['Maximum flow with minimum cost=' num2str(mf) '\n']);
fprintf(['Minimum cost=' num2str(mmf) '\n']);
```

3 Application Example

Suppose there are 5 nodes and 7 links in a network. The data are as follows

From-Node	To-Node	Flow capacity	Cost per unit flow of the link
1	2	7	3
1	4	5	6
2	3	7	3
2	5	3	4
3	4	6	5
3	5	4	2
4	5	8	4

Using the algorithm above, we achieve the maximum flow matrix with minimum cost as the following

0	7	0	5	0
0	0	4	0	3
0	0	0	0	4
0	0	0	0	5
0	0	0	0	0

The maximum flow with minimum cost is 12 and the minimum cost is 103.

References

Chan SB, et al. 1982. Graph Theory and Its Applications. Science Press, Beijing, China

Zhang WJ. 2012. Computational Ecology: Graphs, Networks and Agent-based Modeling. World Scientific, Singapore

Zhang WJ. 2016. A Matlab program for finding shortest paths in the network: Application in the tumor pathway. Network Pharmacology, 1(1): 42-53

Zhang WJ. 2017. Finding maximum flow in the network: A Matlab program and application. Network Biology, 7(3)