Article

A new chaotic system's qualitative analysis, with a focus on Hopf bifurcation analysis

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Abstract
In this study, a brand-new chaotic system with a particular set of parameters is presented. In addition, this study offers instructions about how to resolve the new system. Also demonstrated and stated are some of the system's dynamical characteristics. In essence, this work demonstrates the identification of the fixed points for the system, dynamical analysis utilizing the complementary-cluster energy-barrier criteria (CCEBC), eigenvalue discovery for stability, and Lyapunov exponent discovery to examine some of the dynamical behaviors of the system. Additionally, this research uses the first Lyapunov coefficient to determine the Hopf bifurcation of the new three-dimensional system. With the help of MATLAB, many parameters are changed to display images and diagrams for chaotic systems.

Keywords Hopf bifurcation; Lyapunov exponent; first Lyapunov coefficient; Routh-Hurwitz Criterion.

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1 Introduction

The dynamical system is well-known in the real world for its many applications, including in a population growth model. We may examine how populations have changed through time and anticipate how many people there will be in the future.When Lorenz was making a weather forecast in the early 1960s, he introduced the chaotic dynamical system. He understood that even slight variations in the fundamental elements of the weather system could have unexpected effects on the outcome.Since the system is reliant on its initial conditions, he later gave it the moniker (Chandler and Lorenz, 2008) "butterfly effect". Mathematical study has been highly interested in chaos because it may provide fresh insights into all facets of life. Over the past 20 years, chaos in engineering systems, such as nonlinear circuits, has gradually gone from being only a fascinating phenomenon to one with practical significance and uses. Numerous disciplines have found use for or great potential in chaos, including applications of biomedical engineering to the human heart and brain, efficient liquid mixing, high-performance telecommunication circuit design, and power system collapse prevention, to name a few (Chen, 1999; Chen and Dong, 1998). Therefore, in such applications where chaos is significant and valuable, creating chaos becomes a critical issue.

Some researchers who have modified the Lorenz system have accidentally discovered or tried to learn about the practical uses. The modified Lorenz system has been given in some earlier work by Zhou et al. (2008), Qi et al. (2005), and Yan (2007), who have also examined the system's stability and dynamical behavior.Other chaotic models exist as well, including the Rösler system, the four-wing hyper chaotic attractor, and transient chaos produced by a brand-new 4-D quadratic autonomous system by (Cang et al., 2010). One of the modified chaotic dynamical systems has been discovered by (Lü and Chen, 2002; Lü et al., 2002) from the Lorenz system itself. Zhou et al (2008) has recently conducted some study on a novel chaotic system that has since been given the term Zhou system. The article displays some fundamental dynamical features, including the continuous spectrum, Lyapunov exponents, Poincaré mapping, fractal dimensions, bifurcation diagrams, and chaotic Zhou system dynamical behaviors. Roslan (Tee et al., 2013) examined the issue more recently in relation to solving the Zhou chaotic system using Euler's approach. One of the simplest methods for finding a differential equation's numerical solution is said to exist.

Synchronization was thoroughly researched in the early stages of chaos study. The foundations of the Fitz-Hugh Nagumo neural system and the Hindermarsh-Rose neuron model have been successfully established by Mamat et al. (2011, 2012). Additionally, the synchronizations of periodic and chaotic bursting neurons are investigated.Some new 3D chaotic system was extensively investigated by (Benkouider et al., 2021; Vaidyanathan et al., 2019).

Moreover, Hopf bifurcation is extensively researched in order to better understand how dynamical systems behave. According to Salleh et al. (2011) the dynamical model of a three-species food chain with Lotka-Volterra linear functional response has been investigated. They also investigated the interplay of a three-species feeding chain with a Michaelis-Menten type functional response. In these works, analytical and numerical studies of Hopf bifurcation points have been conducted. MataSanjaya et al. (2012) are more articles that focus on dynamical models and Hopf bifurcation. Yan (2007) has conducted additional studies on Hopf bifurcation in his investigations of the dynamical behavior of Lorenz-type systems using the complementary-cluster energy-barrier criterion. The first Lyapunov coefficient is also used to study the system's Hopf bifurcation. In a research by Li et al. (2007), his group looked at the generalized Lorenz canonical form (GLCF), which is the Hopf bifurcation of a unified chaotic system.

Chaos is applicable to a wide range of situations. A chaotic dynamical system may also bring a new technique or system that will aid in the understanding of mathematics, making it not only significant and useful but also crucial. The Lorenz system, which has been modified from the Lorenz system, will serve as the main foundation for this research (Lorenz, 1963; Sparrow, 1982; Robinson, 2004; Curry, 1978). The following is a description of the Lorenz system equations:

$$\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - y - xz \\ \dot{z} &= x^2 - bz \end{aligned} \qquad (1)$$

where $a = 10, b = 8/3$ and $c = 28$ causes chaos in the system, and a, b and c, respectively, are real constant parameters.

2 A New Chaotic System

A brand-new three-dimensional continuous system was put forth as follows by Lei and Wang (2014):

$$\dot{x} = a(y - x)$$
$$\dot{y} = cx - dy - xz \qquad (2)$$
$$\dot{z} = x^2 - bz$$

when variables x, y, and z are present, and real parameters a, b, c, and d.

2.1 Mathematical properties

Some fundamental dynamical characteristics of equation (2) are covered in this subsection.

- **Symmetry**

  When using the coordinate transform $(x, y, z) \to (-x, -y, z)$, the reflection about the z axis, system (2) exhibits a natural symmetry.

- **The Z axis**

- The z axis,$x = y = 0$ is invariant. All trajectories that begin on the z axis will stay on it and move in the direction of the origin (0,0,0). In addition, when viewed from above the plane $z = 0$, the trajectories that rotate around the z axis do so in a clockwise direction.

- **Dissipativity**

  The flow's divergence $\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = a - b - d$. As a result, the flow contracts the volume element V into a volume element $Ve^{(a-b-d)t}$in time t.Since the system is dissipative, neither unstable periodic orbits nor unstable stationary points may exist. Any stationary points and orbits that are unstable must also be unstable. The presence of unstable objects would suggest the presence of a flow-expanded volume close to the object. The following inequality, $a < b + d$, must be satisfied in order to make the divergence negative. In that sense, the system (2)'s dissipativeness is conditional.

2.2 Dynamical analysis by CCEBC method

Now, using the complementary-cluster energy-barrier criteria (CCEBC) (Xue, 1999), we will ascertain the dynamical behaviors of the dynamical system (2). This approach works well for telling chaotic orbits apart from periodic and quasi-periodic ones. The three dimensional system is

$$\dot{x} = a(y - x)$$
$$\dot{y} = cx - dy - xz$$
$$\dot{z} = x^2 - bz$$

which has a chaotic attractor like Chen system shown in Fig. 1 when $a = 35$, $b = 3, c = -20, d = -28.3$.

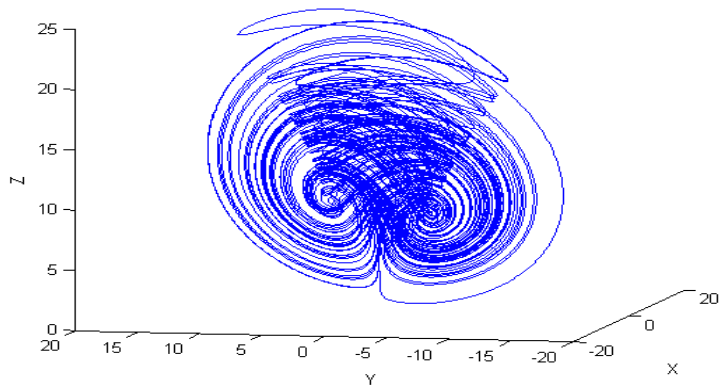


Fig. 1 System(2)'s chaotic attractor.

Let system (2)'s first two differential equations be

$$\dot{x} = a(y - x)$$
$$\dot{y} = cx - dy - xz \qquad (3)$$

where z is a recognized function of the variable t, the time. A linear, two-dimensional system with constant coefficients is system (3) when $t = t_0$. Therefore, its dynamical behavior is quite basic and universal.

We obtain the following characteristic equation by linearizing system (3) about its (0,0) equilibrium point.

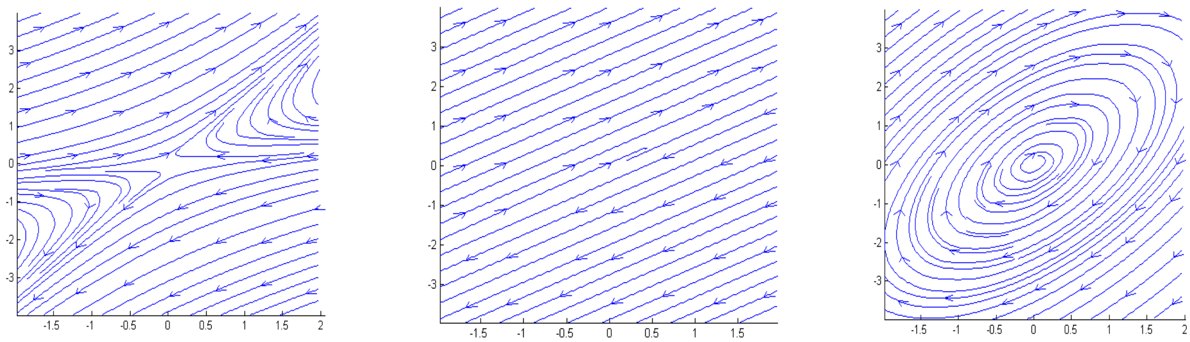
$$f(\lambda) = \lambda^2 + \lambda(a + d) + a(z - c + d) \qquad (4)$$

The eigenvalues λ_1, λ_2 of (3) are used to base the next observations.

1. If $z < c - d$ and $a > 0$ then the eigenvalues satisfies $\lambda_1 > 0 > \lambda_2$. This demonstrates that the equilibrium point $(0, 0)$ in the two-dimensional plane is a saddle point. Fig. 2(a) depicts the solution curve in the $x - y$ plane, with the direction arrow pointing in the direction of the orbit as t grows.

2. If $c - d < z < c - d + \frac{(a+d)^2}{4a}$ then equation (3) has two different negative real roots. In this instance, the (0,0) equilibrium point is a node. Fig. 2(b) displays the $x - y$ plane solution curve.

3. If $z > c - d + \frac{(a+d)^2}{4a}$ then the complex conjugate eigenvalues of equation (3) have a negative real portion. Here, the single equilibrium point (0,0) is the focus. Fig. 2(c), where the direction arrow points in the direction of the orbit as t increases, displays the solution curve in the $x - y$ plane. All orbits spiral back towards their origins when t reaches infinity.



(a) (b) (c)

Fig. 2 The phase portrait of system (3) (a) $z < c - d$, (b) $c - d < z < c - d + \frac{(a+d)^2}{4a}$, (c) $z > c - d + \frac{(a+d)^2}{4a}$.

The time series $x(t), y(t), z(t)$ of system (2) is generated for any initial condition $x(0) = x_0, y(0) = y_0, z(0) = z_0$ and time step 0.5.

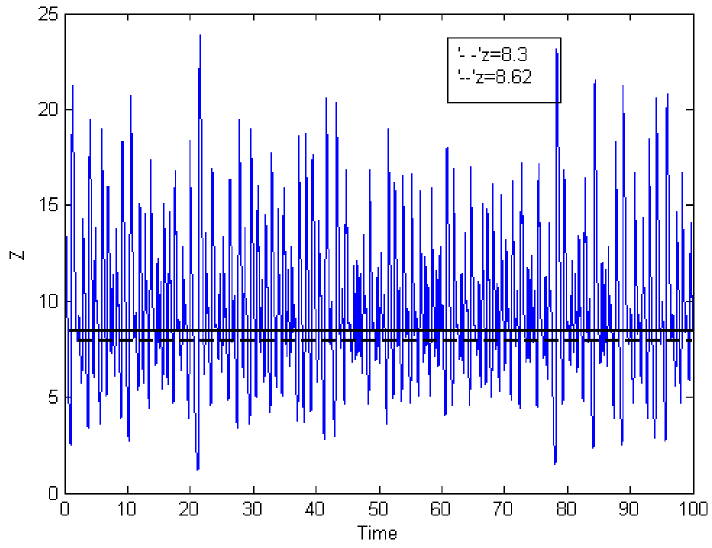


Fig. 3 The chaotic time series of $z(t)$ with $a = 35, b = 3, c = -20$ and $d = -28.3$.

According to Fig. 3, when the $z(t)$ orbit crosses the straight lines, $z = (c - d)$ and $z = c - d + \frac{(a+d)^2}{4a}$ alternately and repeatedly for a number of times $t \to \infty$. These two lines divide the z −axis into three distinct domains: $(-\infty, c - d)$, $\left(c - d, c - d + \frac{(a+d)^2}{4a}\right)$, and $(c - d + \frac{(a+d)^2}{4a}, \infty)$.In these three domains, system (3) exhibit various dynamical behaviors. $Z(t)$orbit's frequent passage through these two points causes complex dynamical behavior, bifurcations, and chaos.

2.3 Bifurcation of equilibrium

Three equilibrium points exist in system (2). They are

- $O = (0,0,0)$
- $S^+ = (\sqrt{b(c-d)}, \sqrt{b(c-d)},\ c - d)$
- $S^- = (-\sqrt{b(c-d)}, -\sqrt{b(c-d)},\ c - d)$

Now we will define some bifurcations depending on the equilibrium points. The jacobian matrix at O of system (2) is

$$J = \begin{pmatrix} -a & a & 0 \\ c & -d & 0 \\ 0 & 0 & -b \end{pmatrix}$$

and this jacobian matrix's eigenvalues

$$-b, -\frac{a}{2} - \frac{d}{2} - \frac{\sqrt{a^2 - 2ad + 4ca + d^2}}{2}, -\frac{a}{2} - \frac{d}{2} + \frac{\sqrt{a^2 - 2ad + 4ca + d^2}}{2}$$

For $a > a_n = d - 2c + \sqrt{2}c(c - d)$ equilibrium point O is a sink and if $a = a_n$ then equilibrium point O is a node.At $d = c$, a pitchfork bifurcation for O appears. We now have three equilibrium points for $d < c$ as a result of O changing into an unstable saddle and two symmetric sinks being formed simultaneously. These bifurcation does not depend on the parameter b.

Hopf bifurcation emerges from the value of $a_h = \frac{-b+2c-4d+\sqrt{(b-2c+4d)^2-4(bd+d^2)}}{2}$ where the complex cojugate eigenvalues are $\mu = \pm i\omega$ and ω is a real number.

2.4 Linear stability analysis

From the basic dynamical behavior of the system (2) it can be concluded that

1. If $d > c$ then system (2) has one real equilibrium point $O(0,0,0)$ and it is asymptotic stable.
2. If $d < c$ then system (2) has three real equilibrium points $O = (0,0,0), S^+ = (\sqrt{b(c-d)}, \sqrt{b(c-d)},\ c-d), S^- = (-\sqrt{b(c-d)}, -\sqrt{b(c-d)},\ c-d)$ and $O(0,00)$ is a saddle point.

Now an interesting thing is for S^+, S^- we get the same characterstic polynomial of system (2).

$$f(\lambda) = \lambda^3 + (a+b+d)\lambda^2 + (ab+bd)\lambda + 2abc - 2abd \qquad (5)$$

So both S^+, S^- have same stability.

Let

$$A = (a+b+d)$$
$$B = ab + bd$$
$$C = 2abc - 2abd$$

The real parts of the roots λ are therefore determined to be negative under Routh-Hurwitz criteria if and only if

$$(a+b+d) > 0,$$
$$2abc - 2abd > 0$$
$$(a+b+d) - 2abc + 2abd > 0$$

The coefficients of equation (5) are all positive. So, $f(\lambda) > 0$ for all $\lambda > 0$. If equation (5) has two complex conjugate roots and real part of the roots are positive then there is instability. Let $\lambda_1, \lambda_2, \lambda_3$ are three roots of equation (5) where λ_1 and λ_2 are complex conjugate such that $\lambda_1 = i\omega$ and $\lambda_2 = -i\omega$ for some real number ω. Since the sum of three roots is

$$\lambda_1 + \lambda_2 + \lambda_3 = -(a+b+d)$$

So we have $\lambda_3 = -(a+b+d)$,which is the stability margin. On the margin

$$0 = f(-(a+b+d)) = 2abc - (ab + bd)(a + b + d) - 2abd$$

That is,

$$a_{1,2} = \frac{-b+2c-4d \pm \sqrt{(b-2c+4d)^2 - 4(bd+d^2)}}{2}$$

3 Hopf Bifurcation Analysis

At the location (x, y, z), the Jacobian of system (2) is given

$$J = \begin{pmatrix} -a & a & 0 \\ c-z & -d & -x \\ 2x & 0 & -b \end{pmatrix}$$

This Jacobian, at $(0,0,0)$ has no imaginary characteristic eigenvalues. Therefore $(0,0,0)$ are not Hopf points of the system.

Notice that the Jacobians of system (2) at S^+, S^-

$$J = \begin{pmatrix} -a & a & 0 \\ d & -d & -\sqrt{b}(c-d) \\ 2\sqrt{b}(c-d) & 0 & -b \end{pmatrix}$$

And

$$J = \begin{pmatrix} -a & a & 0 \\ d & -d & \sqrt{b}(c-d) \\ -2\sqrt{b}(c-d) & 0 & -b \end{pmatrix}$$

respectively. They have the same characteristic polynomial:

$$|\lambda I - J| = \lambda^3 + (a+b+d)\lambda^2 + (ab+bd)\lambda + 2abc - 2abd \qquad (6)$$

Suppose that $|\lambda I - J|$ has a pure imaginary root$\lambda = \pm i\omega$, $\omega \epsilon R^+$. Substituting into (6) yields

$$-\omega^2(a+b+d) + 2ab(c-d) = 0$$
$$-\omega^3 + \omega(ab+bd) = 0$$

Putting the second equation into the first one gives

$$(a+d)(a+b+d) + 2a(c-d) = 0 \qquad (7)$$

This is the bifurcation surface and $\omega^2 = b(a+d)$

Since the equilibrium points$S^+$and $S^-$are not the origin (0,0,0). Therefore, firstly, utilizing the change of variables, we must translate the origin of the coordinates to S^+.

$$x(t) = X(t) + \sqrt{b}(c-d)$$
$$y(t) = Y(t) + \sqrt{b}(c-d)$$
$$z(t) = Z(t) + (c-d)$$

This transforms system (2) into equivalent system.

$$\dot{X} = a(Y - X)$$
$$\dot{Y} = c\left(X + \sqrt{b(c-d)}\right) - d(Y + \sqrt{b(c-d)} - \left(X + \sqrt{b(c-d)}\right)(Z + c - d) \qquad (8)$$
$$\dot{Z} = (X + \sqrt{b(c-d)})^2 - b(Z - c - d)$$

The characteristic equation of the above system is same of (8). Consequently, Hopf bifurcation occurs at the origin (0,0,0) of (8). Hopf bifurcation analysis is particularly challenging of system (8), on the bifurcation surface (7).We just consider the case out of convenience$a = -4c, b = -2c$and $d = -4c$ which leads to $\omega = \sqrt{b(a+d)} = 4c$.

Thus the Jacobian matrix at origin is,

$$A = \begin{pmatrix} 4c & -4c & 0 \\ -4c & 4c & -i\sqrt{10}c \\ 2i\sqrt{10}c & 0 & 2c \end{pmatrix}$$

The eigenvalues of this matrix are $-2c, 6c-2ic, 6c+2ic$.
By rigorous calculation we get these four vectors.

$$q=\begin{pmatrix}1\\ \frac{1}{2}(i-1)\\ \frac{\sqrt{10}}{5}(2i-1)\end{pmatrix}$$

$$\bar{q}=\begin{pmatrix}1\\ \frac{1}{2}(-i-1)\\ \frac{\sqrt{10}}{5}(-2i-1)\end{pmatrix}$$

$$p=\begin{pmatrix}1-i\\ 1\\ \frac{\sqrt{10}}{5}(i+1)\end{pmatrix}$$

$$\bar{p}=\begin{pmatrix}1+i\\ 1\\ \frac{\sqrt{10}}{5}(1-i)\end{pmatrix}$$

The system (8) solely contains bilinear terms.Therefore the bilinear matrix $B(\epsilon,\eta)$ defined for two vectors $\epsilon=(\epsilon_1,\epsilon_{2,}\epsilon_3)^T\epsilon R^3$ and $\eta=(\eta_1,\eta_{2,}\eta_3)^T\epsilon R^3$ can be expressed as

$$B(\epsilon,\eta)=(0,2\epsilon_1\eta_3,2\epsilon_1^2)^T$$
$$C(\epsilon,\eta,\tau)=(0,0,0)^T$$

$$A^{-1}=\begin{pmatrix}-1/(10c) & -1/(10c) & -(\sqrt{10}i)/(20c)\\ -7/(20c) & -1/(10c) & -(\sqrt{10}i)/(20c)\\ (\sqrt{10}i)/(10c) & (\sqrt{10}i)/(10c) & 0\end{pmatrix}$$

$$B(q,q)=\begin{pmatrix}0\\ -1.2649+2.5298i\\ 2\end{pmatrix}$$

$$B(q,\bar{q})=\begin{pmatrix}0\\ -1.2649-2.5298i\\ 2\end{pmatrix}$$

$$s=A^{-1}B(q,\bar{q})=\frac{1}{10c}\begin{pmatrix}0.1265-0.0632i\\ 0.1265-0.0632i\\ 0.8-0.4i\end{pmatrix}$$

$$B(q,s)=\frac{1}{10c}\begin{pmatrix}0\\ 1.6-0.8i\\ 2\end{pmatrix}$$

$$<p,B(q,s)>\frac{2.8649+2.0649i}{10c}$$

$$(2i\omega E - A)^{-1} = \frac{1}{c}\begin{pmatrix} -0.0329 - 0.0842i & 0.0271 - 0.0300i & -0.0073 + 0.0137i \\ 0.0487 - 0.0185i & -0.0329 - 0.0842i & 0.0201 + 0.0283i \\ -0.0401 - 0.0565i & 0.0146 - 0.0274i & -0.0323 - 0.1061i \end{pmatrix}$$

$$r = (2i\omega E - A)^{-1}B(q,q) = \frac{1}{c}\begin{pmatrix} 0.0270 + 0.1339i \\ 0.2947 + 0.0799i \\ -0.0138 - 0.1407i \end{pmatrix}$$

$$B(q,\bar{r}) = \frac{1}{c}\begin{pmatrix} 0 \\ -0.0277 - 0.2814i \\ 2 \end{pmatrix}$$

$$< p, B(q,r) \geq \frac{1.2372 + 1.5463i}{c}$$

Now, the first Lyapunov coefficient is given by

$$\ell_1(0) = \frac{1}{2\omega}Re[< p, C(q,q,\bar{q}) > -2 < p, B(q, A^{-1}B(q,\bar{q})) > + < p, B(\bar{q}(2i\omega E - A)^{-1}B(q,q)) >]$$

$$= \frac{1}{8c}[-2 < p, B(q,s) > + < p, B(\bar{q},r) >]$$

$$= \frac{0.138}{c^2} > 0$$

So, the Hopf bifurcation of system (2) for the equilibrium points $S^+ \, and \, S^-$ are of subcritical type. From the following graph it is verified that system (2) has subcritical type Hopf bifurcation.

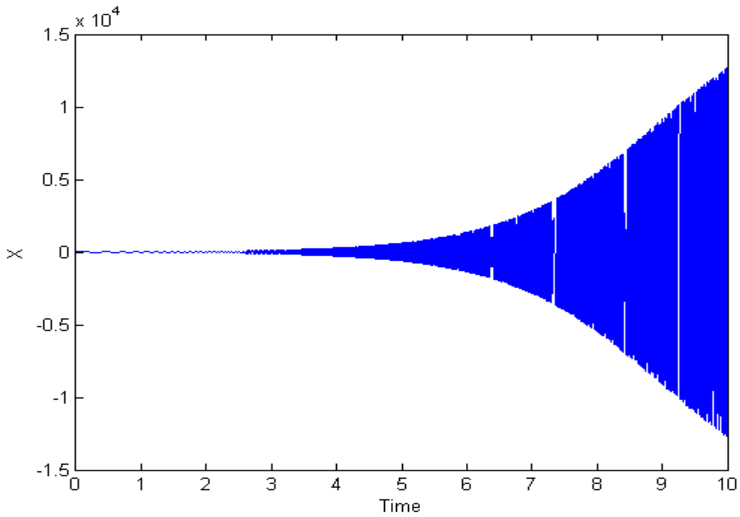


Fig. 4 Subcritical Hopf bifurcation of system (1).

5 Bifurcation By Varying Parameters

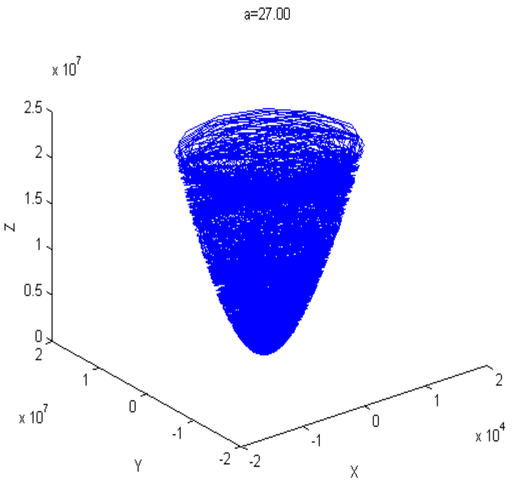
Now we will show some bifurcation by varying parameters of system (2). Here we have showed bifurcations by keeping fixed three parameters and varying one parameter. In next two subsections bifurcation by varying parameter a and d are shown.

5.1 Bifurcation by varying parameter *a*

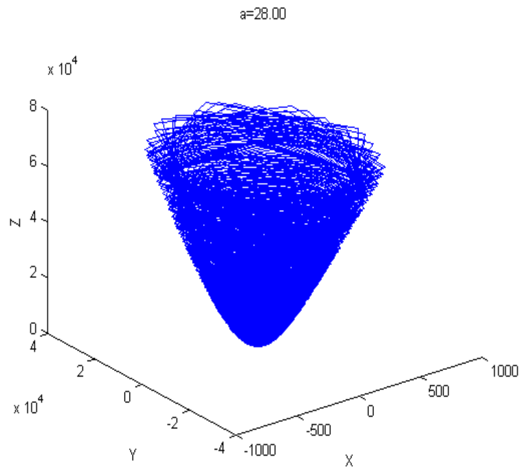
Here we have fixed the value of parameters $b = 3, c = -20, d = -28.3$ and changed the value of a in the range $[27,35]$. We conclude the following things

- When $a\epsilon[27,28]$ the system has hopf bifurcation
- For $a\epsilon[29,34]$ the system (2) changed it's behaviour dynamically.
- When $a = 35$ we get a chaotic Chen type attractor

The maximum lyapunov exponent shows the change of the system (2)'s behaviour.



(a)



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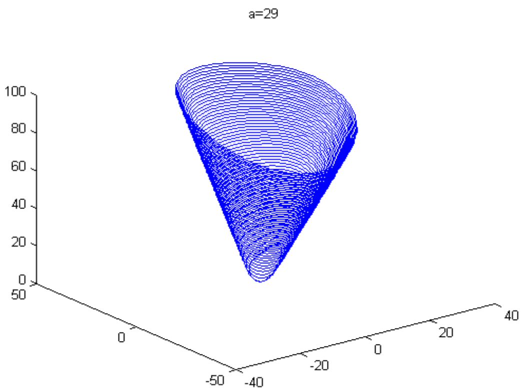
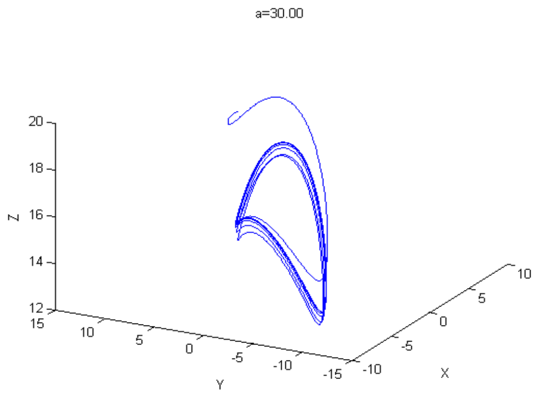


(c)



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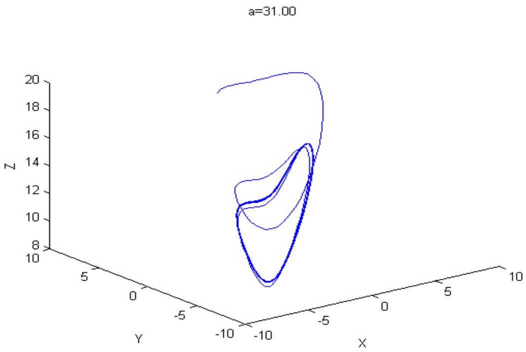

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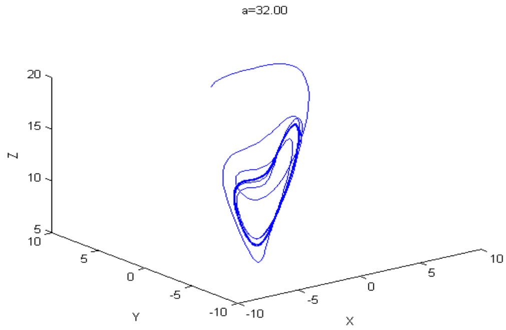
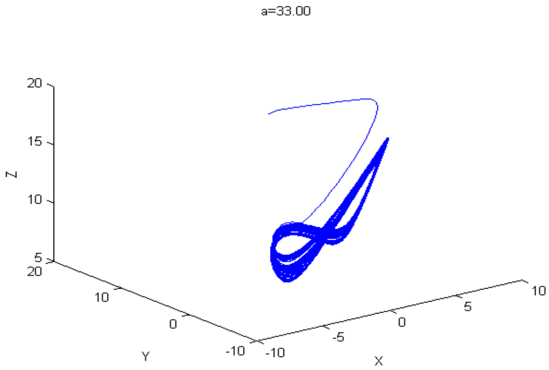
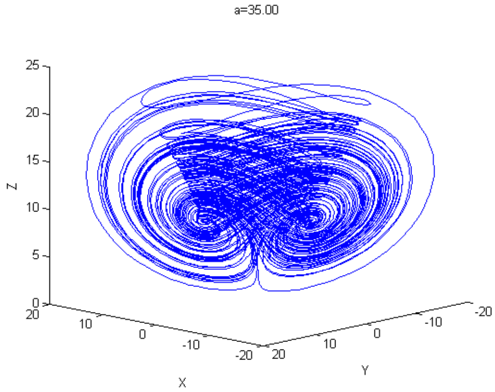


(f)



(g)



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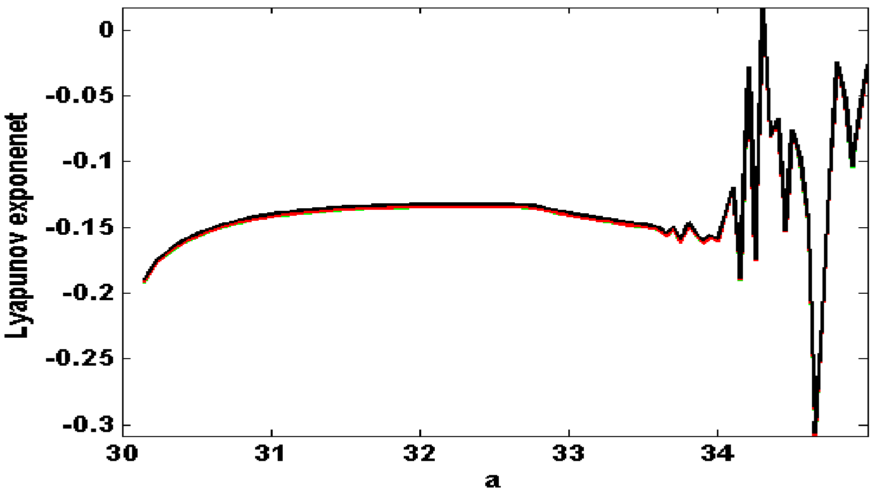


(i)

Fig. 5 (a)-(h) Dynamical change of system (1) with change of parameter a. (i) Maximum lyapunov exponent of system (1) with respect to parameter a.

5.2 Bifurcation by varying parameter d

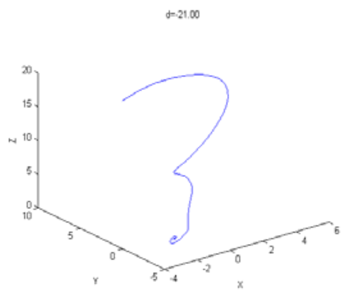
Here we have fixed the value of parameters $a = 35, c = -20, b = 3$ and changed the value of d in the range $[-21, -35]$. We conclude the following things –

- When $d\epsilon[21,22]$ the system is not chaotic.
- For $d\epsilon[-23, -30]$ the system (2) changed it's behaviour dynamically and have some chaotic

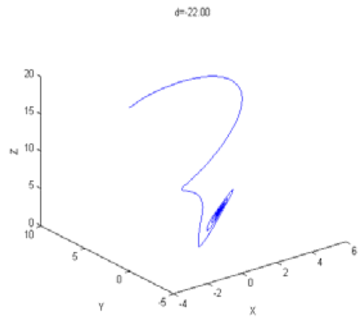
attractors.

- When $d = -35$ the system meet it's hopf bifurcation.

The maximum lyapunov exponent shows the change of the system (2)'s behaviour with the change of parameter d.



(a)



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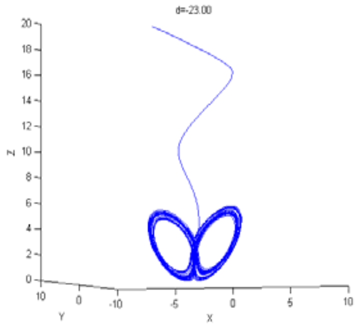
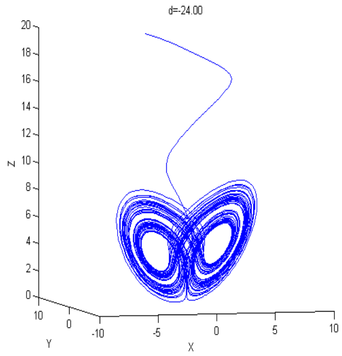


(c)



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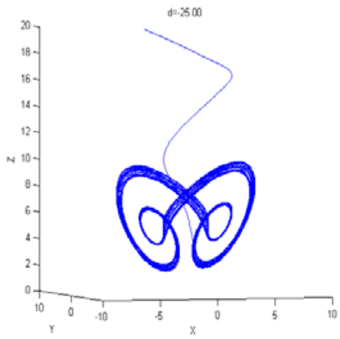
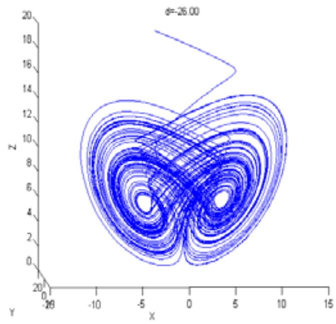
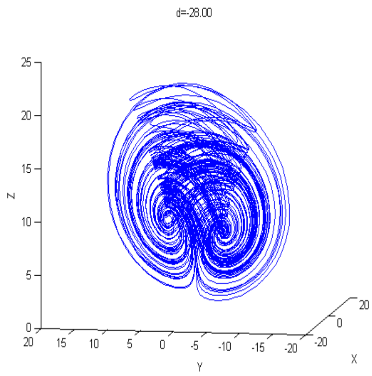


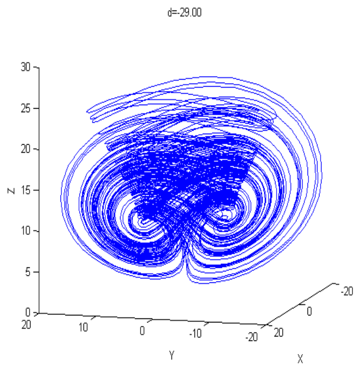
(e)



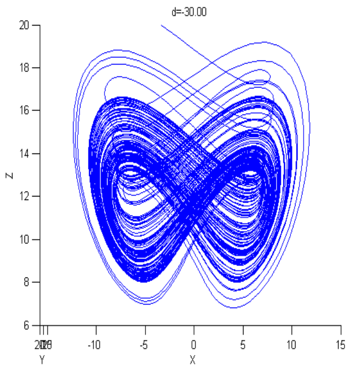
(f)



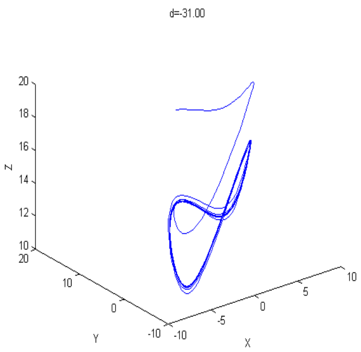
(g)



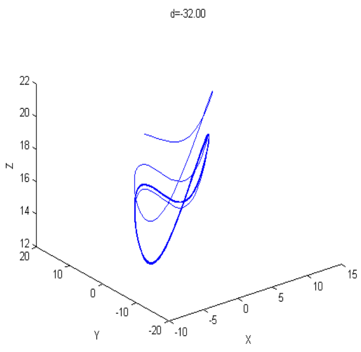
(h)



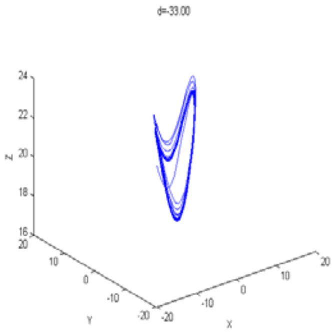
(i)



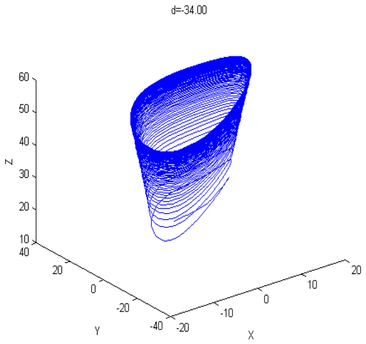
(j)



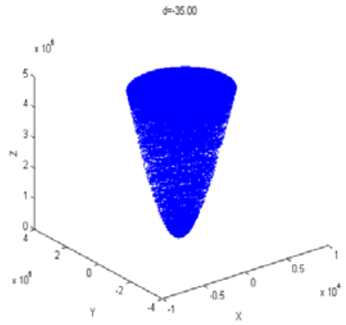
(k)



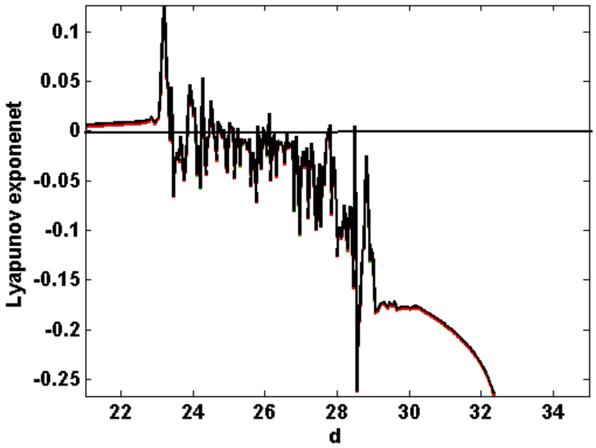
(l)



(m)



(n)



(o)

Fig. 6 (a)-(n)represents dynamical change of system (1) with change of parameter d. (o) Maximum lyapunov exponent of system (1) with respect to parameter d.

6 Conclusions

In this paper, we looked at the system's dynamic behavior and fundamental dynamic analysis. Additionally, we conducted a complementary-cluster energy-barrier criterion (CCEBC) analysis and used the Lyapunov exponent to examine the chaotic behavior.The system exhibited chaotic behavior under a specific set of conditions.To ascertain the system's behavior, we looked at the system (2) using the Routh-Hurwitz criterion. Furthermore, we have demonstrated that the system does experience Hopf bifurcation using the first Lyapunov coefficient. We have demonstrated that the system contains two non-zero fixed points, of which the Hopf bifurcations are subcritical, for the observed results.

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