Article

Inauguration of Kifilideen theorem of matrix transformation of negative power of – n of trinomial expression in which three variables x, y and z are found in parts of the trinomial expression with some other applications

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Abstract
Kifilideen trinomial theorem of negative power of – n is theorem which is used to generate the series and terms of a trinomial expression of negative power of – n in an orderly and periodicity manner that is based on standardized and matrix methods. Negative power of Newton binomial theorem had been used to produce series of partial fractions of a compound fraction. The establishment of the negative power of − n of trinomial theorem would extend the number of compound fraction in which series (expansion) can be produced. This study applied Kifilideen expansion of negative power of – n of Kifilideen trinomial theorem for the transformation of compound fraction into series of partial fractions with other developments. A theorem of matrix transformation of negative power of – n of trinomial expression in which three variables x, y and z are found in parts of the trinomial expression was inauguration. The development would ease the process of evaluating such trinomial expression of negative power of – n. This standardized and matrix method used in arranging the terms of the Kifilideen expansion of negative power of − n of trinomial expression yield an interesting results in which it is utilized in transforming compound fraction into series of partial fractions in a unique way.

Keywords compound fraction; series; partial fraction; combination; kif matrix; Kifilideen standardized method.

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1 Introduction

Sir Isaac Newton (1642–1727) was the first mathematician and physicist to inaugurate negative and fraction power of binomial theorem (Dennis and Addington, 2009; Goss, 2011; Youngmee and Sangwook, 2014;

Schwart, 2015; Cooley, 2019; Osanyinpeju, 2019a). He established formula for binomial theorem that could work for negative, fraction powers of binomial expression (Francia, 2000; Haggstrom, 2012; Aljohani, 2016; Anekwe, 2018; Tavora, 2020). The binomial theorem is a general expression for any power of the sum or difference of any two things, terms or quantities (Godman et al., 1984; Talber et al., 1995; Bird, 2003; Stroud and Booth, 2007; Tuttuh et al., 2014; Bunday and Mulholland, 2014). Binomial is widely used in the field of Physics, Biology and Engineering to expand power of binomial expression (Costa, 2017; Gavrikov, 2018). Binomial series (expansions) are utilized in area of mathematical analysis link to data modeling, probability theory, algebra and approximation techniques (Yang, 2017).

Kifilideen trinomial theorem of negative power of – n is theorem which is used to generate the series and terms of a trinomial expression of negative power of – n in an orderly and periodicity manner that is based on standardized and matrix method (Osanyinpeju, 2021a, 2021b). Negative power of Newton binomial theorem had been used to produce series of partial fractions of a compound fraction (Horn, 2020). The establishment of the negative power of $-$n of trinomial theorem would extend the number of compound fraction in which series can be produced. The negative power of – n of Kifilideen trinomial theorem can be used to generate the series of compound fraction number (examples are partial fraction series of $\frac{13}{9}, \frac{1}{1.75}, \frac{729}{811}$).

The terms of negative power of – n of trinomial theorem produce an infinite series unlike the terms of power of n of trinomial theorem which generate finite series (Osanyinpeju, 2020a). The terms of negative power of – n of trinomial theorem when arranged in the form of matrix, the number of elements in each column has finite value which is also the same for positive power of n of trinomial theorem. Although, the column of any negative power of – n of trinomial theorem is infinite while that of positive power of n of trinomial theorem is finite.

The power combination of each term of the negative power of – n of Kifilideen trinomial theorem is arranged in groups (columns) and periods (rows) in the kif matrix where each power combination takes a designated position. No two power combinations have the same position in the kif matrix for a particular negative power of – n of Kifilideen trinomial theorem. This is also applied to positive power of trinomial theorem (Osanyinpeju, 2020b). The power combination of each term; down the group (column) and across the period (row) decreases and increases respectively at regular pattern for both positive and negative powers of Kifilideen trinomial theorem in the kif matrix format. Also, to transit from one group to another the power combination progress with unique addition of regular figure which occur for both positive and negative power of trinomial theorem.

This standardized and matrix method used in arranging the terms of the Kifilideen expansion of negative power of $-$n of trinomial expression yield an interesting results in which it is utilized in transforming compound fraction into series of partial fractions in a unique way. Kifilideen Matrix approach had been extended in evaluating and computation of power of base of eleven, other bi–digits and tri–digit numbers (Osanyinpeju, 2019b, 2020c, 2020d). This study applied Kifilideen expansion of negative power of – n of Kifilideen trinomial theorem for the transformation of compound fraction into series of partial fraction with other development.

2 Materials and Methods

The Kifilideen coefficient table of negative power of Kifilideen trinomial theorem was used in generating the power coefficient of each term of series of the trinomial expansion.

2.1 Application of Kifilideen series (expansion) of negative power of – n of trinomial theorem using standardized method

[a] Generate the series of $[1+x+x^3]^{-1}$ using Kifilideen standardized method. Otherwise determine the series of $\frac{343}{393}$ (Hint: take x = $\frac{1}{7}$).

Solution

$[a]$ Using Kifilideen coefficient table of negative power of – 1 of trinomial theorem, the coefficient in ascending order 1, - 1, - 1, 1, 2, 1, - 1, - 3, - 3, - 1, 1, 4, 6, 4, 1, -1, - 5, - 10, - 10, - 5, - 1, 1, 6, 15, 20, 15, 6, 1, ….

$$[1+x+x^3]^{-1} = 1 - x^1[x^3]^0 - x^0[x^3]^1 + x^2[x^3]^0 + 2x^1[x^3]^1 + x^0[x^3]^2 - x^3[x^3]^0 - 3x^2[x^3]^1 - 3x^1[x^3]^2 - x^0[x^3]^3 + x^4[x^3]^0 + 4x^3[x^3]^1 + 6x^2[x^3]^2 + 4x^{`1}[x^3]^3 + x^0[x^3]^4 - x^5[x^3]^0 - 5x^4[x^3]^1 - 10x^3[x^3]^2 - 10x^2[x^3]^3 - 5x^1[x^3]^4 - x^0[x^3]^5 + x^6[x^3]^0 + 6x^5[x^3]^1 + 15x^4[x^3]^2 + 20x^3[x^3]^3 + 15x^2[x^3]^4 + 6x^1[x^3]^5 + x^0[x^3]^6 - x^7[x^3]^0 - 7x^6[x^3]^1 - 21x^5[x^3]^2 - 35x^4[x^3]^3 - 35x^3[x^3]^4 - 21x^2[x^3]^5 - 7x^1[x^3]^6 - x^0[x^3]^7 + x^8[x^3]^0 + 8x^7[x^3]^1 + 28x^6[x^3]^2 + 56x^5[x^3]^3 + 70x^4[x^3]^4 + 56x^3[x^3]^5 + 28x^2[x^3]^6 + 8x^1[x^3]^7 + x^0[x^3]^8 - x^9[x^3]^0 - 9x^8[x^3]^1 - 36x^7[x^3]^2 - 84x^6[x^3]^3 - 126x^5[x^3]^4 - 126x^4[x^3]^5 - 84x^3[x^3]^6 - 36x^2[x^3]^7 - 9x^1[x^3]^8 - x^{10}[x^3]^0 + 10x^9[x^3]^1 + 45x^8[x^3]^2 + 120x^7[x^3]^3 + 210x^6[x^3]^4 + 252x^5[x^3]^5 + 210x^4[x^3]^6 + 120x^3[x^3]^7 + 45x^2[x^3]^8 + 10x^1[x^3]^9 + x^0[x^3]^{10} - x^{11}[x^3]^0 - \cdots \quad [1]$$

$$[1+x+x^3]^{-1} = 1 - x - x^3 + x^2 + 2x^4 + x^6 - x^3 - 3x^5 - 3x^7 - x^9 + x^4 + 4x^6 + 6x^8 + 4x^{10} + x^{12} - x^5 - 5x^7 - 10x^9 - 10x^{11} - 5x^{13} - x^{15} + x^6 + 6x^8 + 15x^{10} + 20x^{12} + 15x^{14} + 6x^{16} + x^{18} - x^7 - 7x^9 - 21x^{11} - 35x^{13} - 35x^{15} - 21x^{17} - 7x^{19} - x^{21} + x^8 + 8x^{10} + 28x^{12} + 56x^{14} + 70x^{16} + 56x^{18} + 28x^{20} + 8x^{22} + x^{24} - x^7 - 9x^{11} - 36x^{13} - 84x^{15} - 126x^{17} - 126x^{19} - 84x^{21} - 36x^{23} - 9x^{25} - x^{27} + x^{10} + 10x^{12} + 45x^{14} + 120x^{16} + 210x^{18} + 252x^{20} + 210x^{22} + 120x^{24} + 45x^{26} + 10x^{28} + x^{30} - x^{11} - \cdots \quad [2]$$

$$[1+x+x^3]^{-1} = 1 - x + x^2 - 2x^3 + 3x^4 - 4x^5 + 6x^6 - 9x^7 + 13x^8 - 19x^9 + 28x^{10} - 41x^{11} + 59x^{12} - \cdots$$

[ii] $\frac{343}{393} = [1 + \frac{1}{7} + (\frac{1}{7})^3]^{-1} = 1 - \frac{1}{7} + \frac{1}{49} - \frac{2}{343} + \frac{3}{2401} - \frac{4}{16807} + \frac{6}{117649} - \frac{9}{823543} + \frac{13}{5764801} - \frac{19}{50353607} + \frac{28}{282475249} - \frac{41}{1977326743} + \frac{59}{13841287200} - \cdots$ [3]

The evaluation of the above series gives 0.872774 to 6 decimal places. Also, using calculator $\frac{343}{393}$ gives 0.872774 to 6 decimal places. This indicates that the negative power of -1 of Kifilideen trinomial theorem and coefficient of negative power of -1 from the Kifilideen coefficient table are valid.

[b] Produce the Kifilideen expansion of $[3-x+x^4]^{-2}$ using Kifilideen standardized method. Use the expansion to generate the series of $\frac{6561}{47089}$ (Hint: take x = $\frac{1}{3}$)

Solution

[b] Using Kifilideen coefficient table of negative power of – 2 of trinomial theorem, the coefficient in ascending order 1, - 2, - 2, 3, 6, 3, - 4, - 12, - 12, - 4, 5, 20, 30, 20, 5, - 6, - 30, - 60, - 60, - 30, - 6, 7, 42, 105, 140, 105, 42, 7, - 8, - 56, - 168, - 280, - 280, - 168, - 56, - 8, …

$$[3-x+x^4]^{-2} = [3]^{-2}[-x]^0[x^4]^0 - 2[3]^{-3}[-x]^1[x^4]^0 - 2[3]^{-3}[-x]^0[x^4]^1 + 3[3]^{-4}[-x]^2[x^4]^0 + 6[3]^{-4}[-x]^1[x^4]^1 +$$

$$3[3]^{-4}[-x]^0[x^4]^2 - 4[3]^{-5}[-x]^3[x^4]^0 - 12[3]^{-5}[-x]^2[x^4]^1 - 12[3]^{-5}[-x]^1[x^4]^2 - 4[3]^{-5}[-x]^0[x^4]^3 + 5[3]^{-6}[-x]^4[x^4]^0$$

$$+20[3]^{-6}[-x]^3[x^4]^1 + 30[3]^{-6}[-x]^2[x^4]^2 + 20[3]^{-6}[-x]^1[x^4]^3 + 5[3]^{-6}[-x]^0[x^4]^4 - 6[3]^{-7}[-x]^5[x^4]^0$$

$$-30[3]^{-7}[-x]^4[x^4]^1 - 60[3]^{-7}[-x]^3[x^4]^2 - 60[3]^{-7}[-x]^2[x^4]^3 - 30[3]^{-7}[-x]^1[x^4]^4 - 6[3]^{-7}[-x]^0[x^4]^5 +$$

$$7[3]^{-8}[-x]^6[x^4]^0 + 42[3]^{-8}[-x]^5[x^4]^1 + 105[3]^{-8}[-x]^4[x^4]^2 + 140[3]^{-8}[-x]^3[x^4]^3 + 105[3]^{-8}[-x]^2[x^4]^4$$

$$+42[3]^{-8}[-x]^1[x^4]^5 + 7[3]^{-8}[-x]^0[x^4]^6 - \cdots \quad [4]$$

$$[3-x+x^4]^{-2} = \frac{1}{9} + \frac{2x}{27} - \frac{2x^4}{27} + \frac{3x^2}{81} - \frac{6x^5}{81} + \frac{3x^8}{81} + \frac{4x^3}{243} - \frac{12x^6}{243} + \frac{12x^9}{243} - \frac{4x^{12}}{243} + \frac{5x^4}{729} - \frac{20x^7}{729} + \frac{30x^{10}}{729} - \frac{20x^{13}}{729} + \frac{5x^{16}}{729} + \frac{6x^5}{2187} - \frac{30x^8}{2187} +$$

$$\frac{60x^{15}}{2187} - \frac{60x^{14}}{2187} + \frac{30x^{17}}{2187} - \frac{6x^{20}}{2187} + \cdots \quad [5]$$

$$[3-x+x^4]^{-2} = \frac{1}{9} + \frac{2x}{27} + \frac{3x^2}{81} + \frac{4x^3}{243} - \frac{2x^4}{27} + \frac{5x^4}{729} - \frac{6x^5}{81} + \frac{6x^5}{2187} - \frac{12x^6}{243} - \frac{20x^7}{729} + \frac{3x^8}{81} - \frac{30x^8}{2187} + \frac{12x^9}{243} + \frac{30x^{10}}{729} - \frac{4x^{12}}{243} - \frac{20x^{13}}{729} -$$

$$\frac{60x^{14}}{2187} + \frac{60x^{15}}{2187} + \frac{5x^{16}}{729} + \frac{30x^{17}}{2187} - \frac{6x^{20}}{2187} + \cdots \quad [6]$$

[ii] $\frac{6561}{47089} = \left[3 - \frac{1}{3} + \left(\frac{1}{3}\right)^4\right]^{-2} = \frac{1}{9} + \frac{2}{81} + \frac{3}{729} + \frac{4}{6561} - \frac{2}{2187} + \frac{5}{59049} - \frac{6}{19683} + \frac{6}{531441} - \frac{12}{177147} - \frac{20}{1594323} + \frac{3}{531441} - \frac{30}{14348907} + \frac{12}{4782969} + \frac{30}{43046721} - \frac{4}{129140163} \dots$ [7]

The evaluation of the above series gives 0.13933 to 5 decimal places. Also, using calculator $\frac{6561}{47089}$ gives 0.13933 to 5 decimal places. This indicates that the negative power of - 2 of Kifilideen trinomial theorem and coefficient of negative power of - 2 from the Kifilideen coefficient table are valid.

2.2 Generating series of partial fractions of compound fraction which denominator is decimal number

[d] Produce the series of $[1 + x + x^2]^{-1}$ using Kifilideen standardized method. Otherwise determine the series of

$\frac{1}{1.75}$ $(Hint: x = \frac{1}{2})$.

Solution

Using Kifilideen coefficient table of negative power of – 1 of trinomial theorem, the coefficient in ascending order 1, - 1, - 1, 1, 2, 1, - 1, - 3, - 3, - 1, 1, 4, 6, 4, 1, -1, - 5, - 10, - 10, - 5, - 1, 1,6,15,20,15,6,1,….

$$[1 + x + x^2]^{-1} = 1 - x^1[x^2]^0 - x^0[x^2]^1 + x^2[x^2]^0 + 2x^1[x^2]^1 + x^0[x^2]^2 - x^3[x^3]^0 - 3x^2[x^2]^1 - 3x^1[x^2]^2 - x^0[x^2]^3 + x^4[x^2]^0 + 4x^3[x^2]^1 + 6x^2[x^2]^2 + 4x^{`1}[x^2]^3 + x^0[x^2]^4 - x^5[x^2]^0 - 5x^4[x^2]^1 - 10x^3[x^2]^2 - 10x^2[x^2]^3 - 5x^1[x^2]^4 - x^0[x^2]^5 + x^6[x^2]^0 + 6x^5[x^2]^1 + 15x^4[x^2]^2 + 20x^3[x^2]^3 + 15x^2[x^2]^4 + 6x^1[x^2]^5 + x^0[x^2]^6 - x^7[x^2]^0 - 7x^6[x^2]^1 - 21x^5[x^2]^2 - 35x^4[x^2]^3 - 35x^3[x^2]^4 - 21x^2[x^2]^5 - 7x^1[x^2]^6 - x^0[x^2]^7 + x^8[x^2]^0 + 8x^7[x^2]^1 + 28x^6[x^2]^2 + 56x^5[x^2]^3 + 70x^4[x^2]^4 + 56x^3[x^2]^5 + 28x^2[x^2]^6 + 8x^1[x^2]^7 + x^0[x^2]^8 - x^9[x^2]^0 - 9x^8[x^2]^1 - 36x^7[x^2]^2 - 84x^6[x^2]^3 - 126x^5[x^2]^4 - 126x^4[x^2]^5 - 84x^3[x^2]^6 - 36x^2[x^2]^7 - 9x^1[x^2]^8 - x^{10}[x^2]^0 + 10x^9[x^2]^1 + 45x^8[x^2]^2 + 120x^7[x^2]^3 + 210x^6[x^2]^4 + 252x^5[x^2]^5 + 210x^4[x^2]^6 + 120x^3[x^2]^7 + 45x^2[x^2]^8 + 10x^1[x^2]^9 + x^0[x^2]^{10} - x^{11}[x^2]^0 - \cdots \quad [8]$$

$$[1 + x + x^3]^{-1} = 1 - x - x^2 + x^2 + 2x^3 + x^4 - x^3 - 3x^4 - 3x^5 - x^6 + x^4 + 4x^5 + 6x^6 + 4x^7 + x^8 - x^5 - 5x^6 - 10x^7 - 10x^8 - 5x^9 - x^{10} + x^6 + 6x^7 + 15x^8 + 20x^9 + 15x^{10} + 6x^{11} + x^{12} - x^7 - 7x^8 - 21x^9 - 35x^{10} - 35x^{11} - 21x^{12} - 7x^{13} - x^{14} + x^8 + 8x^9 + 28x^{10} + 56x^{11} + 70x^{12} + 56x^{13} + 28x^{14} + 8x^{15} + x^{16} - x^9 - 9x^{10} - 36x^{11} - 84x^{12} - 126x^{13} - 126x^{14} - 84x^{15} - 36x^{16} - 9x^{17} - x^{18} + x^{10} + 10x^{11} + 45x^{12} + 120x^{13} + 210x^{14} + 252x^{15} + 210x^{16} + 120x^{17} +$$

$45x^{18} + 10x^{19} + x^{20} - x^{11} - 11x^{12} - 55x^{13} - 165x^{14} - 330x^{15} - 462x^{16} - 462x^{17} - 330x^{18} - 165x^{19} - 55x^{20} - 11x^{21} - x^{22} + x^{12} + 12x^{13} + 66x^{14} + 220x^{15} + 495x^{16} + 792x^{17} + 924x^{18} + 792x^{19} + 495x^{20} + 220x^{21} + 66x^{22} + 12x^{23} + x^{24} - x^{13} + \cdots$ [9]

$$[1 + x + x^3]^{-1} = 1 - x + x^3 - x^4 + x^6 - x^7 + x^9 - x^{10} + x^{12} - x^{13} \ldots. \quad [10]$$

[ii] $\frac{1}{1.75} = [1 + 0.5 + (0.5)^2]^{-1} = 1 - \frac{1}{2} + \frac{1}{8} - \frac{1}{16} + \frac{1}{64} - \frac{1}{128} + \frac{1}{512} - \frac{1}{1024} + \frac{1}{4096} - \frac{1}{8192} + \cdots$ [11]

The evaluation of the above series gives 0.5714 to 4 decimal places. Also, using calculator $\frac{1}{1.75}$ gives 0.5714 to 4 decimal places. This indicates that the negative power of -1 of Kifilideen trinomial theorem and coefficient of negative power of -1 from the Kifilideen coefficient table are valid.

2.3 General application of negative power of – n of Kifilideen trinomial theorem using standardized method

[e] If a t^th term in the Kifilideen expansion of $[1 - 2x + ay^2]^{-3}$ yield $-7680xy^6$. Determine the following:

[i] the power combination of the term

[ii] the t^th term

[iii] the value of a

Solution

[i] For any given term in the Kifilideen expansion of $[1 - 2x + ay^2]^{-3}$ is given as

$$^{-3}_{kif}C\ [1]^k\ [-2x]^i\ [ay^2]^f \quad [12]$$

$$^{-3}_{kif}C\ [1]^k\ [-2]^i\ [x]^i\ [a]^f\ [y^2]^f \quad [13]$$

Comparing [13] with $-7680xy^6$, so

$$i = 1 \quad [14]$$

$$2f = 6 \quad [15]$$

Therefore, $$f = 3 \quad [16]$$

Also,

$$k + i + f = n \quad [17]$$

where, $k, i,$ and f are the component parts of the power combination
$n -$ the negative power of – n of the trinomial expression

$$k + i + f = n \quad [18]$$

$$k + 1 + 3 = -3 \quad [19]$$

$$k = -7 \quad [20]$$

$$C_P = kif = -713 \quad [21]$$

[ii]

$$t = \frac{[n-k]^2 + [n-k] + 2f + 2}{2} \quad [22]$$

$k = -7, i = 1, f = 3$ and $n = -3$ [23]

$$t = \frac{[-3 - -7]^2 + [-3 - -7] + 2 \times 3 + 2}{2} \quad [24]$$

$t = 14^{\text{th}}$ term [25]

[iii] $${}_{kif}^{-3}C\ [1]^k\ [-2]^i\ [x]^i\ [a]^f\ [y^2]^f\ = -\ 7680xy^6 \quad [26]$$

$${}_{-713}^{-3}C\ [1]^{-7}\ [-2]^1\ [x]^1\ [a]^3\ [y^2]^3\ = -\ 7680xy^6 \quad [27]$$

From the Kifilideen coefficient table, the coefficient of the 14th term on the n = −3 column is + 60 or

$$\frac{-3!}{-7!1!3!}[1]^{-7}\ [-2]^1\ [x]^1\ [a]^3\ [y^2]^3\ = -\ 7680xy^6 \quad [28]$$

$$\frac{-3\times-4\times-5\times-6\times-7!}{-7!1!3!}[1]^{-7}\ [-2]^1\ [x]^1\ [a]^3\ [y^2]^3\ = -\ 7680xy^6 \quad [29]$$

$$\frac{-3\times-4\times-5\times-6}{1!3!}[1]^{-7}[-2]^{1}[x]^{1}[a]^{3}[y^{2}]^{3} = -7680xy^{6} \qquad [30]$$

$$+60 \times [1]^{-7}[-2]^{1}[x]^{1}[a]^{3}[y^{2}]^{3} = -7680xy^{6} \qquad [31]$$

$$+60 \times -2 \times a^{3} \times xy^{6} = -7680xy^{6} \qquad [32]$$

$$a^{3} = \frac{-7680}{-120} \qquad [33]$$

$$a^{3} = 64 \qquad [34]$$

$$a = 4 \qquad [35]$$

[f] If a term of the Kifilideen expansion of $\left[1 - \frac{bx^{2}}{y^{4}} + 2yz^{3}\right]^{-4}$ is 7560 $x^{4}y^{-6}z^{6}$. Find

[i] the power combination of the term

[ii] the tth term

[iii] the value of b

Solution

$$^{-4}_{kif}C\ [1]^{k}\left[-\frac{bx^{2}}{y^{4}}\right]^{i}[2yz^{3}]^{f} \qquad [36]$$

$$^{-4}_{kif}C\ [1]^{k}[-b]^{i}[x^{2}]^{i}[y^{-4}]^{i}[2]^{f}[y]^{f}[z^{3}]^{f} \qquad [37]$$

$$^{-4}_{kif}C\ [1]^{k}[-b]^{i}[2]^{f}[x^{2}]^{i}[y]^{-4i+f}[z^{3}]^{f} \qquad [38]$$

Comparing [38] with 7560 $x^{4}y^{-6}z^{6}$, so

$$2i = 4 \qquad [39]$$

$$i = 2 \qquad [40]$$

$$-4i + f = -6 \qquad [41]$$

$$-4 \times 2 + f = -6 \quad [42]$$

$$f = 2 \quad [43]$$

Therefore,

Also,

$$k + i + f = n \quad [44]$$

$$k + 2 + 2 = -4 \quad [45]$$

$$k = -8 \quad [46]$$

$$C_P = kif = -822 \quad [47]$$

[ii] $k = -8,\ i = 2\ and\ f = 2$ [48]

Note, $$x = n - k \quad [49]$$

$$t = y + f \text{ and } n = k + i + f \quad [50]$$

So,

$$x = -4 - -8 = 4 \quad [51]$$

$$y = \frac{x^2+x+2}{2} \quad [52]$$

$$y = \frac{4^2+4+2}{2} \quad [53]$$

$$y = 11 \quad [54]$$

$$t = y + f = 11 + 2 = 13^{\text{th}} \text{ term} \quad [55]$$

[iii] Comparing [38] with $7560\ x^4y^{-6}z^6$, then

$$_{kif}^{-4}C\ [1]^k\ [-b]^i\ [2]^f [x^2]^i\ [y]^{-4i+f}\ [z^3]^f\ = 7560\ x^4y^{-6}z^6 \quad [56]$$

$$_{-822}^{-4}C\ [1]^{-8}\ [-b]^2\ [2]^2[x^2]^2\ [y]^{-4\times2+2}\ [z^3]^2\ = 7560\ x^4y^{-6}z^6 \quad [57]$$

$$ {}_{-822}^{-4}C\ [1]^{-8}\,[-b]^2\,[2]^2 x^4 y^{-6} z^6 = 7560\, x^4 y^{-6} z^6 \quad [58]$$

From the Kifilideen coefficient table, the coefficient of the 13th term on the n = − 4 column is + 210

$$+210\ [1]^{-8}\,[-b]^2\,[2]^2 x^4 y^{-6} z^6 = 7560\, x^4 y^{-6} z^6 \quad [59]$$

$$840\, b^2 = 7560 \quad [60]$$

$$b^2 = \frac{7560}{840} \quad [61]$$

$$b^2 = 9 \quad [62]$$

$$b = 3 \quad [63]$$

[g] If a term in the Kifilideen expansion of of $\left[\frac{cx^2}{z^5} + 5x^3y - 3yz^2\right]^{-5}$ is $\frac{850500}{1048576} x^{-14} y^5 z^{56}$. Determine the value of

[i] the power combination

[ii] show that the negative power of – n of the trinomial expression is – 5

[iii] the tth term

[iv] the value of c.

Solution

[i]

$$ {}_{kif}^{-5}C\ \left[\frac{cx^2}{z^5}\right]^k\,[5x^3y]^i\,[-3yz^2]^f \quad [64]$$

$$ {}_{kif}^{-4}C\ [c]^k\,[x^2]^k\,[z^{-5}]^k\,[5]^i\,[x^3]^i\,[y]^i\,[-3]^f[y]^f\,[z^2]^f \quad [65]$$

$${}_{kif}^{-5}C\ [c]^{k}\ [5]^{i}\ [-3]^{f}[x]^{2k+3i}\ [y]^{i+f}\ [z]^{-5k+2f} \quad [66]$$

Comparing [67] with $\frac{850500}{1048576}x^{-4}y^{5}z^{56}$, so

$$2k+3i=-14 \quad [67]$$

$$i+f=5 \quad [68]$$

$$-5k+2f=56 \quad [69]$$

Also,

$$k+i+f=n \quad [70]$$

$$k+i+f=-5 \quad [71]$$

Put [68] in [67]

$$2k+3[5-f]=-14 \quad [72]$$

From [72] and [69], then

$$2k-3f=-29\times 5 \quad [73]$$

$$-5k+2f=56\ \ \times 2 \quad [74]$$

$$10k-15f=-145$$

$$-10k+4f=\ 112$$

$$-11f=\ \ -33 \quad [77]$$

$$f=3 \quad [78]$$

From [68] put [78] in [79], $i+f=5$ [79]

$$i + 3 = 5 \quad [80]$$

$$i = 2 \quad [81]$$

From [67] put [81] in [82] $2k + 3i = -14$ [82]

$$2k + 3[2] = -14 \quad [83]$$

$$k = -10 \quad [84]$$

Power combination $= kif = -10, 2, 3$ [85]

[ii] From [70], $n =$ negative power of – n of the trinomial expression $= k + i + f$ [86]

Put [78], [81] and [84] in [86], so $n = n = -10 + 2 + 3$ [87]

$$n = -5 \quad [88]$$

Proved

[iii] $t = \frac{[n-k]^2 + [n-k] + 2f + 2}{2}$ [89]

$k = -10, i = 2, f = 3$ and $n = -5$ [90]

$$t = \frac{[-5--10]^2 + [-5--10] + 2\times3 + 2}{2} \quad [91]$$

$t = 19^{th}$ term [92]

[iv] Comparing [67] with $\frac{850500}{1048576} x^{-4} y^5 z^{56}$, so

$$ {}_{kif}^{-5}C\ [c]^{k}\ [5]^{i}\ [-3]^{f}[x]^{2k+3i}\ [y]^{i+f}\ [z]^{-5k+2f}\ = \frac{850500}{1048576}x^{-4}y^{5}z^{56} \quad [93] $$

$$ {}_{-10,2,3}^{-5}C\ [c]^{-10}\ [5]^{2}\ [-3]^{3}[x]^{-4}\ [y]^{5}\ [z]^{56}\ = \frac{850500}{1048576}x^{-4}y^{5}z^{56} \quad [94] $$

From the Kifilideen coefficient table, the coefficient of the 19th term on the n = -5 column is -1260

$$ -1260\ [c]^{-10}\ [5]^{2}\ [-3]^{3} = \frac{850500}{1048576} \quad [95] $$

$$ c^{-10} = \frac{850500}{1048576\times27\times25\times1260} \quad [96] $$

$$ c^{-10} = [1048576]^{-1} \quad [97] $$

$$ c^{-10} = [4]^{-10} \quad [98] $$

$$ c = 4 \quad [99] $$

2.4 Theorem of matrix transformation of negative power of – n of trinomial expression

If three variables $x,\ y$ and z are found in each part of trinomial expression of negative power of – n such as

$$ \left[sx^{d}y^{e}z^{g} + ux^{h}y^{j}z^{l} + vx^{m}y^{r}z^{p}\right]^{-n} \quad [100] $$

and the power combination of any term in the Kifilideen expansion of that kind of negative power of – n of the trinomial expression is set as kif while the value of this term is designated as $wx^a y^b z^c$.

Then, the Kifilideen matrix transformation of such negative power of – n of trinomial expression is of the form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}: \qquad \begin{bmatrix} d & h & m \\ e & j & r \\ g & l & p \end{bmatrix} \begin{bmatrix} k \\ i \\ f \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad [101]$$

Thus $k + i + f = n$ and where s, u, v and w are constants

More so,

$$^{-n}_{kif}C s^k u^i v^f = w \qquad [102]$$

Illustration

Given that a term in the Kifilideen expansion of $\left[\frac{dx^5}{z^3} + \frac{z^7}{x^2 y} + \frac{8y}{z}\right]^{-n}$ is $\frac{-z^{16}}{32x^7 y}$ where d is a constant value. Using Kifilideen matrix transformation method of negative power of – n of trinomial expression. Find

[i] the power combination

[ii] the power of the trinomial expression

[iii] the tth term

[iv] the value of d.

Solution

[i] Trinomial expression: $[dx^5y^0z^{-3}+x^{-2}y^{-1}z^7+8x^0y^1z^{-1}]^{-n}$ [103]

Power combination to be obtained: $k \quad i \quad f$ [104]

t[th] term of the power combination: $\frac{-1}{32}x^{-17}y^{-1}z^{16}$ [105]

Using the Kifilideen matrix transformation method, so

$$\begin{bmatrix}x\\y\\z\end{bmatrix}: \qquad \begin{bmatrix}5 & -2 & 0\\0 & -1 & 1\\-3 & 7 & -1\end{bmatrix}\begin{bmatrix}k\\i\\f\end{bmatrix}=\begin{bmatrix}-17\\-1\\16\end{bmatrix}$$

[106]

Also, $k+i+f=n$ [107]

Using Crammer's rule, so

$$k=\frac{\Delta k}{\Delta}=\frac{\begin{vmatrix}-17 & -2 & 0\\-1 & -1 & 1\\16 & 7 & -1\end{vmatrix}}{\begin{vmatrix}5 & -2 & 0\\0 & -1 & 1\\-3 & 7 & -1\end{vmatrix}}$$ [108]

$$k=-3$$ [109]

$$i=\frac{\Delta i}{\Delta}=\frac{\begin{vmatrix}5 & -17 & 0\\0 & -1 & 1\\-3 & 16 & -1\end{vmatrix}}{\begin{vmatrix}5 & -2 & 0\\0 & -1 & 1\\-3 & 7 & -1\end{vmatrix}}$$

[110]

$$i=1$$ [111]

$$f = \frac{\Delta f}{\Delta} = \frac{\begin{vmatrix} 5 & -2 & -17 \\ 0 & -1 & -1 \\ -3 & 7 & 16 \end{vmatrix}}{\begin{vmatrix} 5 & -2 & 0 \\ 0 & -1 & 1 \\ -3 & 7 & -1 \end{vmatrix}} \quad [112]$$

$$f = 0 \quad [113]$$

So, the power combination $= kif = -310$ [114]

[ii] the negative power of – n of the trinomial expression $= n = k + i + f = -3 + 1 + 0$ [115]

$$n = -2 \quad [116]$$

[iii] $$t = \frac{[n-k]^2 + [n-k] + 2f + 2}{2} \quad [117]$$

$$t = \frac{[-2--3]^2 + [-2--3] + 2 \times 0 + 2}{2} \quad [118]$$

$t = 2^{nd}$ term [119]

[iv] from the question, $s = d, u = 1, v = 8$ and $w = \frac{-1}{32}$ [120]

Using Kifilideen matrix transformation method,

$$_{kif}^{-n}C s^k u^i v^f = w \quad [121]$$

$$_{-310}^{-2}C[d]^{-3}[1]^1[8]^0 = \frac{-1}{32} \quad [122]$$

From the Kifilideen coefficient table, the coefficient of the 2^{nd} term on the n = -2 column is -2

$$-2 \times [d]^{-3} = \frac{-1}{32} \quad [123]$$

$$[d]^{-3} = [64]^{-1} \quad [124]$$

$$[d]^{-3} = [4]^{-3} \quad [125]$$

$$d = 4 \quad [126]$$

3 Conclusion

This study applied Kifilideen expansion of negative power of – n of Kifilideen trinomial theorem for the transformation of compound fraction into series of partial fraction with other development. A theorem of matrix transformation of negative power of – n of trinomial expression in which three variables x, y and z are found in part of the trinomial expression was developed. The development would ease the process of evaluating such trinomial expression of negative power of – n. The standardized and matrix method used in arranging the terms of the Kifilideen expansion of negative power of $-$n of trinomial expression yield an interesting results in which it is utilized in transforming compound fraction into series of partial fractions in a unique way.

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